Defining Argumentation Semantics under a Claim-centric View

Anna Rapberger

TU Wien, Institute of Logic and Computation

Abstract

Claim-augmented argumentation frameworks (CAFs) constitute a generic formalism for conflict resolution of conclusion-oriented problems in argumentation. CAFs extend Dung argumentation frameworks (AFs) by assigning a claim to each argument; so far, semantics for CAFs have been defined by considering the semantics for AFs and interpreting the extensions in terms of the claims of the arguments. However, certain semantics of the originally considered (instantiated) problem which involve maximization of the range on conclusion-level cannot be not captured by performing maximization on argument-level. In this paper, we propose therefore an alternative way of defining range-based semantics for CAFs in order to mimic the behavior of the respective semantics of the instantiated problems; we investigate the relation of the newly introduced semantics to their argument-level based counterparts.

1 Introduction

Abstract argumentation frameworks (AFs) as introduced by Dung [6] provide a general schema for analyzing discourses by treating arguments as abstract entities while an attack relation encodes conflicts between them; the acceptance status of arguments is evaluated with respect to different semantics. Moreover, AFs exhibit a close connection to logic programming and other non-monotonic reasoning formalisms by allowing for an alternative way of representing inconsistent and conflicting information. The instantiation of logic programs (LPs) into AFs and generalizations thereof has been frequently discussed in the literature [6, 13, 5] and reveals the close connection of both formalisms in particular by comparing the respective semantics; the correspondence of stable model semantics for LPs with stable semantics in AFs is probably the most fundamental example [6], but also 3-valued stable model semantics or well-founded model semantics admit equivalent argumentation semantics [13].

In a nutshell, an *instantiation procedure* into AFs includes (1) extraction of arguments and conflicts among them; (2) identification of jointly acceptable arguments (extensions) based on a particular argumentation semantics; (3) inspection of claims of the acceptable arguments in order to draw conclusions about the original system. Instantiation procedures for different formalisms have been established, see e.g. [11, 10, 4, 5]. A generalization of AFs which is ideally suited for instantiation procedures in this spirit are claim-augmented argumentation frameworks (CAFs) [8] which extend AFs by assigning a claim to each argument. In [8], semantics for CAFs are evaluated with respect to the underlying AF, the extensions are then interpreted in terms of the claims of the arguments (*inherited semantics*). We furthermore mention a particular restriction on the attack relation of CAFs which is satisfied by many instantiation procedures: A CAF is well-formed iff arguments having the same claim attack the same argument. In the following example, we will adapt an instantiation of logic programs to AFs due to [5] by defining an appropriate claim-function for the generated arguments.

Example 1. *Consider the logic program P from Figure 1, we will construct a CAF $CF = (A, R, claim)$ by instantiating the AF (A, R) following [5] and extracting the claim-function claim for the constructed arguments*

$$
\begin{array}{llll}
r_0: & a \leftarrow \texttt{not}\ d & r_3: & c \leftarrow \texttt{not}\ a, \texttt{not}\ b \\
r_1: & d \leftarrow \texttt{not}\ a & r_4: & f \leftarrow \texttt{not}\ f \\
r_2: & b \leftarrow \texttt{not}\ a & r_5: & f \leftarrow \texttt{not}\ a, \texttt{not}\ f
\end{array}
$$

Figure 1: Logic program P.

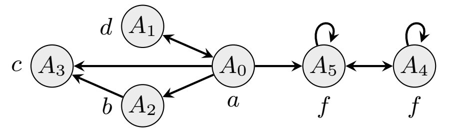


Figure 2: Resulting CAF $CF = (A, R, claim)$.

as follows: Each rule $r_i : c \leftarrow \texttt{not}\ b_1, \ldots, \texttt{not}\ b_m$ is interpreted as argument $A_i \in A$ where the head c or r_i corresponds to the claim of A_i (that is, we define $claim(A_i) = c$). Moreover, the negated atoms determine the potential attackers of A_i in CF, that is, an argument A_j attacks A_i, i.e. $(A_j, A_i) \in R$, iff A_j has claim b_k for some $k \leq m$. The resulting CAF is depicted in Figure 2. Evaluating CF with respect to stable semantics[1] yields no extension; also, P does not possess a stable model. Observe that the procedure yields a well-formed CAF.

Although the CAF CF in Example 1 yields the same results as the original problem with respect to most of the semantics, certain irregularities may arise when it comes to so-called *range-based semantics*, which take arguments (atoms) into account that are defeated (set to false) in the particular extension (model): Semi-stable semantics [12, 3], which yield admissible sets[2] with $\subseteq$-maximal range, potentially leads to a different outcome than the corresponding LP-variant, namely L-stable semantics [9], which maximize the set of all ground atoms which are either considered true or false in a 3-valued stable model. Indeed, in Example 1, evaluation of P with respect to L-stable semantics yields $\{d, b\}$; whereas the semi-stable extensions of CF are given by $\{a\}, \{d, b\}$.

While it has been shown that inherited semantics for CAFs are adequate for standard Dung semantics, the example above reveals that for range-based semantics, results may deviate from the expected outcome of the original problem. A crucial observation is that semantics for LPs operate on conclusion (claim) level while extensions in AFs as well as in CAFs are evaluated on argument level. We are thus interested in developing adequate variants of range-based semantics for CAFs which mimic the behavior of semantics performing maximization on conclusion-level of the original problem (e.g. L-stable model semantics for LPs).

