



Investigating the Relationship between Argumentation Semantics via Signatures

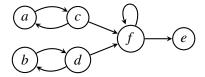
Paul E. Dunne¹, Thomas Linsbichler², Christof Spanring^{1,2}, Stefan Woltran²

¹ University of Liverpool, UK ² TU Wien, Austria

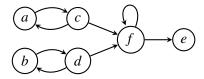
July 14, 2016



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 - legal reasoning, online debates, medicine, . . .
- Abstract Argumentation Framework (AF) [Dung, AlJ 1995]:

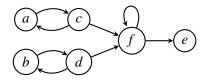


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- Evaluation: argumentation semantics
- Extension: set of jointly acceptable arguments

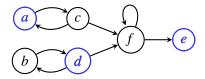
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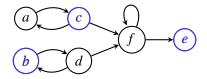
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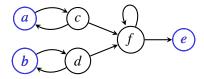
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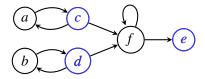
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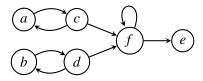
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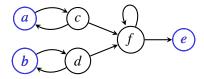


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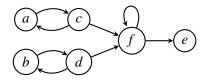


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• Further semantics: preferred, complete, grounded, semi-stable, ...

- Systematic comparison of semantics [Baroni and Giacomin, AlJ 2007]
- Expressive power of semantics via realizability [Dunne et al., AlJ 2015].

Question

What sets of extensions can be the outcome of the evaluation of an arbitrary AF under semantics σ ?

- Systematic comparison of semantics [Baroni and Giacomin, AlJ 2007]
- Expressive power of semantics via realizability [Dunne et al., AlJ 2015].

Question

What sets of extensions can be the outcome of the evaluation of an arbitrary AF under semantics σ ?

- Integral to AGM-style revision of AFs [Diller et al., IJCAI 2015]
 - Argumentation as inherently dynamic process
- Pruning of search space in solvers
 - Increasing interest in systems for solving reasoning tasks

Example

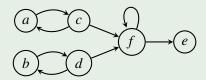
Given $\mathbb{S} = \{\{a,b\}, \{a,d,e\}, \{b,c,e\}, \{c,d,e\}\}.$

• $\exists F \text{ s.t. } prf(F) = \mathbb{S}$?

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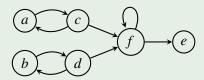
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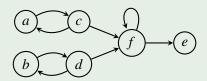


• $\exists F \text{ s.t. } stb(F) = \mathbb{S}$?

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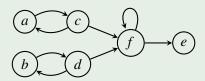


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- $\exists F \text{ s.t. } stb(F) = \mathbb{S} ? \text{ No.}$
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Signature of semantics σ :

$$\Sigma_{\sigma} = \{ \sigma(F) \mid F \text{ is an AF} \}$$

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Theorem [Dunne et al., 2015]

$$\begin{split} &\Sigma_{\textit{nai}} = \{ \mathbb{S} \neq \emptyset \mid \mathbb{S} = \textit{bd}(\mathbb{S}) \} \\ &\Sigma_{\textit{stb}} = \{ \mathbb{S} \mid \mathbb{S} \subseteq \textit{bd}(\mathbb{S}) \} \\ &\Sigma_{\textit{prf}} = \{ \mathbb{S} \neq \emptyset \mid \mathbb{S} \text{ incomparable and } \mathbb{S} \rtimes \mathbb{S} \} \\ &\Sigma_{\textit{sem}} = \{ \mathbb{S} \neq \emptyset \mid \mathbb{S} \text{ incomparable and } \mathbb{S} \rtimes \mathbb{S} \} \end{split}$$

Definition

Given semantics $\sigma_1, \ldots, \sigma_n$, their n-dimensional signature is defined as

$$\Sigma_{\sigma_1,...,\sigma_n} = \{ \langle \sigma_1(F), \ldots, \sigma_n(F) \rangle \mid F \text{ is an AF} \}.$$

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- $\Rightarrow \mathbb{S} \in \Sigma_{\sigma}, \mathbb{T} \in \Sigma_{\tau}$
- ⇒ Well-known semantics relations:
 - $stb \subseteq sem \subseteq prf \subseteq com \subseteq adm \subseteq cf$, $stb \subseteq nai \subseteq cf$

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- ⇒ Well-known semantics relations:
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 - Other conditions?
- ⇒ Measure of the independence of semantics.
- ⇒ Useful for the enumeration of multiple sets of extensions.

