On the Relation between Claim-augmented Argumentation Frameworks and Collective Attacks

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Abstract. Dung's abstract argumentation frameworks (AF) are a popular conceptual tool to define semantics for advanced argumentation formalisms. Hereby, arguments representing a possible inference of a claim are constructed and an attack relation between arguments indicates certain conflicts between the claim of one argument and the inference of another. Based on this abstract model, sets of jointly acceptable arguments are then gathered and finally interpreted in terms of their claims. Argumentation formalisms following this type of instantiating Dung AFs naturally produce several arguments with the same claim. This causes several issues and challenges for argumentation systems: on the one hand, the relation between claims remains implicit and, on the other hand, determining the acceptance of claims requires additional computations on top of argument acceptance. An instantiation that avoids this situation could provide additional insights and advantages, thus complementing the standard instantiation process via Dung AFs. Consequently, the research question we tackle is as follows: Can one combine different arguments sharing the same claim to a single abstract argument without affecting the overall results (and which abstract formalisms can serve such a purpose)? As a main result we show that a certain class of frameworks, where arguments with the same claim have the same outgoing attacks, can be equivalently (for all standard semantics) represented as argumentation frameworks with collective attacks where each claim occurs in exactly one argument. We further identify a class of frameworks where one even obtains an equivalent Dung AF with just one argument per claim.

1 Introduction

The formal analysis of human reasoning facing uncertain information and conflicting beliefs is an important research area within AI. Abstract argumentation, as introduced by Dung [16], has been established as an important tool to analyze and evaluate the structure of argumentation systems by treating arguments as abstract entities. Depending on the particular task, various instantiation processes are used to model discourses, medical and legal cases [4], but also logic programs and non-monotonic reasoning formalisms [16, 13].

The general schema is often referred to as the argumentation pipeline and involves: (1) instantiation of a problem (which is given in terms of a knowledge base) into an abstract argumentation framework (AF); (2) application of semantics yielding sets of collectively acceptable arguments (the extensions of the AF); (3) re-interpretation of the extensions in terms of the original problem. Different instantiation processes have been established, see e.g. [22, 26, 13]. They all have in common that each generated argument stands for a statement

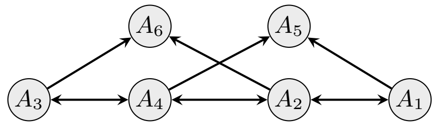

Figure 1: Resulting AF from Example 1.

(*claim*) that follows from elements of the initial knowledge base. Step (3) hence usually consists of inspecting the claims of the arguments occurring in the extensions; for instance, one might ask whether a particular claim is justified in each extension of the AF which is the case if each extension contains an argument featuring that claim.

Example 1. *We consider an instantiation procedure using ASPIC*$^+$ *[26]. Let* $\mathcal{K}_p = \{b, \overline{b}, c, \overline{c}\}$ *be the set of premises,* $\mathcal{K}_s = \{\overline{b} \to a, \overline{c} \to a\}$ *be the set of strict rules and let the pairs* $(b, \overline{b})$*,* $(c, \overline{c})$*,* $(\overline{b}, \overline{c})$ *be contradictory. We instantiate this knowledge base into a Dung AF which consists of abstract arguments and an attack from argument* A *to argument* B *is present if the claim of* A *is contradictory to the support of* B*. The arguments are listed in the table below and the resulting Dung AF is depicted in Figure 1.*

Argument	A_1	A_2	A_3	A_4	A_5	A_6
Structure	b	$\overline{b}$	c	$\overline{c}$	$A_2 \to a$	$A_4 \to a$
Claim	b	$\overline{b}$	c	$\overline{c}$	a	a

The evaluation of this AF under stable semantics[2] *yields the sets* $\{A_1, A_3\}$*,* $\{A_2, A_3, A_5\}$ *and* $\{A_1, A_4, A_6\}$*. The re-interpretation in terms of claims gives us the sets* $\{b, c\}$*,* $\{a, \overline{b}, c\}$ *and* $\{a, b, \overline{c}\}$*. The arguments* A_5 *and* A_6 *both refer to the same claim* a *and while the second and third extensions are disjoint on their arguments they are not on their claims. That is, multiple arguments with the same claim can lead to subtle differences in reasoning about claim acceptance. It is thus a natural question whether or not we can avoid multiple occurrences of claims in the abstract arguments.*

In this paper we are interested under which conditions an instantiation procedure can be modified such that it results in an abstract representation that requires only one argument per claim and that is equivalent to the standard attack-model. Some clarifications are in order: (1) When we talk about *abstract representations*, we mean any formalism that abstracts away from the contents of the arguments, and allows for an evaluation that is solely depending on the relations between arguments. We have already seen the simple Dung AFs at work. A generalization which we use in this paper are SETAFs (AFs with collective attacks [25]), where arguments can be jointly attacked

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[2] A set S of arguments is stable if it attacks exactly those arguments which do not belong to S.

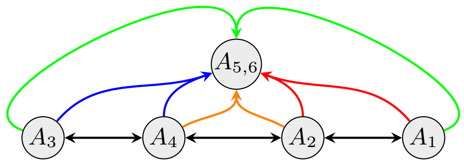

Figure 2: Example 1 expressed as SETAF.

by other arguments. (2) The term *equivalent* refers to the fact that two abstractions deliver the same result when their extensions are interpreted in terms of their claims.

While undoubtedly there are good reasons to keep several arguments with the same claim in the abstract representation (in particular, to preserve the different lines of support for a specific claim), there are also advantages of having only one argument per claim on the abstract level:

- In a representation with only one argument per claim we have an explicit representation of the conflict relation between the claims, which remains implicit in the standard case.
- In abstract representations with several arguments per claim we need to add an additional layer in argumentation systems to obtain the acceptance status of the claims from the acceptance status of the arguments. This increases not only the complexity of argumentation systems in practice; recent results also show higher computational complexity of certain tasks [20] compared to the standard acceptance problems on the argument level.
- When allowing for several arguments with same claim, the number of arguments might be exponentially larger than in abstract representations with only one argument per claim, e.g. in the instantiations for LPs discussed below. Again, this might have significant impact on computational aspects.