The discrepancy concerning range-based semantics has been already observed by Caminada et al. [5, 4]; they showed that the realization of L-stable semantics on argument level is in fact impossible under standard instantiation methods. We will therefore propose a variant of range-based semantics for CAFs which performs maximization on claim-level (*cl-semantics*). That is, instead of evaluating the underlying AF with respect to semi-stable semantics, we will consider admissible claim-sets and identify the set of claims they defeat. Hereby, we require that each occurrence of a claim is attacked. In Example 1, the claim-set $\{b, d\}$ defeats the claims $\{a, c\}$ while $\{a\}$ defeats $\{b, d\}$; observe that the argument A_0 does not attack the argument A_4, thus f is not defeated by A_0. As a consequence we have that the set $\{a\}$ is no longer semi-stable since $\{a, b, d\}$ is a proper subset of $\{a, b, c, d\}$, thus the evaluation matches the outcome of P with respect to L-stable model semantics.

We introduce alternatives based on maximization on claim-level and investigate their relation to inherited semantics in the spirit of [8] which perform maximization on argument-level. The main results of our paper are:

- We introduce alternative definitions for semi-stable and stage semantics for CAFs by shifting maximization of extensions from argument-level to claim-level. A crucial notion therefore is the *defeat of claims*, where one requires that a claim c is defeated iff every occurrence of c is attacked.

- We propose two variants of stable semantics, based on conflict-free, respectively, admissible sets. We show that for well-formed CAFs, both variants of stable semantics as well as inherited stable semantics coincide.

- We compare inherited semantics with cl-semantics. We show that they exhibit similar behaviour concerning incomparability: For general CAFs, incomparability of claim-sets is not guaranteed, whereas for well-formed CAFs, every semantics under consideration yields incomparable claim-sets; moreover, we show that even for well-formed CAFs, both variants of semi-stable and stage semantics potentially yield different claim-sets.

2 Preliminaries

We introduce argumentation frameworks [6] (for a comprehensive introduction, see [2, 1]). We fix U as countable infinite domain of arguments.

[1] A set S is stable iff it is conflict-free and attacks every argument in $A \setminus S$.

[2] A set S is admissible in an AF F iff it is conflict-free and attacks all attackers of S.

Definition 1. *An* argumentation framework (AF) *is a pair $F = (A, R)$ where $A \subseteq U$ is a finite set of arguments and $R \subseteq A \times A$ is the attack relation. We say that $S \subseteq A$* attacks *b if $(a, b) \in R$ for some $a \in S$. Moreover, an argument $a \in A$ is* defended *(in F) by $S \subseteq A$ if each b with $(b, a) \in R$ is attacked by S in F.*

Furthermore we denote by $S^+_F = \{b \in A \mid (a, b) \in R\}$ the set of attacked arguments of S. If no ambiguity arises, we drop the subscript F. We call $S \cup S^+_F$ the range *of S in F.*

Semantics for AFs are defined as functions σ which assign to each AF $F = (A, R)$ a set $\sigma(F) \subseteq 2^A$ of extensions. We consider for σ the functions *cf*, *adm*, *stb*, *sem* and *stg* which stand for conflict-free, admissible, stable, semi-stable and stage extensions, respectively.

Definition 2. *Let $F = (A, R)$ be an AF. A set $S \subseteq A$ is* conflict-free (in F)*, if there are no $a, b \in S$, such that $(a, b) \in R$. $cf(F)$ denotes the collection of sets being conflict-free in F. For a conflict-free set $S \in cf(F)$, we say $S \in adm(F)$, if each $a \in S$ is defended by S in F; $S \in stb(F)$, if each $a \in A \setminus S$ is attacked by S in F; $S \in sem(F)$, if $S \in adm(F)$ and there is no $T \in adm(F)$ with $S \cup S^+_F \subset T \cup T^+_F$; $S \in stg(F)$, if there is no $T \in cf(F)$, with $S \cup S^+_F \subset T \cup T^+_F$.*

We recall that for each AF F, $stb(F) \subseteq stg(F) \subseteq cf(F)$ and $stb(F) \subseteq sem(F) \subseteq adm(F)$; also $stb(F) = sem(F) = stg(F)$ in case $stb(F) \neq \emptyset$. Moreover, semantics $\sigma \in \{stg, stb, sem\}$ deliver *incomparable* sets, i.e. for all $S, T \in \sigma(F)$, $S \subseteq T$ implies $S = T$; the property is also referred to as *I-maximal*.

Next we define claim-augmented argumentation frameworks according to [8].

Definition 3. *A* claim-augmented argumentation framework (CAF) *is a triple $(A, R, claim)$ where (A, R) is an AF and $claim : A \to C$ is a function which assigns a claim to each argument in A; C is a set of possible claims. The claim-function is extended to sets in the following way: For a set $E \subseteq A$, $claim(E) = \{claim(a) \mid a \in E\}$.*

The CAF CF is called well-formed *if $\{a\}^+_{(A,R)} = \{b\}^+_{(A,R)}$ for all $a, b \in A$ such that $claim(a) = claim(b)$.*

In [8], semantics of CAFs are defined based on the standard semantics of the underlying AF. The extensions are interpreted in terms of the claims of the arguments. We call this variant *inherited semantics* (i-semantics).

