Trumpet Reincarnations: Proofs from the Book

Intertranslatability Results for Abstract Argumentation Semantics

Christof Spanring

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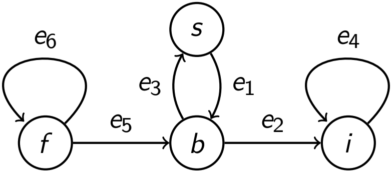
Building a Zoo

Which animals can share a pen?

e_1: $s \rightarrowtail b$, snakes eat birds, e_2: $b \rightarrowtail i$, birds eat insects,
e_3: $b \rightarrowtail s$, birds eat snakes, e_4: $i \rightarrowtail i$, insects eat insects,
e_5: $f \rightarrowtail b$, felines eat birds, e_6: $f \rightarrowtail f$, felines eat felines.



Groups with biggest impact

$sem(F) = \{\{s\}\}$
snakes defend themselves

$stg(F) = \{\{b\}\}$
birds have biggest impact

Abstract Argumentation

Argumentation Frameworks (AF) [Dung, 1995]

- An AF $F = (A, R)$ is composed of a set A of arguments and a set $R \subseteq A \times A$ of directed conflicts.
- Extensions $E \subseteq A$ are specified by conditions such as conflict-freeness and maximality, a semantics $\sigma(F)$ is a specific collection of extensions.

Language in Use

- For $a, b \in A$, $(a, b) \in R$ we say that *a attacks b* and write $a \rightarrowtail b$.
- For $E \subseteq A, a \in A$ we have $E \rightarrowtail a$ (resp. $a \rightarrowtail E$) if there is some $e \in E$ such that $e \rightarrowtail a$ (resp. $a \rightarrowtail e$).
- For any set $E \subseteq A$ we call $E^+ = E \cup \{a \in A \mid E \rightarrowtail a\}$ the *range* of E.

Argumentation Semantics

Extension-Based Semantics

For any given AF $F = (A, R)$ and some set $E \subseteq A$ we call E

- conflict-free, iff there is no conflict between the arguments in E;
- admissible, iff E is conflict-free and defends itself against all attacks from the outside;
- a stage extension, iff E is conflict-free and maximal with respect to range;
- a semi-stable extension, iff E is admissible and maximal with respect to range.

For any given AF we call a collection of some specific extensions a semantics, e.g. for semi-stable semantics we write $sem(F)$ for the collection of all semi-stable extensions (which is a set of sets).

Intertranslatability

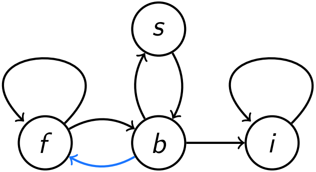
Definition

A *Translation* from some semantics σ to some semantics σ' is a function Tr mapping AFs to AFs, we call Tr

- exact: $\sigma(F) = \sigma(Tr(F))$,
  in words: the original extensions and the new extensions are exactly the same;
- faithful: $E \in \sigma(F)$ iff $\exists E'$, $E \subseteq E'$, $E' \in \sigma'(Tr(F))$ and $|\sigma(F)| = |\sigma'(Tr(F))|$,
  in words: we allow new arguments to expand the new extensions.

Sample Translations

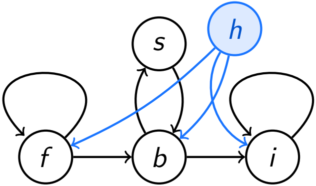
exact Tr: $stg \Rightarrow sem$



$$stg(F) = \{\{b\}\}$$
$$sem(Tr_1(F)) = \{\{b\}\}$$

If birds can eat feline then stage becomes also semi-stable

faithful Tr: $sem \Rightarrow stg$



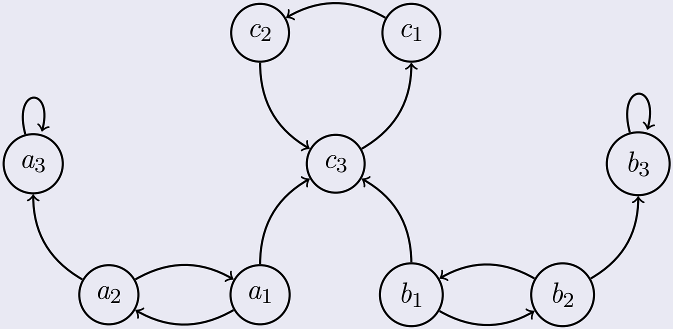
$$sem(F) = \{\{s\}\}$$
$$stg(Tr_2(F)) = \{\{s, h\}\}$$

If humans eat every animal but snakes then semi-stable becomes also stage.