To exemplify the research question, we show in Figure 2 a SETAF that fulfills above criteria for Example 1. Hereby, arguments A_1 to A_4 have the same claims as the Dung AF; argument $A_{5,6}$ combines the two arguments A_5 and A_6 with claim a. A collective attack $(\{A_1, A_2\}, A_{5,6})$ is interpreted as neither A_1 nor A_2 attacks $A_{5,6}$ but when both are considered together they jointly attack $A_{5,6}$. It can be shown that the stable extensions of this SETAF are equivalent to the stable extensions of the Dung AF in Example 1 when interpreted in terms of claims but duplicate claims are avoided. Notice, that this cannot by accomplished with Dung AFs as one would need an AF such that the stable extensions are given by the sets $\{b, c\}$, $\{a, \bar{b}, c\}$ and $\{a, b, \bar{c}\}$. However, such an AF does not exist as follows from results on expressiveness of AF semantics by Dunne *et al.* [17]. Notice, that the attack $(\{A_1, A_3\}, A_{5,6})$ in Figure 2 reflects the fact that the three statements $\{a, b, c\}$ are in conflict although there is no pairwise conflict. Also observe that this relation remains implicit in Figure 1.

Similar investigations have been undertaken for concrete instantiation procedures of abstract argumentation, in particular, instantiations for logic programs (LPs) into Dung AFs have been studied in the literature [13]. Recently, Alcantara *et al.* [1] proposed an equivalent (under certain semantics) instantiation procedure that results in a specific class of ADFs (which in the light of [24] can also be interpreted as SETAFs) that avoids duplicates of claims.

In order to base our studies independently from concrete instantiations, we utilize the concept of *claim-augmented argumentation frameworks*, CAFs for short, as introduced by Dvořák and Woltran [20]. CAFs are AFs where arguments are associated with claims and semantics deliver those sets of claims attached to the arguments in the extensions under standard AF semantics. CAFs provide a natural intermediate layer between structured and purely abstract argumentation formalisms, since they carry the necessary information to compute the extensions and at the same time re-interpret them in terms of the instantiated problem. This is exactly the level of detail we require for our investigations. Our research question now amounts to the question of translating CAFs into purely abstract formalisms such that duplicate claims are avoided, hence allowing for a one-to-one correspondence between argument names and their claims. We will focus on the most common semantics, namely preferred, stable, complete, admissible and grounded semantics [16]. A crucial notion for our purpose relies on the following restriction on the attack relation: A CAF is *well-formed* if arguments with the same claim attack the same arguments (the AF from Example 1 is indeed well-formed). This constraint naturally occurs in instantiation processes: attacks are usually generated under consideration of the claim of the attacker.[3]

Translating CAFs into an abstract representation without an additional claim layer allows to investigate the capabilities of abstract argumentation formalisms from different viewpoints. (1) Several approaches to extend Dung AFs have been discussed in the literature. However, the question on their advantages in the instantiation process has often been neglected. Our research targets this question by investigating whether such generalizations are capable of representing scenarios not directly expressible via Dung AFs. Our main focus here will be on SETAFs, which have received significant interest in the last years [21, 31, 18]. (2) From a theoretical perspective, we study under which circumstances different arguments $S_1 : c, \ldots, S_n : c$ (with the same claim but different support) and their relation to other arguments can be combined into a single argument $\bigvee_i S_i : c$ such that this amalgamation can still be represented on the abstract level.

The main results of our paper are:

- We show that well-formed CAFs can be equivalently expressed via SETAFs where the argument names in the SETAF refer to the claims in the CAF, thus providing a single argument for each claim (in fact, we show that well-formed CAFs and SETAFs are equally powerful w.r.t. the semantics under consideration). This result demonstrates that *all* instantiation procedures that result in Dung structures where the arguments' claims imply well-formedness can be defined in terms of SETAFs where each argument stands for a different claim.
- We complement the above results by showing that each SETAF can be equivalently represented by a well-formed CAF. It thus follows that SETAFs and well-formed CAFs have the same expressiveness.
- We strengthen the result for well-formed CAFs by characterizing a class of CAFs, so-called attack-unitary CAFs, that, under admissibility-based semantics, can be transformed to an equivalent standard Dung AF.
- We give a rewriting technique for CAFs in order to reduce the number of arguments with the same claim, even in case when a full translation to SETAFs or AFs is not possible.
- Finally, we show that for the classes of well-formed CAFs and attack-unitary CAFs only one attack violating the respective property can lead to a situation where a translation to SETAFs (resp. AFs) is not possible anymore.

Some technical details are omitted due to space constraints. A version with full proofs is available at `www.dbai.tuwien.ac.at/research/argumentation/ecai2020-full.pdf`.

[3] Exceptions are instantiation procedures which allow rule and claim preferences (cf. ASPIC$^+$).

2 Preliminaries

In this section, we first introduce standard abstract argumentation frameworks [16] and recall the semantics we study (for a comprehensive introduction, see [5]). We then introduce their generalization to collective attacks.

Dung AFs. We start with some basic definitions.

Definition 1. *A* Dung argumentation framework (AF) *is a pair $F = (A, R)$ where A is a finite set of arguments and $R \subseteq A \times A$ is the attack relation. The pair $(a, b) \in R$ means that a attacks b. Given an argument a, we use $a_R^+ = \{b \mid (a, b) \in R\}$ and $a_R^- = \{b \mid (b, a) \in R\}$; we extend both notions to sets S as expected: $S_R^+ = \bigcup_{a \in S} a_R^+$, $S_R^- = \bigcup_{a \in S} a_R^-$. We say that an argument $a \in A$ is* defended *(in F) by $S \subseteq A$ if $a_R^- \subseteq S_R^+$.*

Semantics for AFs are defined as functions σ which assign to each AF $F = (A, R)$ a set $\sigma(F) \subseteq 2^A$ of extensions. We consider for σ the functions cf, adm, com, grd, stb, and prf, which stand for conflict-free, admissible, complete, grounded, stable, and preferred extensions, respectively.

Definition 2. *Let $F = (A, R)$ be an AF. A set $S \subseteq A$ is* conflict-free (in F)*, if there are no $a, b \in S$, such that $(a, b) \in R$. $cf(F)$ denotes the collection of sets being conflict-free in F. For a conflict-free set $S \in cf(F)$, it holds that*

- *$S \in adm(F)$, if each $a \in S$ is defended by S in F;*
- *$S \in com(F)$, if $S \in adm(F)$ and each $a \in A$ defended by S in F is contained in S;*
- *$S \in grd(F)$, if $S = \bigcap_{T \in com(F)} T$;*
- *$S \in stb(F)$, if each $a \in A \setminus S$ is attacked by S in F;*
- *$S \in prf(F)$, if $S \in adm(F)$ and there is no $T \supset S$ such that $T \in adm(F)$.*

We recall that for each AF F, $stb(F) \subseteq prf(F) \subseteq com(F) \subseteq adm(F)$, and $grd(F)$ yields a unique extension. Moreover, semantics $\sigma \in \{stb, prf\}$ deliver *incomparable* sets, i.e. for all $S, T \in \sigma(F)$, $S \subseteq T$ implies $S = T$.