Definition 4. *For a CAF $CF = (A, R, claim)$, for a semantics σ, we define* i-semantics *$\sigma_c(CF) = \{claim(E) \mid E \in \sigma((A, R))\}$. We call a set $E \in \sigma((A, R))$ with $claim(E) = S$ a σ-realization of S in CF.*

Basic relations between different semantics carry over from standard AFs, i.e. for any CAF CF, $stb_c(CF) \subseteq sem_c(CF) \subseteq adm_c(CF)$ and $stb_c(CF) \subseteq stg_c(CF) \subseteq cf_c(CF)$; moreover, if $stb(CF) \neq \emptyset$ then $stb_c(CF) = sem_c(CF) = stg_c(CF)$. However, the next example shows that we lose fundamental properties of semantics like I-maximality of stable, semi-stable and stage semantics.

Example 2. *Let $CF = (A, R, claim)$ with $(A, R) = (\{x_1, x_2, y\}, \{(x_1, x_2), (x_2, x_1), (x_2, y)\})$ and $claim(x_i) = x$, $i \leq 2$, $claim(y) = y$. Then $stb_c(CF) = sem_c(CF) = stg_c(CF) = \{\{x\}, \{x, y\}\}$. Note that CF is not well-formed.*

3 Range-based Semantics in CAFs

For standard argumentation frameworks, the range of a set E of arguments is defined as the union of E together with all aruments it attacks; hence a claim-centered variant of range-based semantics requires explicit concepts for the defeat of claims. In the current section, we will discuss defeat on claim-level and the range of a claim-set which both exhibit certain differences to its argument-based counter-parts. In Sections 3.1, 3.2 and 3.3, we will discuss claim-centered variants of stable, semi-stable and stage semantics, respectively.

We will introduce the range of a claim-set $S \subseteq claim(A)$ in a CAF $CF = (A, R, claim)$, that is, we will define, for any claim-set S, the set of all claims it defeats. Since each claim-set depends on a particular realization in the underlying AF (A, R), we will first introduce claim-defeat on argument-level.

Definition 5. *Let $CF = (A, R, claim)$, $E \subseteq A$ and $c \in claim(A)$. We say that E* defeats *c iff E attacks every $a \in A$ with $claim(a) = c$. We define $dis_{CF}(E) = \{c \in claim(A) \mid \forall x \in A, claim(x) = c\ \exists y \in E \text{ s.t. } (y, x) \in R\}$. If no ambiguity arises, we drop the subscript CF.*

Observe that $dis_{CF} : A \to claim(A)$ is monotone, i.e. if $E \subseteq E'$ then $dis_{CF}(E) \subseteq dis_{CF}(E')$ for any $E, E' \subseteq A$.

Next we will consider claim-defeat with respect to a claim-set S independently of a particular realization. The general idea is to consider, for each realization E of S, the set of defeated claims $dis_{CF}(E)$ as potential candidate to identify the range of S. Observe that, in contrast to the range of a set of arguments, the range of a set of claims S is in general not unique since S can possess multiple realizations; moreover, we restrict ourselves to σ-realizations of S for some semantics σ in order to exclude for example conflicting realizations.

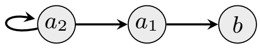


Figure 3: Example of a CAF $CF = (A, R, claim)$ with $claim(a_1) = claim(a_2) = a$, $claim(b) = b$.

Definition 6. *Let $CF = (A, R, claim)$, $S \subseteq claim(A)$ and consider a semantics σ. Then $\mathcal{D}_{\sigma,CF}(S) = \{dis_{CF}(E) \mid E \in \sigma((A, R)), claim(E) = S\}$; moreover, $\mathcal{R}_{\sigma,CF}(S) = \{S \cup S' \mid S' \in \mathcal{D}_{\sigma,CF}(S)\}$ represents every possible range of S with respect to σ. If no ambiguity arises, we drop the subscript CF.*

Observe that for every claim-set S and two semantics σ, σ' with $\sigma((A, R)) \subseteq \sigma'((A, R))$ it holds that $\mathcal{D}_{\sigma,CF}(S) \subseteq \mathcal{D}_{\sigma',CF}(S)$. Indeed, if $dis_{CF}(E) \in \mathcal{D}_{\sigma,CF}(S)$ for some $E \subseteq A$, then $E \in \sigma((A, R)) \subseteq \sigma'((A, R))$, and thus $dis_{CF}(E) \in \mathcal{D}_{\sigma',CF}(S)$. Moreover notice that, in general, $|\mathcal{R}_{CF}(S)| \geq 1$, that is, the range of a claim-set potentially consists of multiple alternatives. However, for well-formed CAFs CF, it holds that for every two sets $E, E' \subseteq A$ with $claim(E) = claim(E')$, $E^+ = E'^+$, thus $dis_{CF}(E) = dis_{CF}(E')$. It follows that the range of a claim-set S is unique if the CAF is well-formed. This also implies that, for well-formed CAFs, the range is independent of the particular realization with respect to a semantics σ.

Lemma 1. *Let $CF = (A, R, claim)$ be well-formed and let $S \subseteq claim(A)$. Then $|\mathcal{R}_{\sigma,CF}(S)| = 1$.*

3.1 Stable Semantics

We will introduce two variants of stable semantics based on maximization on claim-level. The first variant requires the underlying realization of a claim-set S to be conflict-free, while the second variant requires admissibility. We clarify the relation between both variants as well as the relation to i-stable semantics and compare them also with regard to I-maximality of their extensions.