There is no Exact Translation for *sem* $\Rightarrow$ *stg*

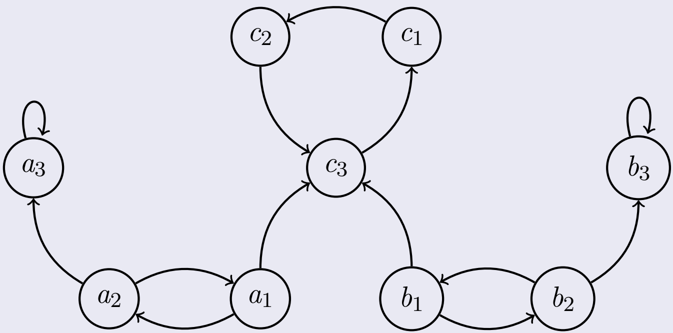
$$E \in stg(F) \iff E \in cf(F) \wedge \nexists B \in cf(F) : E^+ \subsetneq B^+$$
$$E \in sem(F) \iff E \in adm(F) \wedge \nexists B \in adm(F) : E^+ \subsetneq B^+$$

Counterexample



There is no Exact Translation for *sem* $\Rightarrow$ *stg*

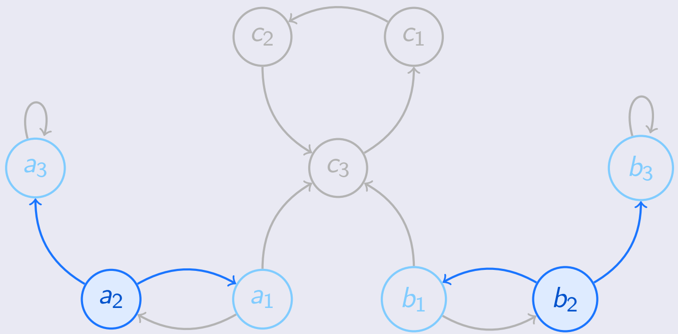
Counterexample, Semi-Stable Extensions



$$sem(F) = \{\, \{a_2, b_2\}, \{a_1, b_2, c_1\}, \{a_2, b_1, c_1\} \}$$

There is no Exact Translation for *sem* $\Rightarrow$ *stg*

Counterexample, Semi-Stable Extensions



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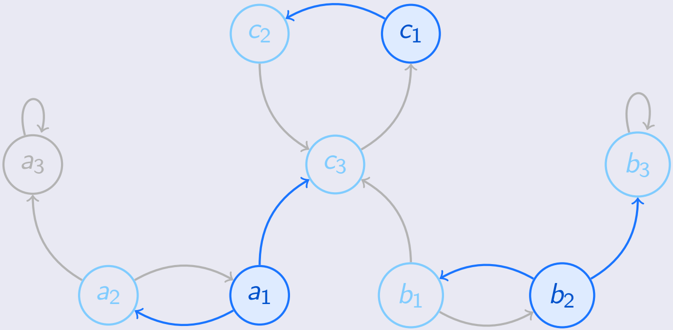

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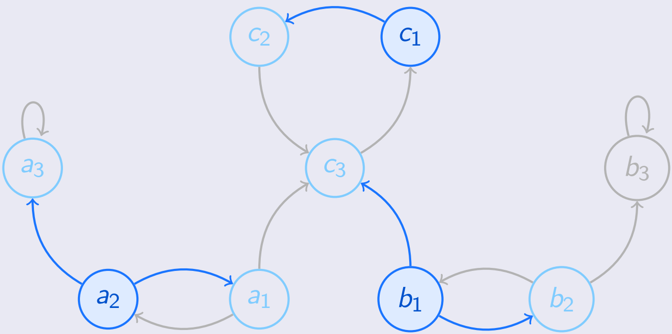

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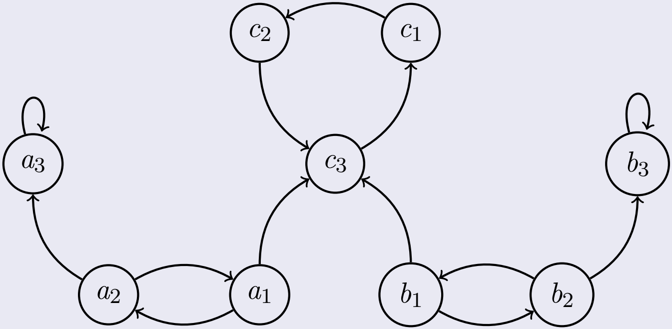