SETAFs and Collective Attacks. SETAFs, as introduced by Nielsen and Parsons [25], generalize the binary attack-relation in AFs to collective attacks of arguments. The formalism captures situations in which a single argument might be too weak to attack more powerful statements.

Definition 3. *A SETAF is a pair $SF = (A, R)$ where A finite, and $R \subseteq (2^A \setminus \{\emptyset\}) \times A$ is the attack relation.*

Given a SETAF $SF = (A, R)$, then $S \subseteq A$ *attacks* a if there is a set $S' \subseteq S$ with $(S', a) \in R$. S is *conflicting* in SF if S attacks some $a \in S$; S is *conflict-free* in SF, if S is not conflicting in SF, i.e. $S' \cup \{a\} \not\subseteq S$ for each $(S', a) \in R$. Finally, $a \in A$ is *defended* by S if for each set $B \subseteq A$ with $(B, a) \in R$, there is some $b \in B$ such that S attacks b. With these extended notions of conflict and defense at hand, the semantics of AFs generalize to SETAFs as follows.

Definition 4. *Given a SETAF $SF = (A, R)$, we denote the set of all conflict-free sets in SF as $cf_s(SF)$. For $S \in cf_s(SF)$, it holds that*

- *$S \in adm_s(SF)$ if each $a \in S$ is defended by S in SF;*
- *$S \in com_s(SF)$, if $S \in adm_s(SF)$ and $a \in S$ for all $a \in A$ defended by S in SF;*
- *$S \in grd_s(SF)$, if $S = \bigcap_{T \in com_s(SF)} T$;*
- *$S \in stb_s(SF)$, if each $a \in A \setminus S$ is attacked by S in SF;*
- *$S \in prf_s(SF)$, if $S \in adm_s(SF)$ and there is no $T \supset S$ such that $T \in adm_s(SF)$.*

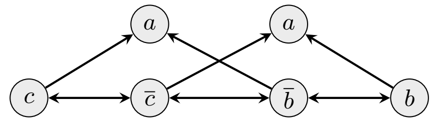

Figure 3: The AF F from Example 1 as CAF; here, claims are depicted instead of argument names.

We introduce attack formulas as an alternative formalisation of the attack-structure in SETAFs.

Definition 5. *For any SETAF $SF = (A, R)$ and $a \in A$, let*

$$\mathcal{D}_a^{SF} = \bigvee_{B \subseteq A, (B,a) \in R} \quad \bigwedge_{b \in B} b$$

denote the attack-formula *of a in SF.*

For each $s \in A$, the models of the attack-formula $\mathcal{D}_s^{SF}$ coincide with the sets $S \subseteq A$ such that S attacks s in SF. Using this identity, the semantics for SETAFs can be rephrased in terms of attack-formulas. For example, a set S is stable in SF, if for each $s \in A$, we have that $s \in S$ if and only if $\delta \not\subseteq S$ for all $\delta \in \mathcal{D}_s^{SF}$ (following standard conventions, we will occasionally identify formulas in CNF or DNF as a collection of sets of literals; in our case, atoms).

3 Argumentation Frameworks with Claims

In order to abstract from a concrete instantiation we use an augmentation of Dung AFs that allows for a uniform representation of the outcome of a wide variety of instantiation procedures. That is we consider *claim-augmented argumentation frameworks* (CAFs) as recently introduced by Dvořák and Woltran [20]. The idea is to assign each argument in an AF a claim, i.e. an element from a countable infinite domain of claims C. Hence, different arguments can have the same claim, but no further information about claims is available. Notice that CAFs provide exactly the necessary information to compute the extensions and re-interpret them in terms of the instantiated problem.

Definition 6. *A* Claim-augmented Argumentation Framework (CAF) *is a triple $(A, R, claim)$ where (A, R) is an AF and $claim : A \to C$ assigns a claim to each argument of A.*

Figure 3 shows the CAF for Example 1. Notice that CAFs cannot only model the outcome of ASPIC style instantiations but is also applicable to e.g. instantiations from logic programming [20] and assumption-based argumentation (ABA).

Semantics of CAFs are defined from the standard semantics of the underlying AF, but interpret the extensions in terms of the claims of their arguments. To this end, we extend the claim function to sets, i.e. $claim(S) = \{claim(s) \mid s \in S\}$.

Definition 7. *For a semantics σ, we define its claim-based variant σ_c as follows. For any CAF $CF = (A, R, claim)$, $\sigma_c(CF) = \{claim(S) \mid S \in \sigma((A, R))\}$. Given $S \in \sigma_c(CF)$, we say that $E \subseteq A$ is a σ-*realization *of S in CF if $claim(E) = S$ and $E \in \sigma((A, R))$.*

Example 2. *Let $CF = (A, R, claim)$ be given with (A, R) as depicted in Figure 4, and $claim(x_i) = x$, $claim(y_i) = y$ for $i = 1, 2$. We have $com_c(CF) = adm_c(CF) = \{\emptyset, \{x\}, \{x, y\}\}$, $grd_c(CF) = \{\emptyset\}$, and $stb_c(CF) = prf_c(CF) = \{\{x\}, \{x, y\}\}$. Note that $\{x\} \in adm_c(CF)$ has two adm-realizations, namely $E_1 = \{x_1\}$ and $E_2 = \{x_2\}$.*

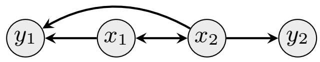

Figure 4: The AF from Example 2.

Some basic relations between different semantics carry over from standard AFs. In fact, we have for any CAF CF

$$stb_c(CF) \subseteq prf_c(CF) \subseteq com_c(CF) \subseteq adm_c(CF) \qquad (1)$$

and $grd_c(CF)$ is unique and contained in $com_c(CF)$. Moreover, the claim-based grounded extension $S \in grd_c(CF)$ is still the unique minimal claim-based complete extension.

As Example 2 shows, claim-based semantics loose some basic properties, for instance incomparability of stable and preferred extensions. However, this can be circumvented with a particular subclass called *well-formed CAFs* which has been defined in [20]; a similar property has been studied in [22, 3].

Definition 8. *A CAF $(A, R, claim)$ is called* well-formed *if $a^+_R = b^+_R$ for any $a, b \in A$ with $claim(a) = claim(b)$, i.e. arguments with the same claim attack the same arguments.*

Proposition 1. *For any well-formed CAF CF, $S \in prf_c(CF)$ iff $S \in com_c(CF)$ and there is no $T \in com_c(CF)$ with $T \supset S$.*

Proof. Consider a well-formed CAF $CF = (A, R, claim)$. We show that for $D, E \in com((A, R))$, $D \subseteq E$ iff $claim(D) \subseteq claim(E)$. The assertion then follows immediately. As $claim(.)$ is monotone we have that $D \subseteq E$ implies $claim(D) \subseteq claim(E)$. We next show the converse, i.e. that $claim(D) \subseteq claim(E)$ implies $D \subseteq E$. As CF is well-formed, $claim(D) \subseteq claim(E)$ implies $D^+_R \subseteq E^+_R$. That is all arguments defended by D in (A, R) are also defended by E in (A, R). Finally as D defends all its arguments and E contains all arguments it defends we have $D \subseteq E$. □

Notice that the above result generalises similar observations for instantiations from logic programming [13] and ABA [12], which both result in well-formed CAFs. The following statement follows from Proposition 1 and (1).