Definition 7. *Let $CF = (A, R, claim)$ and $S \subseteq claim(A)$. S is a cf-*cl-stable claim-set*, in symbols $S \in$ cl-stb$_{cf}(CF)$, iff there exists $S' \in \mathcal{D}_{cf,CF}(S)$ such that $S \cup S' = claim(A)$.*

The proposed variant of claim-based stable semantics relaxes the definition of inherited stable semantics in the way that it is no longer required that a *stb*-realization of a *cf*-cl-stable claim-set exists. Consider the CAF $CF = (A, R, claim)$ from Figure 3 with $claim(a_1) = claim(a_2) = a$, $claim(b) = b$. Here, $stb_c(CF) = \emptyset$ but $cl\text{-}stb_{cf}(CF) = \{\{a\}\}$: The *cf*-realization $E = \{a_1\}$ satisfies $dis_{CF}(E) = \{b\}$ and therefore, $claim(E) \cup dis_{CF}(E) = claim(A)$. Observe that CF is not well-formed. Furthermore notice that the *cf*-cl-stable claim-set $\{a\}$ is in fact not *adm*-realizable in (A, R). Thus in contrast to standard AF semantics where each stable extension satisfies admissibility, a $cl\text{-}stb_{cf}$-realization in the underlying AF is not necessarily admissible. Thus we consider also a stronger notion of stable semantics which requires *adm*-realizability in the underlying AF.

Definition 8. *Let $CF = (A, R, claim)$ and $S \subseteq claim(A)$. S is an adm-*cl-stable set*, in symbols $S \in$ cl-stb$_{adm}(CF)$, if there exists $S' \in \mathcal{D}_{adm,CF}(S)$ such that $S \cup S' = claim(A)$.*

Proposition 1. *For any $CF = (A, R, claim)$, $stb_c(CF) \subseteq cl\text{-}stb_{adm}(CF) \subseteq cl\text{-}stb_{cf}(CF)$.*

Proof. Let $S \in stb_c(CF)$ and consider a *stb*-realization $E \subseteq A$. Observe that $E \in adm((A, R))$. Let $c \in claim(A) \setminus S$, then for all $x \in A$ with $claim(x) = c$, $x \in A \setminus E$. Since E is stable in (A, R) we have that E attacks each argument $x \in A \setminus E$, therefore $c \in dis_{CF}(E)$. Thus $dis_{CF}(E) = claim(A) \setminus S$ and therefore we have found a set $T = dis_{CF}(E) \in \mathcal{D}_{adm,CF}(S)$ with $S \cup T = claim(A)$, i.e. $S \in cl\text{-}stb_{adm}(CF)$. Moreover, $cl\text{-}stb_{adm}(CF) \subseteq cl\text{-}stb_{cf}(CF)$ follows from the fact that each admissible set is also conflict-free. $\square$

In the CAF $CF = (A, R, claim)$ from Figure 3 we have $cl\text{-}stb_{adm}(CF) \neq cl\text{-}stb_{cf}(CF)$ since $cl\text{-}stb_{adm}(CF) = \emptyset$ but $cl\text{-}stb_{cf}(CF) = \{\{a\}\}$. A small modification of the CAF CF also shows that $cl\text{-}stb_{adm}(CF) \neq stb_c(CF)$: Let $CF_1 = (A, R \setminus \{(a_2, a_1)\}, claim)$, then $cl\text{-}stb_{adm}(CF_1) = \{\{a\}\}$ (witnessed by the *adm*-realization $\{a_1\}$ in (A, R)) but $stb_c(CF_1) = \emptyset$. Observe that both CF and CF_1 are not well-formed. We will show next that for well-formed CAFs, all considered variants of stable semantics are in fact equal.

Proposition 2. *For any well-formed CAF $CF = (A, R, claim)$, $cl\text{-}stb_{adm}(CF) = cl\text{-}stb_{cf}(CF) = stb_c(CF)$.*

Proof. We will show that $cl\text{-}stb_{cf}(CF) \subseteq stb_c(CF)$, the other direction is due to Proposition 1.

Let $S \in cl\text{-}stb_{cf}(CF)$, then there is some set $S' \in \mathcal{D}_{cf,CF}(S)$ such that $S \cup S' = claim(A)$ (recall that $|\mathcal{D}_{cf,CF}(S)| = 1$ by Lemma 1). We consider a maximal *cf*-realization $E \subseteq A$ of S, that is, $E \in cf((A, R))$ with $E = claim(S)$ and for every set $E' \in cf((A, R))$ with $E' = claim(S)$, $E' \subseteq E$. We show that $E^+_R = A \setminus E$. Let

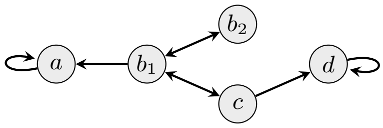


Figure 4: CAF $CF = (A, R, claim)$ with $claim(b_1) = claim(b_2) = b$ and $claim(x) = x$ for $x \in A \setminus \{b_1, b_2\}$.