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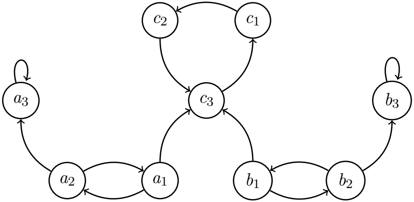

$$sem(F) = \{\{a_2, b_2\}, \{a_1, b_2, c_1\}, \{a_2, b_1, c_1\}\}$$

Proof

- Assume there is an exact translation $Tr\colon sem \Rightarrow stg$, then $stg(Tr(F)) = sem(F)$.
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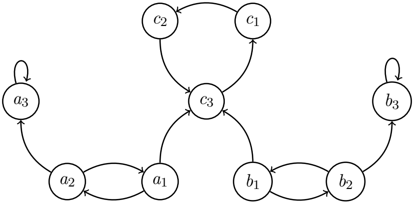
There is no Exact Translation for *sem* $\Rightarrow$ *stg*



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- It follows that c_1 is not in conflict with neither a_2 nor b_2 in $Tr(F)$.
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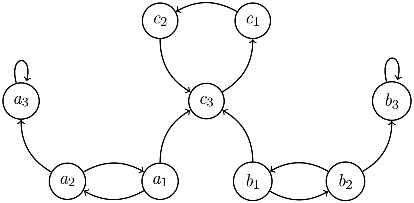
There is no Exact Translation for $sem \Rightarrow stg$



$$sem(F) = \{\{a_2, b_2\}, \\ \{a_1, b_2, c_1\}, \\ \{a_2, b_1, c_1\}\}$$

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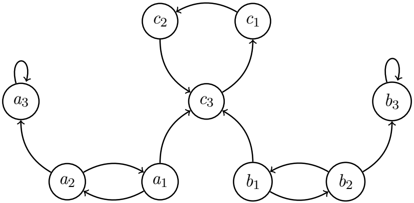
There is no Exact Translation for *sem* $\Rightarrow$ *stg*



$$sem(F) = \{\{a_2, b_2\},\ \{a_1, b_2, c_1\},\ \{a_2, b_1, c_1\}\}$$

Proof

- Assume there is an exact translation Tr: *sem* $\Rightarrow$ *stg*, then $stg(Tr(F)) = sem(F)$.
- It follows that c_1 is not in conflict with neither a_2 nor b_2 in $Tr(F)$.
- Now $\{a_2, b_2, c_1\}$ is conflict-free in $Tr(F)$.
- But then $\{a_2, b_2\}^+_{Tr(F)} \subsetneq \{a_2, b_2, c_1\}^+_{Tr(F)}$, and thus $\{a_2, b_2\}$ can not be a stage extension in $Tr(F)$, a contradiction. $\square$

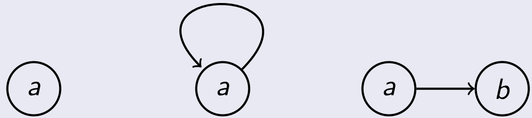
There is no modular faithful translation for *sem* $\Rightarrow$ *stg*

Definition

A translation Tr is called *modular* iff from $F = F_1 \cup F_2$ it follows that also $Tr(F) = Tr(F_1) \cup Tr(F_2)$, in words if we can build the translated framework by translating arbitrary parts, which is useful for distributed computing.

Observation

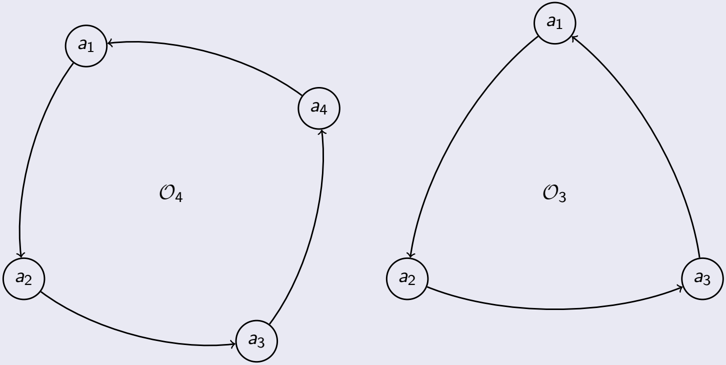
We observe that a translation is called modular iff it is fully defined by translating the following frameworks:



$(\emptyset, \emptyset)$ $\quad$ $(\{a\}, \emptyset)$ $\quad$ $(\{a\}, \{(a,a)\})$ $\quad$ $(\{a,b\}, \{(a,b)\})$

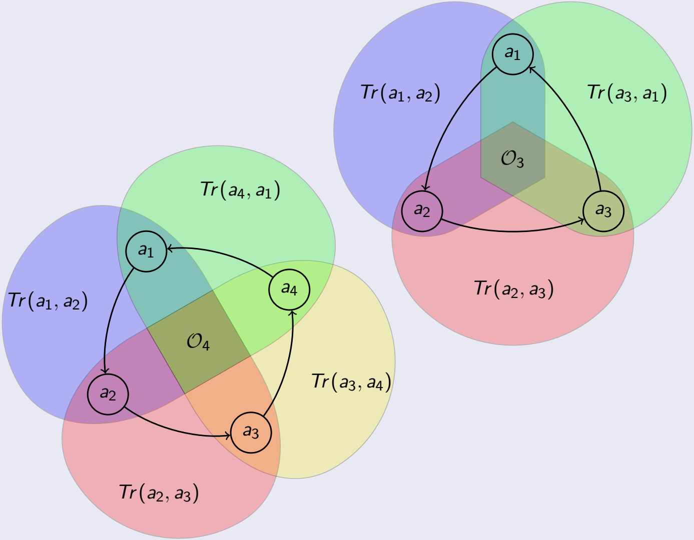
There is no modular faithful translation for *sem* $\Rightarrow$ *stg*

Counterexample



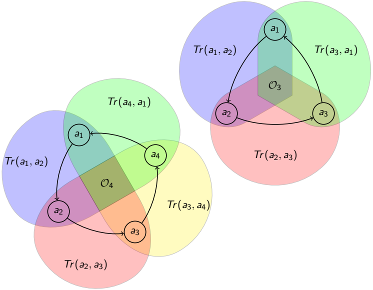
There is no modular faithful translation for *sem* ⇒ *stg*

Counterexample



There is no modular faithful translation for $sem \Rightarrow stg$

$$stg(Tr(\mathcal{O}_3)) = \{E\}$$
$$stg(Tr(\mathcal{O}_4)) = \{E_1, E_2\}$$
$$E \cap A_{\mathcal{O}_3} = \emptyset$$
$$E_1 \cap A_{\mathcal{O}_4} = \{a_1, a_3\}$$
$$E_2 \cap A_{\mathcal{O}_4} = \{a_2, a_4\}$$



Proofsketch

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There is no modular faithful translation for *sem* $\Rightarrow$ *stg*

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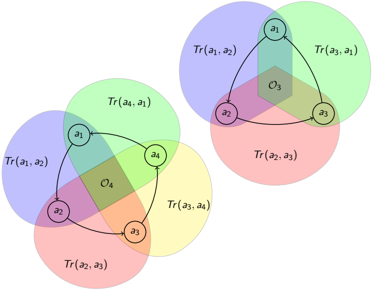

Proofsketch

- We observe that $E \cap Tr\left(\left(\{a_i, a_j\}, \{(a_i, a_j)\}\right)\right)$ must be strictly isomorphic for all attacks $(a_i, a_j) \in \mathcal{O}_3$. Simply because there is only one extension of $Tr(\mathcal{O}_3)$.
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There is no modular faithful translation for *sem* $\Rightarrow$ *stg*

$$
\begin{aligned}
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E \cap A_{\mathcal{O}_3} &= \emptyset \\
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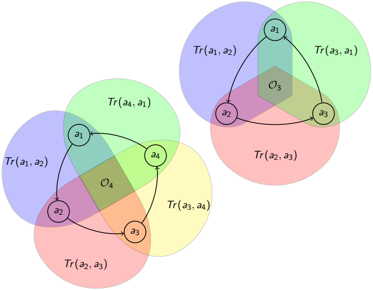

Proofsketch

- We observe that $E \cap Tr\left(\left(\{a_i, a_j\}, \{(a_i, a_j)\}\right)\right)$ must be strictly isomorphic for all attacks $(a_i, a_j) \in \mathcal{O}_3$. Simply because there is only one extension of $Tr(\mathcal{O}_3)$.
- But then we can move E in an isomorphic extending way to $Tr(\mathcal{O}_4)$ receiving an unwanted extension. Namely an extension that does not contain any of a_1, a_2, a_3, a_4. $\square$

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