Proposition 2. *For $\sigma \in \{stb, prf\}$ and every well-formed CAF $CF = (A, R, claim)$, we have (a) $\sigma_c(CF)$ is incomparable, and (b) $|\sigma((A, R))| = |\sigma_c(CF)|$.*

4 Well-formed CAFs and SETAFs

Our aim is to translate CAFs to argumentation frameworks such that each claim that appears in the CAF corresponds to exactly one argument in the argumentation framework. Thus we do not allow additional auxiliary arguments that lack a counterpart in their claims with respect to the original domain. The two main results of this section will show that CAFs and SETAFs can be translated to each other while preserving the outcome in terms of claims. This shows that in the abstract representation of an instantiation procedure one can trade multiple claims for collective attacks and vice versa.

4.1 Expressing Well-formed CAFs as SETAFs

We show that each well-formed CAF can be reduced to an equivalent SETAF by identifying claims in well-formed CAFs with arguments in SETAFs. In well-formed CAFs, arguments with the same claim are indistinguishable in terms of their outgoing attacks. Hence, in contrast to general CAFs, one can speak about claims attacking arguments. We will introduce attack formulas for each claim c. These formulas intuitively capture all possible sets of claims which jointly contradict each occurrence of claim c.

Definition 9. *Given a well-formed CAF $CF = (A, R, claim)$, then for each claim $c \in claim(A)$, the* CNF-attack-formula of c in CF *is defined as*

$$\mathcal{CD}^{CF}_c = \bigwedge_{a \in A, claim(a)=c} \quad \bigvee_{(x,a) \in R} claim(x).$$

$\mathcal{D}^{CF}_c$ denotes any equivalent DNF-formula over the same set of variables and is called DNF-attack-formula of c in CF.

Note that the attack-formula $\mathcal{CD}^{CF}_c$ is unsatisfiable iff there exists an argument x in CF with $claim(x) = c$ such that $x^-_R = \emptyset$.

Similarly to SETAFs, attack formulas allow for an exact characterization of well-formed CAFs, i.e. each well-formed CAF $CF = (A, R, claim)$ is uniquely determined (modulo argument names) via its attack formulas $\mathcal{CD}^{CF}_c$.

Example 3. *Consider the following CNF-attack formulas:*

$$\begin{array}{lll} \mathcal{CD}^{CF}_a & = & (c \lor \overline{b}) \land (\overline{c} \lor b) \\ \mathcal{CD}^{CF}_b & = & \overline{b} \qquad \mathcal{CD}^{CF}_c \ = \ \overline{c} \\ \mathcal{CD}^{CF}_{\overline{b}} & = & b \lor \overline{c} \quad\ \ \mathcal{CD}^{CF}_{\overline{c}} \ = \ \overline{b} \lor c \end{array}$$

The two conjuncts of $\mathcal{CD}^{CF}_a$ determine that we have two arguments with claim a, while the remaining claims appear only once. The disjunctions refer to the attackers of the arguments. In fact, we obtain the CAF as depicted in Figure 3.

We introduce the translation $\mathtt{T}_{cts}$, which maps each well-formed CAF CF to a corresponding SETAF $\mathtt{T}_{cts}(CF)$. Each claim c in the original framework CF corresponds to an argument in $\mathtt{T}_{cts}(CF)$, furthermore we identify each disjunct of the DNF-attack-formula $\mathcal{D}^{CF}_c$ with a collective attack against c. Consequently, the formula $\mathcal{D}^{CF}_c$ coincides with the attack-formula $\mathcal{D}^{\mathtt{T}_{cts}(CF)}_c$ of the resulting SETAF $\mathtt{T}_{cts}(CF)$. Therefore, the SETAF $\mathtt{T}_{cts}(CF)$ solely depends on the DNF-attack-formulas $\mathcal{D}^{CF}_c$.

Translation 1. *For a well-formed CAF $CF = (A, R, claim)$ we define $\mathtt{T}_{cts}(CF) = (A', R')$ with $A' = claim(A)$ and*

$$R' = \{(\delta, c) \mid c \in A', \delta \in \mathcal{D}^{CF}_c\}.$$

Example 4. *In Example 3 we have already provided the attack formulas for the CAF CF depicted in Figure 3. The attack formula $\mathcal{CD}^{CF}_a$ for claim a in DNF representation yields $\mathcal{D}^{CF}_a = (c \land \overline{c}) \lor (c \land b) \lor (\overline{b} \land \overline{c}) \lor (\overline{b} \land b)$. The attack formulas for the remaining claims are readily given in DNF. Applying Translation 1 yields the SETAF given in Figure 5: for a, we need a collective attack for each disjunct in $\mathcal{D}^{CF}_a$; for the remaining arguments, each disjunct contains one atom, thus the incoming attacks remain binary.*

Notice that the translation links multiple occurrences of a claim with collective attacks on a corresponding single argument. Therefore, it interlinks claim-based extensions of CAFs with extensions

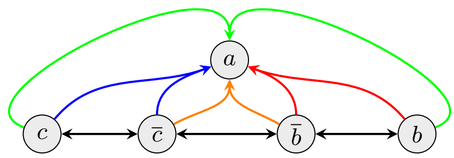

Figure 5: SETAF $\mathtt{T}_{cts}(CF)$ from Example 4.

(on the argument level) of SETAFs. As we show next, this is performed in a faithful way, that is, the reviewed semantics of well-formed CAFs can be reduced to their counterparts in SETAFs.

Theorem 1. *For each well-formed CAF CF and* $\sigma \in \{cf, adm, com, grd, prf, stb\}$*, Translation 1 provides an equivalent SETAF* $\mathtt{T}_{cts}(CF)$ *such that* $\sigma_c(CF) = \sigma_s(\mathtt{T}_{cts}(CF))$.