$x \in A \setminus E$ and let $claim(x) = c$. If $c \notin S$, then $c \in S'$ by definition of cf-cl-stable semantics, thus E attacks x. Consider now the case $c \in S$, i.e. there is an argument $y \in E$ such that $claim(y) = c$ and observe that $E \cup \{x\}$ is not conflict-free by maximality of E; thus either (a) $(x, x) \in R$ or there is $z \in E$ such that either (b) $(z, x) \in R$ or (c) $(x, z) \in R$. In case (a) then also $(y, x) \in R$ by well-formedness; in case (b) we are done; in case (c) we have $(y, z) \in R$ by well-formedness and therefore E is not conflict-free, contradiction. □

Recall that i-stable claim-sets are not necessarily I-maximal (c.f. Example 2). As a consequence of Proposition 1 we deduce that cf-cl-stable claim-sets are not I-maximal for arbitrary CAFs. In [7] it has been shown that i-stable semantics yield I-maximal claim-sets for well-formed CAFs. By Proposition 2, we conclude that cl-stable claim-sets satisfy I-maximality if well-formedness is guaranteed.

Proposition 3. *For any well-formed CAF CF, both* $cl\text{-}stb_{cf}(CF)$ *and* $cl\text{-}stb_{adm}(CF)$ *are I-maximal.*

3.2 Semi-stable Semantics

We consider the following claim-based variant of semi-stable semantics which relaxes adm-cl-stable semantics by dropping the requirement that the range of a claim-set must consist of all claims in the framework. Instead, we consider claim-sets with maximal range.

Definition 9. *Let* $CF = (A, R, claim)$*,* $S \subseteq claim(A)$ *is a* cl-semi-stable claim-set*, in symbols* $S \in cl\text{-}sem(CF)$*, iff there exists* $S' \in \mathcal{D}_{adm,CF}(S)$ *such that there is no* $T \in adm_c(CF)$*,* $T' \in \mathcal{D}_{adm,CF}(T)$ *with* $S \cup S' \subset T \cup T'$*.*

As an example, consider the CAF $CF = (A, R, claim)$ from Figure 4 with $claim(b_1) = claim(b_2) = b$ and $claim(x) = x$ for $x \in A \setminus \{b_1, b_2\}$. First notice that $stb_c(CF) = cl\text{-}stb_{cf}(CF) = cl\text{-}stb_{adm}(CF) = \emptyset$ since b_1 and c are mutually attacking, thus either a or d are not attacked. Admissible claim-sets are $S_1 = \{b\}$, $S_2 = \{c\}$ and $S_3 = \{b, c\}$; then $\mathcal{D}_{adm}(S_1) = \{\{\emptyset, \{a, c\}\}$ and $\mathcal{D}_{adm}(S_2) = \mathcal{D}_{adm}(S_3) = \{\{d\}\}$. Observe that S_2 is not cl-semi-stable, since $S_2 \cup \{d\} \subseteq S_3 \cup \{d\}$; moreover, S_1 is cl-semi-stable, since $S_1 \cup \{a, c\} = \{a, b, c\} \not\subseteq S_3 \cup \{d\}$ S_3 is cl-semi-stable, since $S_3 \cup \{d\} = \{b, c, d\} \not\subseteq S_1 \cup \{a, c\}$. It follows that cl-semi-stable claim-sets are not necessarily I-maximal. Notice that CF is not well-formed.

Since for well-formed CAFs, the range is unique and moreover, the function dis_{CF} is monotone, we conclude that cl-semi-stable semantics yields I-maximal claim-sets if well-formedness is satisfied.

Proposition 4. *For any well-formed CAF CF,* $cl\text{-}sem(CF)$ *is I-maximal.*

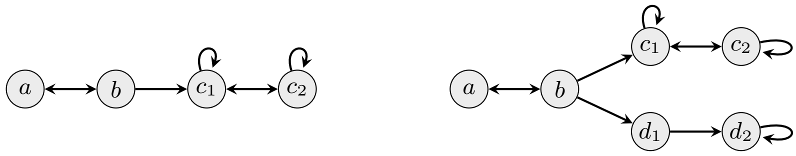
This observation accords with the analysis of i-semi-stable claim-sets: I-maximality of i-semi-stable claim-sets is not guaranteed in the general case but for well-formed CAFs, as we show next.

Proposition 5. *For any well-formed CAF CF,* $sem_c(CF)$ *is I-maximal.*

Proof. Towards a contradiction, assume that there are two semi-stable claim-sets $S, S' \in sem_c(CF)$ such that $S \subseteq S'$. We consider sem-realizations E, E' for S, S' respectively and recall that semi-stable extensions are I-maximal on argument level, i.e. there is $E \Delta E' \neq \emptyset$. Observe that $E^+ \subseteq E'^+$ holds by well-formedness: Let $x \in E^+$, then there is $y \in E$ such that $(y, x) \in R$. By assumption $S \subseteq S'$, there exists $z \in E'$ such that $claim(y) = claim(z)$, thus $(z, x) \in R$ by well-formedness. It follows that every argument $x \in E \setminus E'$ is defended by E' and thus $E' \cup \{x\} \cup (E' \cup \{x\})^+ \supset E' \cup E'^+$, contradiction to E' being semi-stable. □

However, a closer comparison of cl-semi-stable and i-semi-stable semantics reveals the inherent difference between maximization on claim- vs. argument-level. As already discussed in the introduction, the well-formed CAF CF from Example 1 yields $sem_c(CF) = \{\{a\}, \{d, b\}\}$ while $cl\text{-}sem(CF) = \{d, b\}$, thus $sem_c(CF) \not\subseteq cl\text{-}sem(CF)$. The following example extends Example 1 in order to show $cl\text{-}sem(CF) \not\subseteq sem_c(CF)$.