Theorem

$$\Sigma_{\textit{nai},\textit{stb}} = \{ \langle \mathbb{S}, \mathbb{T} \rangle \mid \mathbb{S} \in \Sigma_{\textit{nai}}, \mathbb{T} \in \Sigma_{\textit{stb}}, \mathbb{T} \subseteq \mathbb{S} \}$$

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 $F_{\textit{nai,stb}}(\mathbb{S},\mathbb{T}) = (A,R)$ with

- \bullet $A = \bigcup \mathbb{S} \cup \{x_S \mid S \in \mathbb{S} \setminus \mathbb{T}\}$ and
- $R = \textit{Confs}_{\mathbb{S}} \cup \{(x_S, x_S), (a, x_S) \mid S \in \mathbb{S} \setminus \mathbb{T}, a \in \bigcup \mathbb{S} \setminus S\}$

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Example

 $F_{\textit{nai,stb}}(\{\{a,b\},\{a,d\},\{b,c\}\},\{\{a,d\}\})$:



Theorem

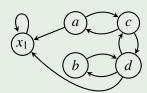
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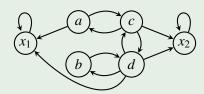
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 $F_{nai,stb}(\{\{a,b\},\{a,d\},\{b,c\}\},\{\{a,d\}\}):$



- $S = \{\{a,b\}, \{a,d,e\}\}$
- $\mathbb{T} = \{\{a,b\}, \{a,d,e\}, \{b,c,e\}, \{c,d,e\}\}$
- $\langle \mathbb{S}, \mathbb{T} \rangle \in \Sigma_{\textit{stb,prf}}$?

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- ullet $\mathbb{S}\in\Sigma_{\mathit{stb}}$
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- However, $\langle \mathbb{S}, \mathbb{T} \rangle \notin \Sigma_{stb,prf} X$

Example - Stable vs. Preferred

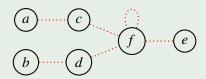
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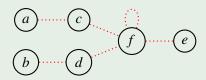
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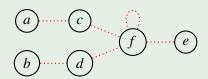
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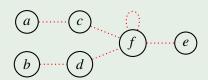
• $bd(\mathbb{T}) = \{\{a, b, e\}, \{a, d, e\}, \{b, c, e\}, \{c, d, e\}\}$

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- $bd(\mathbb{T}) = \{\{a, b, e\}, \{a, d, e\}, \{b, c, e\}, \{c, d, e\}\}$
- $\Rightarrow \langle \mathbb{S}', \mathbb{T} \rangle \in \Sigma_{\textit{stb,prf}} \; \text{iff} \; \mathbb{S}' \subseteq \{\{a, d, e\}, \{b, c, e\}, \{c, d, e\}\}.$

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- $\Rightarrow \langle \mathbb{S}', \mathbb{T} \rangle \in \Sigma_{\textit{stb,prf}} \text{ iff } \mathbb{S}' \subseteq \{\{a, d, e\}, \{b, c, e\}, \{c, d, e\}\}.$
- $\Rightarrow \langle \mathbb{S}, \mathbb{T} \rangle \notin \Sigma_{stb,prf}$

	idl	eag	nai	stb	sem	prf	cf	adm
grd	1	/	1	1	~	/	~	~
idl	-	/	/	~	~	~	/	~
eag		-	~	~	~	?	~	?
nai			-	~	~	/	~	✓
stb				-	~	/	~	✓
sem					-	?	~	?
prf						-	~	~
cf							-	✓

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grd	1	/	1	1	~	1	~	~
idl	-	/	~	~	~	~	~	~
eag		-	~	~	~	?	~	?
nai			-	~	~	~	~	~
stb				-	~	~	~	~
sem					-	?	~	?
prf						-	~	~
cf							-	✓

- Concrete realizations of pairs of extension-sets.
- Exact characterizations: see poster.

	idl	eag	nai	stb	sem	prf	cf	adm
grd	V	V	/	/	~	~	~	~
idl	-	/	~	~	~	~	~	~
eag		-	~	~	✓	?	~	?
nai			-	~	~	~	~	~
stb				-	~	~	~	~
sem					-	?	~	?
prf						-	~	~
cf							-	✓

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- Exact characterizations: see poster.

Conclusion

Summary:

- Exact characterizations of 32 of 36 two-dimensional signatures
- Constructions for standard realizations
- Discussion of the subtle issue of preferred and semi-stable semantics

Future work:

- Complete, stage semantics
- Labelling-based semantics
- Concrete pruning techniques
- n-dimensional signatures (n > 2)
- Other KR formalisms

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