Proof (for $\sigma \in \{cf, stb\}$*).* First, let $\sigma = cf$. We rephrase conflict-freeness in terms of CNF- resp. DNF-attack-formulas. In well-formed CAFs, $S \in cf_c(CF)$ iff for each $s \in S$ there is an $a \in A$ with $claim(a) = s$ such that a is not attacked by any argument b with $claim(b) \in S$. Note that, for any $s \in claim(A)$, $\mathcal{CD}_s^{CF}$ identifies each clause with the set of attacking claims for a particular occurrence of s in CF. That is, $S \in cf_c(CF)$ iff for each $s \in S$,

$$\textit{there is some } \gamma \in \mathcal{CD}_s^{CF} \textit{ such that } \gamma \cap S = \emptyset. \tag{C1}$$

In a SETAF $SF = (A', R')$, a set $S \subseteq A'$ is conflict-free iff for all $S' \subseteq S$ and all $s \in S$, $(S', s) \notin R'$. In terms of attack formulas, $S \in cf_s(SF)$ iff for each $s \in S$, it holds that

$$\textit{for all } \delta \in \mathcal{D}_s^{SF} \textit{ we have } \delta \not\subseteq S. \tag{C2}$$

Now let $SF = \mathtt{T}_{cts}(CF) = (A', R')$. We have (i) $\mathcal{D}_s^{SF} = \mathcal{D}_s^{CF}$ for each $s \in A'$ by construction, and (ii) that no $\delta \in \mathcal{D}_s^{CF}$ is a subset of S iff there exists $\gamma \in \mathcal{CD}_c^{CF}$ such that $\gamma \cap S = \emptyset$, and thus we obtain that (C1) is equivalent to (C2), hence the statement follows.

Let $\sigma = stb$. A set S is stable on claim-level in CF if for each $s \in claim(A)$, it holds that $s \in S$ *if and only if* (C1). Similarly, S is stable in $\mathtt{T}_{cts}(CF)$ if for each $s \in A' = claim(A)$, it holds that $s \in S$ if and only if (C2). Again, the statement follows by the equivalence of (C2) and (C1). □

The result shows that multiple occurrences of claims in CAFs can be equivalently treated as collective attacks, if the framework satisfies well-formedness. Indeed, in our running example (cf. Figure 5), the stable extensions $stb_s(\mathtt{T}_{cts}(CF))$ are given by the sets $\{a, \overline{b}, c\}$, $\{a, b, \overline{c}\}$ and $\{b, c\}$.

A natural question that arises is about the contents the arguments in the resulting SETAF are representing with respect to an initial instantiation. To this end, let us have one more look on the example from the introduction, where argument A_5 for claim a builds on a support consisting of $\overline{b}$ and rule $\overline{b} \to a$ while argument A_6 has support $\overline{c}; \overline{c} \to a$. The combined argument $A_{5,6}$ (see Figure 2) thus can be interpreted as argument for a with *disjunctive* support $(\overline{b};\ \overline{b} \to a) \vee (\overline{c};\ \overline{c} \to a)$. Now, in order to attack this argument we need to find combinations of arguments that are contradictory to each of the disjuncts. Recall that the contradictory relation is given by $\{(b, \overline{b}), (c, \overline{c}), (\overline{b}, \overline{c})\}$ and arguments A_1, A_2, A_3, A_4 have as their respective claims b, $\overline{b}$, c, $\overline{c}$. We thus need together either A_1 and A_3, A_1 and A_4, A_2 and A_3, or A_2 and A_4 to attack $A_{5,6}$. This is exactly the collective attack structure in Figure 2 and likewise the result of our translation which delivers this combination by applying the logical rule of distributivity to the CNF-attack-formula $\mathcal{CD}_a^{CF}$ of claim a in order to obtain the corresponding attack-formula $\mathcal{D}_a^{SF}$ of argument a for constructing the SETAF.[4]

4.2 Expressing SETAFs as well-formed CAFs

Our next result shows that it is equally possible to map each SETAF to a well-formed CAF while preserving the reviewed semantics. That is, we show that well-formed CAFs and and SETAF have the same expressiveness and one can easily translate between them.

We will provide a translation $\mathtt{T}_{stc}$ which maps each SETAF SF to an equivalent well-formed CAF using attack formulas. Each argument a will correspond to a claim in the resulting CAF; furthermore, we introduce for each clause γ in the attack formula $\mathcal{CD}_a^{SF}$ an argument a_γ labeled with claim a. The clause γ also determines the set of attackers of the argument a_γ.

Translation 2. *For each SETAF SF* $= (A', R')$*, we define* $\mathtt{T}_{stc}(SF) = (A, R, claim)$ *as follows:*

$$A = \{a_\gamma \mid a \in A', \gamma \in \mathcal{CD}_a^{SF}\} \cup \{a_\emptyset \mid a \in A', \mathcal{CD}_a^{SF} = \emptyset\},$$
$$R = \{(x, a_\gamma) \mid a \in A', \gamma \in \mathcal{CD}_a^{SF}, claim(x) \in \gamma\},$$
$$claim(a_\gamma) = a.$$

Example 5. *Let SF* $= (A', R')$ *be a SETAF as in Figure 5. In order to apply the translation* $\mathtt{T}_{stc}$ *we first compute the attack formulas of the arguments in SF. For instance, the attack formula* $\mathcal{D}_a^{SF}$ *for the argument* $a \in A'$ *is given by* $(c \wedge \overline{c}) \vee (b \wedge \overline{b}) \vee (\overline{b} \wedge \overline{c}) \vee (b \wedge c)$ *where every disjunct represents a set which attacks a. The corresponding CNF attack formula is* $\mathcal{CD}_a^{SF} = (c \vee \overline{b}) \wedge (\overline{c} \vee b)$*. For the remaining arguments, the CNF and DNF representation coincides. Once every attack formula is constructed, one can apply Translation 2, where the arguments in the SETAF SF correspond to the claims in the CAF* $\mathtt{T}_{stc}(SF)$*: For a, we introduce two arguments* $a_{c\vee\overline{b}}$, $a_{\overline{c}\vee b}$ *with* $claim(a_{c\vee\overline{b}}) = claim(a_{\overline{c}\vee b}) = a$*; the attacks are determined by the claims which appear in the clauses, e.g. every argument x with* $claim(x) = c$ *attacks the argument* $a_{c\vee\overline{b}}$*. The resulting CAF* $\mathtt{T}_{stc}(SF)$ *corresponds to the CAF depicted in Figure 3.*

By applying similar techniques as in the proof of Theorem 1 it can be shown that the reviewed semantics for SETAFs correspond to their counterparts in well-formed CAFs.

Theorem 2. *For each SETAF SF and* $\sigma \in \{cf, adm, com, grd, prf, stb\}$*, Translation 2 provides an equivalent CAF* $\mathtt{T}_{stc}(SF)$ *such that* $\sigma_s(SF) = \sigma_c(\mathtt{T}_{stc}(SF))$.