Example 3. *We extend the CAF* $CF = (A, R, claim)$ *from Example 1: Let* $CF = (A \cup \{b, e\}, R', claim')$ *with* $R' = R \cup \{(e, e), (e, b), (A_3, e), (A_3, b), (b, A_3)\}$ *and* $claim(x) = x$ *for* $x \in \{b, e\}$*. Then* $\{a\}$ *is the only i-semi-stable claim-set. For cl-semi-stable claim-sets, consider* $adm_c(CF) = \{\{d\}, \{b, d\}, \{a\}\}$*; inspecting the range yields* $\{d, a\}$*,* $\{b, d, a, c\}$ *and* $\{a, c, d\}$ *and thus* $cl\text{-}sem(CF) = \{\{b, d\}\}$*. Observe that CF is indeed well-formed.*



(a) CAF CF_1 from Example 4. (b) CAF CF_2 from Example 5.

Figure 5: Examples of CAFs CF_1, CF_2 with $\textit{cl-stg}(CF_1) \not\subseteq \textit{stg}_c(CF_1)$ and $\textit{stg}_c(CF_2) \not\subseteq \textit{cl-stg}(CF_2)$.

3.3 Stage Semantics

We define cl-stage semantics in the spirit of cl-semi-stable semantics.

Definition 10. *Let $CF = (A, R, claim)$, then $S \subseteq claim(A)$ is a* cl-stage claim-set, *in symbols $S \in \textit{cl-stg}(CF)$, there exists $S' \in \mathcal{D}_{cf,CF}(S)$ such that there is no $T \in cf_c(CF)$, $T' \in \mathcal{D}_{cf,CF}(T)$ with $S \cup S' \subset T \cup T'$.*

Recall that i-stage semantics do not satisfy I-maximality in general. Figure 4 shows that also for cf-stage semantics, I-maximality for arbitrary CAFs does not hold (note that $\textit{cl-sem}(CF) = \textit{cl-stg}(CF)$ in this example). However, for well-formed CAFs, I-maximality is guaranteed for cl-stage semantics. The following proposition is an immediate consequence of from Lemma 1.

Proposition 6. *For any well-formed CAF CF, $\textit{cl-stg}(CF)$ is I-maximal.*

We will show that also for i-stage semantics, I-maximality is satisfied if the CAF is well-formed.

Proposition 7. *For any well-formed CAF CF, $\textit{stg}_c(CF)$ is I-maximal.*

Proof. Towards a contradiction, assume that there are $S_1, S_2 \in \textit{stg}_c(CF)$ such that $S_1 \subset S_2$. Consider *stg*-realizations E_1, E_2 of S_1 and S_2. So $E_1 \cup E_1^+$, $E_2 \cup E_2^+$ are incomparable and both subset-maximal. By well-formedness, $E_1^+ \subseteq E_2^+$. Indeed, let $x \in A$ be attacked by E_1, i.e. there is $a \in E_1$ such that $(a, x) \in R$. Since $claim(E_1) \subset claim(E_2)$, there is $b \in E_2$ such that $claim(b) = claim(a)$. By definition of well-formedness, $(b, x) \in R$. Since $E_1^+ \subseteq E_2 \cup E_2^+$, it must be the case that $E_1 \not\subset E_2 \cup E_2^+$, i.e. there exists $a \in E_1$ such that $a \notin E_2$ and $a \notin E_2^+$. Let $E = E_2 \cup \{a\}$, then (i) E is conflict-free since $a \notin E_2^+$ and a does not attack E_2 (assume otherwise, then there is some $b \in E_2$ such that $b \in E_1^+$, but then also $b \in E_2^+$ since $E_1^+ \subseteq E_2^+$, contradiction) and, furthermore, $(a, a) \notin R$ since $a \in E_1$; and (ii) $E_2^+ \subseteq E^+$ by definition of E (actually, $E^+ = E_2^+$ since $claim(a) \in claim(E_2)$). Therefore there is a conflict-free set $E \subseteq A$ such that $E \cup E^+ \supset E_2 \cup E_2^+$, contradiction to the subset-maximality of $E_2 \cup E_2^+$. ☐

The following examples show that even for well-formed CAFs, i-stage and cl-stage semantics potentially yield different claim-sets.

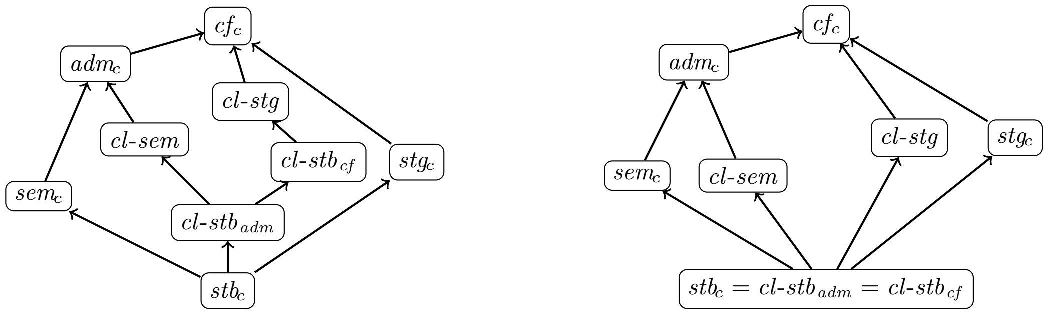
Example 4. *Let $CF_1 = (A, R, claim)$ with (A, R) given in Figure 5a, $claim(c_1) = claim(c_2) = c$, $claim(a) = a$ and $claim(b) = b$. Then $\{b\}$ is the only i-stage claim-set. Observe that CF_1 is indeed well-formed. Consider now the cl-stage claim-sets. The conflict-free sets are $\{a\}$ and $\{b\}$. Inspecting the range yields $\{a, b\}$ in both cases and therefore $\textit{cl-stg}(CF_1) = \{\{a\}, \{b\}\}$, i.e. $\textit{cl-stg}(CF_1) \not\subseteq \textit{stg}_c(CF_1)$.*

