

Complexity Theory

VU 181.142, WS 2019

2. Fundamental Notions and Results

(Short Recapitulation from “Formale Methoden der Informatik”)

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Computation and Computability

- Ability to read and formulate decision/optimization problems
- Several kinds of problems:
decision, function, optimization, enumeration, counting problems
- Problem vs. problem instance
- Problem vs. algorithm vs. program
- Church-Turing thesis
- Halting problem
- Decidability vs. undecidability vs. semi-decidability
- Complement of a decision problem
- Properties of complementation



Outline

2. Fundamental Notions and Results

- 2.1 Computation and Computability
- 2.2 Complexity of Problems and Algorithms
- 2.3 Reductions
- 2.4 NP-Completeness
- 2.5 Other Important Complexity Classes
- 2.6 Turing Machines



Complexity of Problems and Algorithms

- Asymptotic, worst-case complexity vs. other notions of complexity
- Basic understanding of growth rates (polynomial vs. exponential)
- The class P
- The class NP
- Tractability vs. intractability
- Optimization vs. decision problem



Reductions

- Two motivations for reducing one problem (or language) to another.
- Two kinds of reductions (Turing, many-one).
- Limiting the resources used by reductions.
- Cook / Karp reductions.
- **Proving the correctness of problem reductions.**
- The definitions of C -hard and C -complete problems for a complexity class C .
- Understanding the role of complete problems in complexity theory.
- Proving undecidability by reduction from the HALTING problem.
- Proving non-semi-decidability by reduction from the NON-HALTING problem.

Other Important Complexity Classes

- Understanding the definitions of L , $PSPACE$ and $EXPTIME$
- Being aware of the main inclusions between P , NP , and the three classes above.

NP-Completeness

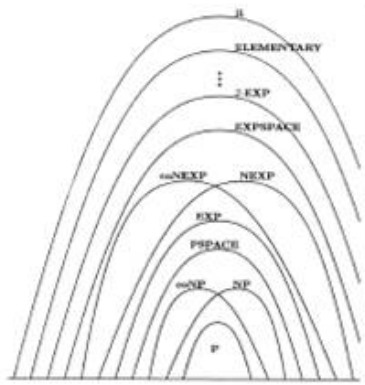
- You should now be familiar with the intuition of NP-completeness (and recognize NP-complete problems).
- Two fundamental NP-complete problems: **SAT** and **3-SAT**.
- Difference between logical equivalence and sat-equivalence.
- Many more examples of NP-complete problems, e.g.: **CLIQUE**, **INDEPENDENT SET**, **VERTEX COVER**, **3-COLORABILITY**, **HAMILTON-PATH**, **HAMILTON-CYCLE**, **TSP(D)**, etc.
- Usefulness of reductions to **SAT**.

Turing Machines

- Definition of Turing machines.
- Turing machines as a reasonable model of computation.
- Formal definition of “deterministic” complexity classes P , $EXPTIME$, L , $PSPACE$, $EXSPACE$.
- Solving problems with Turing machines. (Decision problems can be considered as languages!)
- (Strengthening of) the Church-Turing Thesis
- Nondeterministic Turing machines. Differences between deterministic and nondeterministic TMs
- Nondeterminism as “guess and check” algorithms
- Definitions of NL , NP , $NEXPTIME$ via nondeterministic TMs.
- The definition of complementary problems.
- Summary of important complexity classes: L , NL , $co-NL$, P , NP , $co-NP$, $PSPACE$, $EXPTIME$, $NEXPTIME$, $co-NEXPTIME$, $EXSPACE$.

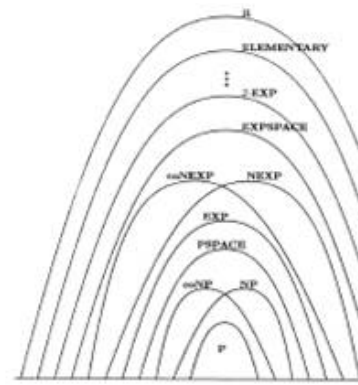
Overview of Complexity Classes

Recursive Problems



Overview of Complexity Classes

Recursive Problems



Inside PSPACE