Thus by the Theorems 1 and 2, we conclude that well-formed CAFs and SETAFs are equally powerful with respect to the semantics under consideration.

Corollary 1. *Let* $\sigma \in \{cf, adm, com, grd, prf, stb\}$*. For any well-formed CAF CF, there is a SETAF SF such that* $\sigma_c(CF) = \sigma_s(SF)$*, and vice versa.*

Given the exact characterizations of the expressiveness of semantics of SETAFs in [18] the above translation results immediately provide exact characterizations of the expressiveness of the respective semantics of well-formed CAFs.

[4] Recall that in our translation we use in the SETAF claims anonymously as argument names.

5 Classes of CAFs Expressible as AFs

In this section we investigate classes of CAFs that can be directly expressed by AFs. Prominently, one can show that for well-formed CAFs $(A, R, claim)$ where the attack relation R is symmetric there is an equivalent Dung AF. This type of CAFs naturally occurs in instantiations with only rebutting attacks (see [29] for a discussion of different types of attacks) which is reflected by work on symmetric AFs [15]. However, as we show next, a weaker condition on the attack structure is already sufficient.

Definition 10. *A CAF $(A, R, claim)$ is called* attacker-unitary (att-unitary) *if, for any $a, b \in A$ with $claim(a) = claim(b)$, it holds that $a^-_R = b^-_R$, i.e. arguments with the same claim are attacked by the same arguments.*

The crucial property is that in att-unitary CAFs, a set of arguments E defends either all or no occurrences of a claim c. Thus, for admissible-based semantics, each claim-extension S can be realized by a maximal representative. That is, the maximal representative of a claim-extension S is given by the set $E_S^{\max}$ containing all arguments with a claim from S.

Lemma 1. *Let $CF = (A, R, claim)$ be att-unitary and let $S \in \sigma_c(CF)$ for $\sigma \in \{adm, com, grd, prf, stb\}$. Then $E_S^{\max} = \{x \in A \mid claim(x) \in S\} \in \sigma((A, R))$.*

Next, we present a translation $\mathtt{T}_{cta}$ from att-unitary CAFs to AFs. Given CAF $CF = (A, R, claim)$, each claim $c \in claim(A)$ is mapped to a single argument c in the resulting AF $\mathtt{T}_{cta}(CF)$, wherein c attacks d if at least one argument with claim c attacks the arguments with claim d in (A, R).

Translation 3. *For an att-unitary CAF $CF = (A,R,claim)$, we define $\mathtt{T}_{cta}(CF) = (claim(A), R')$ with*

$$R' = \{(claim(x), claim(y)) \mid (x, y) \in R\}.$$

Example 6. *Let $CF' = (A, R, claim)$ be an att-unitary CAF, with $A = \{x_1, y_1, x_2\}$, $R = \{(x_1, y_1), (y_1, x_1), (y_1, x_2)\}$ and $claim(x_1) = claim(x_2) = x$, $claim(y_1) = y$. The AF $\mathtt{T}_{cta}(CF')$ constructed by Translation 3 is given by $(\{x, y\}, \{(x, y), (y, x)\})$ and we have $stb_c(CF') = stb_s(\mathtt{T}_{cta}(CF')) = \{\{x\}, \{y\}\}$.*

We next show that CF and $\mathtt{T}_{cta}(CF)$ are equivalent for the admissible-based semantics under our considerations.

Theorem 3. *For each att-unitary CAF CF and $\sigma \in \{adm, com, grd, prf, stb\}$, Translation 3 provides an equivalent AF $\mathtt{T}_{cta}(CF)$ such that $\sigma_c(CF) = \sigma_s(\mathtt{T}_{cta}(CF))$.*

Proof (for $\sigma = stb$). Let $CF = (A, R, claim)$, $F = \mathtt{T}_{cta}(CF) = (A', R')$ and consider $S \in stb_c(CF)$. By Lemma 1, $E_S^{max} \in stb((A, R))$. Note that $S \in cf(F)$ by definition of R'. For each $c \in A' \setminus S$ there is an $x \in A \setminus E_S^{max}$ with $claim(x) = c$ and there is $y \in E_S^{max}$ such that $(y, x) \in R$. Consequently $(claim(y), claim(x)) \in R'$, and S attacks in F all arguments $a \in A' \setminus S$.

Now, let $S \in stb(F)$ and let $c \in A' \setminus S$. Then there is some argument $a \in S$ such that $(a, c) \in R'$. By definition of R', there are arguments $x, y \in A$, $claim(x) = a$, $claim(y) = c$, such that $(x, y) \in R$. By att-unitaryness, $(x, z) \in R$ for each $z \in A$ such that $claim(z) = c$. Hence each argument $y \in A \setminus E_S^{max}$ is attacked by E_S^{max}. □

Notice that Theorem 3 does not extend to cf semantics. For conflict-free semantics well-formed and att-unitary CAFs are intertranslateable, as the orientation of attacks is immaterial. Thus, under conflict-free semantics att-unitary CAFs correspond to SETAFs.

6 General CAFs

This section is concerned with CAFs neither satisfying well-formedness nor att-unitaryness. We show that in general one cannot avoid multiple occurrences of claims. However, we provide a result that allows to reduce the number of arguments having the same claim. On the one hand we show that there is no translation from general CAFs into SETAFs. Moreover, already a small deviation of the respective properties of being well-formed or att-unitary can prohibit a translation to SETAFs or AFs, respectively. On the other hand, we provide a result to reduce the number of arguments having the same claim, even in the case that a translation to a SETAF or an AF is impossible.

We first investigate whether the introduced translation $\mathtt{T}_{cts}$ from well-formed CAFs to SETAFs can be extended to arbitrary CAFs that only slightly deviate from well-formedness. The following example shows that this is not the case by providing an example where already removing one attack suffices to make a semantics-preserving representation as SETAF impossible.

Example 7. *Let $CF = (A, R, claim)$ be the well-formed CAF from Figure 6 with $A = \{x_1, x_2, y_1\}$, $R = \{(x_1, y_1), (y_1, x_1), (x_2, y_1)\}$ and $claim(x_1) = claim(x_2) = x$, $claim(y_1) = y$. Observe that in the CAF $CF_1 = (A, R', claim)$ which arises after removing the attack $a_1 = (x_2, y_1)$ (i.e. $R' = R \setminus \{a_1\}$), we have that $stb_c(CF_1) = \{\{x\}, \{x, y\}\}$. However, recent results on the expressiveness of semantics in SETAFs [18] show that there is no SETAF SF such that $\sigma(SF) = \sigma_c(CF_1)$.*

Example 7 shows that already a deviation of well-formedness by a single attack is sufficient to prohibit a faithful translation from CAFs to SETAFs. We provide a similar result for att-unitary CAFs: A semantics-preserving representation for general CAFs that deviate from att-unitaryness by only one attack is in general not possible.