Example 5. *Let $CF_2 = (A, R, claim)$ with (A, R) as in Figure 5b, $claim(d_i) = d$, $claim(c_i) = c$, $claim(a) = a$, $claim(b) = b$. Then $\textit{stg}_c(CF_2) = \{\{a, d\}, \{b\}\}$ but $\textit{cl-stg}(CF_1) = \{\{a, d\}\}$, that is, $\textit{stg}_c(CF_2) \not\subseteq \textit{cl-stg}(CF_2)$.*

3.4 Relations between Semantics

We start with a general observation which clarifies the relation between inherited and claim-level semantics for CAFs where every argument posses a unique claim. In that case, both variants coincide with the standard AF semantics interpreted in terms of the claims since the claims in the CAF can be identified with the arguments in the underlying AF. It follows that negative results concerning the relations between the semantics carry over from standard AFs, i.e. counter-examples showing that two AF semantics σ, τ are not in a subset-relation can be adapted to CAFs.

Proposition 8. *Let $CF = (A, R, claim)$, let σ, τ be semantics. If there exists an AF F such that $\sigma(F) \not\subseteq \tau(F)$ then there exists a (well-formed) CAF CF such that $\alpha(CF) \not\subseteq \beta(CF)$ for $\alpha \in \{\textit{cl-}\sigma, \sigma_c\}$, $\beta \in \{\textit{cl-}\tau, \tau_c\}$.*



(a) Relations for arbitrary CAFs.

(b) Relations for well-formed CAFs.

Figure 6: Relations between semantics. An arrow from σ to τ indicates that $\sigma(CF) \subseteq \tau(CF)$ for each CAF CF.

Proposition 9. *The relations between the semantics depicted in Figure 6 hold.*

Proof. The relations between inherited semantics have been already discussed in Section 2; moreover, $stb_c(CF) \subseteq cl\text{-}stb_{adm}(CF) \subseteq cl\text{-}stb_{cf}(CF)$ for arbitrary CAFs by Proposition 1 and $stb_c(CF) = cl\text{-}stb_{adm}(CF) = cl\text{-}stb_{cf}(CF)$ for each well-formed CAF CF by Proposition 2. Moreover, for any CAF CF, for every $S \in cl\text{-}stb_{adm}(CF)$ exists $S' \in \mathcal{D}_{adm,CF}(S)$ such that $S \cup S' = A$ and thus $S \in cl\text{-}sem(CF)$; furthermore, since each $S \in cl\text{-}sem(CF)$ is i-admissible by definition, it follows that $cl\text{-}stb_{adm}(CF) \subseteq cl\text{-}sem(CF) \subseteq adm_c(CF)$. A similar reasoning applies for the cf-based counter-parts, i.e. for every $S \in cl\text{-}stb_{cf}(CF)$ exists $S' \in \mathcal{D}_{cf,CF}(S)$ such that $S \cup S' = A$ and thus $S \in cl\text{-}stg(CF)$; moreover, every $S \in cl\text{-}stg(CF)$ is conflict-free, thus $cl\text{-}stb_{cf}(CF) \subseteq cl\text{-}stg(CF) \subseteq cf_c(CF)$.

We present counter-examples for the remaining cases: By Corollary 8, there is a well-formed CAF CF such that $\alpha(CF) \not\subseteq \beta(CF)$ for (a) $\alpha = cf_c$, $\beta \in \{adm_c, cl\text{-}sem, sem_c, cl\text{-}stg, stg_c, cl\text{-}stb_{cf}, cl\text{-}stb_{adm}, stb_c\}$; (b) $\alpha = adm_c$, $\beta \in \{cl\text{-}sem, sem_c, cl\text{-}stg, stg_c, cl\text{-}stb_{cf}, cl\text{-}stb_{adm}, stb_c\}$; (c) $\alpha \in \{cl\text{-}sem, sem_c\}$, $\beta \in \{cl\text{-}stg, stg_c, cl\text{-}stb_{cf}, cl\text{-}stb_{adm}, stb_c\}$ and (d) $\alpha \in \{cl\text{-}stg, stg_c\}$, $\beta \in \{adm_c, cl\text{-}sem, sem_c, cl\text{-}stb_{cf}, cl\text{-}stb_{adm}, stb_c\}$. Example 3 shows that $cl\text{-}sem(CF) \neq sem_c(CF)$ where CF is well-formed; moreover, $cl\text{-}stg(CF) \neq stg_c(CF)$ using (well-formed) CAFs from Example 4 and Example 5. Counter-examples for general CAFs and stable semantics have been discussed in Section 3.1. $\square$

Recall that for inherited semantics, $stb_c(CF) = sem_c(CF) = stg_c(CF)$ in case $stb_c(CF) \neq \emptyset$. One can show that this does not extend to cl-stable semantics. However, we can obtain the following weaker version.