Example 8. *We consider the att-unitary CAF $CF' = (A, R, claim)$ from Figure 6 with $A = \{x_1, x_2, y_1\}$, $R = \{(x_1, y_1), (y_1, x_1), (y_1, x_2)\}$ and $claim(x_1) = claim(x_2) = x$, $claim(y_1) = y$. The CAF $CF'_1 = (A, R', claim)$ which arises when removing the attack $b_1 = (y_1, x_2)$, i.e. $R' = R \setminus \{b_1\}$), possesses the stable extensions $stb_c(CF'_1) = \{\{x\}, \{x, y\}\}$. Thus by similar arguments as in Example 7, we can conclude that there is no AF F such that $\sigma(F) = \sigma_c(CF'_1)$.*

Although CAFs cannot be fully translated to SETAFs or AFs with unique claims in general, there is still potential to reduce the number of arguments having the same claim. Towards such a simplification procedure we introduce the concept of redundant arguments.

Definition 11. *Let $CF = (A, R, claim)$. An argument $a \in A$ is called* redundant *(in CF) w.r.t. argument $b \in A$ if $a \neq b$, $claim(a) = claim(b)$, $a^+_R = b^+_R$, and $a^-_R \supseteq b^-_R$.*

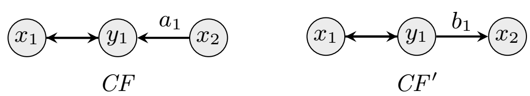

Figure 6: CAFs CF, CF' from Examples 7 and 8.

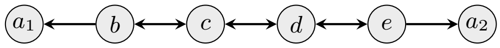

Figure 7: The AF from Example 9.

In the CAF CF from Example 7, the argument x_1 is redundant w.r.t. x_2: Indeed, we have that $x_1^+ = x_2^+ = \{y_1\}$ and $x_1^- = \{y_1\} \supseteq \emptyset = x_2^-$. Observe that the framework $CF' = (\{x_2, y_1\}, \{(x_2, y_1)\}, claim)$ where the redundant argument x_1 is deleted yields the same stable extension $\{x\}$; in fact, $\sigma_c(CF) = \sigma_c(CF')$ for every semantics σ under consideration. The following proposition states that, for an arbitrary CAF, redundant arguments do not influence the outcome of the reviewed semantics.

Proposition 3. *Let $CF = (A, R, claim)$, $a \in A$ be redundant in CF w.r.t. some $b \in A$, and let $CF' = (A', R', claim)$ with $A' = A \setminus \{a\}$, and $R' = R \cap (A' \times A')$. Then, for $\sigma \in \{cf, adm, com, grd, stb, prf\}$, $\sigma_c(CF) = \sigma_c(CF')$.*

It follows that redundant arguments can be safely removed without changing the semantics. One can apply the above proposition iteratively in order to obtain a CAF without redundant arguments. If the resulting CAF can be translated into a SETAF or an AF then also the original CAF can be translated into such a framework. Another consequence of Proposition 3 is that each well-formed CAF which violates att-unitaryness by only one attack can be translated to an equivalent AF. This shows that Translation 3 is indeed robust against minimal deviations if well-formedness is guaranteed. However, well-formed CAFs violating att-unitaryness to a slightly higher extent cannot, in general, be expressed as AFs, as illustrated next.

Example 9. *Let $CF = (A, R, claim)$ with (A, R) as depicted in Figure 7, $claim(a_1) = claim(a_2) = a$, and $claim(x) = x$ for $x \in \{b, c, d, e\}$. CF is well-formed and $stb_c(CF) = \{\{a, b, d\}, \{b, e\}, \{a, c, e\}\}$. Results on signatures for AFs [17] imply that there is no AF F such that $stb(F) = stb_c(CF)$. Observe that retaining att-unitaryness requires the deletion (or addition) of at least two attacks, e.g. removing (b, a_1) and (e, a_2) yields an att-unitary CAF.*

7 Discussion

Related Work. The work by Amgoud *et al.* [3] is probably closest to ours. They investigate equivalence in logic-based argumentation systems and study conditions under which arguments can be removed from a system without affecting its semantics. Their setting is more limiting as they require arguments to have both equivalent support and equivalent claims in order to remove one of them. The notion of CAFs is borrowed from Dvořák and Woltran [20] who introduce CAFs for a different purpose, namely to analyze the complexity of acceptance problems in terms of CAFs. Also related are semantics-preserving translations to standard AFs that have been investigated for several generalizations of AFs, e.g. [14, 8, 6]. In contrast to our main results where claims are mapped to arguments, all of these translations concern the argument level only. An exception is the work by Strass [28] on expressiveness of AFs compared to the expressiveness of logic programs and propositional logic, where arguments are mapped to propositional atoms. Furthermore, we mention studies [9, 22, 2] that analyze whether particular properties on the level of claims (rationality and consistency postulates) are fulfilled by AF extensions in certain instantiation scenarios. Moreover, Caminada and Oren [11] classified arguments that can be neglected when computing the grounded extension of infinitary AFs. Their classification is similar to our notion of redundant arguments but their results are limited to grounded semantics.

Finally, the idea of merging arguments with the same claim has been considered in a different context. Beirlaen *et al.* [7] extend ASPIC+ by a reasoning by cases inference scheme that allows to generate additional arguments; in [23], Heynick and Strasser model case-based reasoning by constructing independent assumption sets of the given arguments (i.e. disjunctive supports) in their argumentative approach on dynamic proof theories for defeasible reasoning. In contrast to our approach both studies do not consider collective attacks; moreover, the newly generated arguments add to the number of arguments featuring the same claim, that is, multiple arguments with the same claim are not avoided in those works.

Summary and Outlook. In this work we tackled the research question under which circumstances instantiation-based argumentation can be achieved in a way where the abstract representation does not require multiple arguments for the same claim. This alternative approach provides an orthogonal view (compared to the standard way of instantiation) that is centered on the relation between claims rather than on the relation between single arguments.

We based our investigations on claim-augmented AFs (CAFs), which reflect the actual structure of the standard instantiation into Dung AFs but carry information about the claims associated to the arguments. Our research question then translates into the question whether (certain sub-classes of) CAFs can be equivalently expressed in a purely abstract setting (where claims equal argument names). We showed that well-formed CAFs are equally powerful to SETAFs with respect to the reviewed semantics and that attacker-unitary CAFs can be represented as Dung AFs with respect to admissible-based semantics. We also argued that these classes are adequately defined in the sense that minimal deviations make these translations invalid.