Lemma 2. *For any CAF $CF = (A, R, claim)$, (a) $cl\text{-}stb_{cf}(CF) \neq \emptyset$ implies $cl\text{-}stb_{cf}(CF) = cl\text{-}stg(CF)$ and (b) $cl\text{-}stb_{adm}(CF) \neq \emptyset$ implies $cl\text{-}stb_{adm}(CF) = cl\text{-}sem(CF)$.*

4 Discussion

In this work, we investigated range-based semantics for claim-augmented argumentation frameworks. We introduced inherited semi-stable and stage semantics in the spirit of [8] which perform maximization on argument-level and developed claim-based alternatives which perform maximization on claim-level. In doing so, we were able to provide a variant of semi-stable semantics which mimics the behavior of L-stable model semantics of LPs; observe that cl-semi-stable semantics in fact corresponds to L-stable model semantics. We furthermore studied two variants of claim-level stable semantics based on conflict-free respectively admissible semantics. Our findings underline the inherent difference of argument-based vs. claim-based maximization of the range: While cf-cl-stable semantics correspond to stable semantics on argument-level for well-formed CAFs, this is not the case for semi-stable and stage semantics; we have shown that both i-semi-stable and cl-semi-stable semantics as well as i-stage and cl-stage semantics are incomparable, even for well-formed CAFs.

For future work, we plan to extend our investigations to other semantics involving maximization, in particular to preferred and naive semantics. Moreover, we want to connect our findings with studies in [7] where it has been shown that well-formed CAFs can be faithfully translated (with respect to standard argumentation semantics) to SETAFs, i.e. AFs which allow for collective attacks of arguments; of particular interest is the behavior of the variants of range-based semantics we have considered in this work. Another direction of future research is to extend our studies to further classes of CAFs, e.g. attacker-unitary CAFs as introduced in [7].

References

[1] Pietro Baroni, Martin Caminada, and Massimiliano Giacomin, 'An introduction to argumentation semantics', *Knowledge Eng. Review*, **26**(4), 365–410, (2011).

[2] Pietro Baroni, Dov M. Gabbay, and Massimiliano Giacomin, *Handbook of Formal Argumentation*, College Publications, 2018.

[3] Martin Caminada, 'Semi-stable semantics', in *Proceedings of the 1st Conference on Computational Models of Argument (COMMA 2006)*, eds., Paul E. Dunne and Trevor J. M. Bench-Capon, volume 144 of *Frontiers in Artificial Intelligence and Applications*, pp. 121–130. IOS Press, (2006).

[4] Martin Caminada, Samy Sá, João Alcântara, and Wolfgang Dvořák, 'On the difference between assumption-based argumentation and abstract argumentation', *IfCoLog Journal of Logic and its Applications*, **2**(1), 15–34, (2015).

[5] Martin Caminada, Samy Sá, João Alcântara, and Wolfgang Dvořák, 'On the equivalence between logic programming semantics and argumentation semantics', *Int. J. Approx. Reasoning*, **58**, 87–111, (2015).

[6] Phan Minh Dung, 'On the acceptability of arguments and its fundamental role in nonmonotonic reasoning, logic programming and n-person games', *Artif. Intell.*, **77**(2), 321–358, (1995).

[7] Wolfgang Dvořák, Anna Rapberger, and Stefan Woltran, 'On the relation between claim-augmented argumentation frameworks and collective attacks.', in *Proc. ECAI*, (2020). To appear - available at www.dbai.tuwien.ac.at/research/report/dbai-tr-2020-118.pdf.

[8] Wolfgang Dvořák and Stefan Woltran, 'Complexity of abstract argumentation under a claim-centric view', in *The Thirty-Third AAAI Conference on Artificial Intelligence, AAAI 2019, The Thirty-First Innovative Applications of Artificial Intelligence Conference, IAAI 2019, The Ninth AAAI Symposium on Educational Advances in Artificial Intelligence, EAAI 2019, Honolulu, Hawaii, USA, January 27 - February 1, 2019.*, pp. 2801–2808. AAAI Press, (2019).

[9] Thomas Eiter, Nicola Leone, and Domenico Saccà, 'On the partial semantics for disjunctive deductive databases', *Ann. Math. Artif. Intell.*, **19**(1-2), 59–96, (1997).

[10] Nikos Gorogiannis and Anthony Hunter, 'Instantiating abstract argumentation with classical logic arguments: Postulates and properties', *Artif. Intell.*, **175**(9-10), 1479–1497, (2011).

[11] Henry Prakken, 'An abstract framework for argumentation with structured arguments', *Argument and Computation*, **1**(2), 93–124, (2010).

[12] Bart Verheij, 'Two approaches to dialectical argumentation: admissible sets and argumentation stages', in *Proceedings of the 8th Dutch Conference on Artificial Intelligence (NAIC'96)*, eds., John Jules C. Meyer and Linda C. van der Gaag, pp. 357–368, (1996).

[13] Yining Wu, Martin Caminada, and Dov M. Gabbay, 'Complete extensions in argumentation coincide with 3-valued stable models in logic programming', *Studia Logica*, **93**(2-3), 383–403, (2009).