In terms of our research question, we thus have shown that (i) *all* instantiation procedures that result in Dung structures where arguments that implicitly share the same claim have the same outgoing attacks can be equivalently mapped to SETAFs where each argument stands for a different claim; (ii) *all* instantiation procedures that result in Dung structures where arguments that implicitly share the same claim have the same incoming attacks can be equivalently mapped to Dung AFs where each argument stands for a different claim. These findings thus give rise to alternative instantiation methods which (a) provide a single argument for each claim and (b) thus reduce the number of arguments on the abstract level. A concrete formalization of such direct instantiation procedures tailored to advanced argumentation formalisms (like ASPIC$^+$ without preferences or logic programs) that result in SETAFs is subject of ongoing work. Investigations of this kind are also relevant from a practical side as nowadays abstract argumentation is not only understood as conceptual tool, but also the base formalism in implementations of argumentation systems, e.g. in the TOAST system [27]. Moreover, first implementations for SETAFs entered the stage [19] and can be employed for direct instantiations into SETAFs.

In this initial study we have focused on two classes of CAFs, well-formed and attacker-unitary. To extend our investigations to further instantiations, for instance full ASPIC$^+$ with preferences, further such sub-classes need to be investigated and corresponding abstract formalisms have to be found which allow for similar translations as we have shown here. We anticipate that AFs with some form of recursive attacks might be the adequate formalism for this purpose. Another direction for future research is to extend our studies to further prominent argumentation semantics, e.g. naïve, stage and semi-stable semantics [30], and labelling-based semantics [10].

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A Appendix

In this appendix we provide full proofs for the main results of the paper. We first show an alternative characterization of grd_c-semantics that we will exploit later on.

Proposition 4. *For any CAF CF, $G \in grd_c(CF)$ iff $G \in com_c(CF)$ and $G \subseteq S$ for all $S \in com_c(CF)$.*

Proof. Let $CF = (A, R, claim)$ and consider the grounded extension $G \in grd((A, R))$. We have that $G \subseteq E$ for all $E \in com((A, R))$ and, by the monotonicity of $claim(.)$, $claim(G) \subseteq S$ for all $S \in com_c(CF)$. □

Proofs of Section 3

Proposition 2 (restated). For $\sigma \in \{stb, prf\}$ and every well-formed CAF $CF = (A, R, claim)$, we have (a) $\sigma_c(CF)$ is incomparable, and (b) $|\sigma((A, R))| = |\sigma_c(CF)|$.

Proof. Let $F = (A, R)$. Then $\sigma(F)$ is an incomparable set, $\sigma(F) \subseteq com(F)$, and, as we have shown in the proof of Proposition 1, for $E, E' \in com(F)$: $E \subseteq E'$ iff $claim(E) \subseteq claim(E')$. Hence, for $S, T \in \sigma(F)$, $S \neq T$ implies $claim(S) \not\subseteq claim(T)$. The result follows. □

Proofs of Section 4.1

Theorem 1 (restated). For each well-formed CAF CF and $\sigma \in \{cf, adm, com, grd, prf, stb\}$, Translation 1 provides an equivalent SETAF $\mathtt{T}_{cts}(CF)$ such that $\sigma_c(CF) = \sigma_s(\mathtt{T}_{cts}(CF))$.

Proof. Let $SF = \mathtt{T}_{cts}(CF) = (A', R')$. We show the result step-by-step for the different semantics.

(1) Let $\sigma = cf$. We rephrase the property of being conflict-free in terms of CNF- resp. DNF-attack-formulas. In well-formed CAFs, $S \in cf_c(CF)$ iff for each $s \in S$ there is an $a \in A$ with $claim(a) = s$ such that a is not attacked by any argument b with $claim(b) \in S$. Note that, for any $s \in claim(A)$, the CNF-attack-formula $\mathcal{CD}_s^{CF}$ identifies each clause with the set of attacking claims for a particular occurrence of s in well-formed CAFs. That is, $S \in cf_c(CF)$ iff for each $s \in S$,

$$\textit{there is some } \gamma \in \mathcal{CD}_s^{CF} \textit{ such that } \gamma \cap S = \emptyset. \tag{C1}$$

In a SETAF (A', R'), a set $S \subseteq A'$ is conflict-free iff for all $S' \subseteq S$ and all $s \in S$, $(S', s) \notin R'$. In terms of attack formulas we have that $S \in cf_s(SF)$ iff for each $s \in S$, it holds that

$$\textit{for all } \delta \in \mathcal{D}_s^{SF} \textit{ it holds that } \delta \not\subseteq S. \tag{C2}$$

We have (i) $\mathcal{D}_s^{SF} = \mathcal{D}_s^{CF}$ for each $s \in A'$ by construction, and (ii) that no $\delta \in \mathcal{D}_s^{CF}$ is a subset of S iff there exists $\gamma \in \mathcal{CD}_c^{CF}$ such that $\gamma \cap S = \emptyset$, and thus we obtain that (C1) is equivalent to (C2), hence the statement follows.

(2) Let $\sigma = adm$. We will translate admissibility in well-formed CAFs resp. SETAFs to CNF- resp. DNF-attack-formulas. Let $S \subseteq claim(A)$. S is adm-realizable in CF if there exists a set $E \subseteq A$, $claim(E) = S$, which is conflict-free and defends itself. Recall that in well-formed CAFs, arguments with the same claim attack the same arguments, which allows for speaking about claims attacking arguments. Using this advantage, we get that $S \in adm_c(CF)$ iff for each $s \in S$, there exists an $a \in A$, $claim(a) = s$, such that $(b, a) \notin R$ for any argument b with $claim(b) \in S$ and for all claims $d \in claim(A)$ which attack a, for each argument with claim d there is a claim $s' \in S$ which attacks the argument. Thus, in terms of CNF-attack-formulas: $S \in adm_c(CF)$ iff for each $s \in S$,

$$\begin{aligned} &\textit{there exists } \gamma \in \mathcal{CD}_s^{CF} \textit{ such that } \gamma \cap S = \emptyset, \\ &\textit{and for all } d \in \gamma, \textit{ for all } \gamma' \in \mathcal{CD}_d^{CF} \\ &\textit{it holds that } \gamma' \cap S \neq \emptyset. \end{aligned} \tag{A1}$$

A set $S \subseteq A'$ is admissible in SF iff it is conflict-free and defends itself. The latter is satisfied iff each attacking set B is attacked by some subset $S' \in S$, i.e. there is some $b \in B$ which gets attacked by S'. Thus, in terms of DNF-attack-formulas, a set S is admissible in SF iff for each $s \in S$, it holds that

$$\begin{aligned} &\textit{for all } \delta \in \mathcal{D}_s^{SF} \textit{ it holds that } \delta \not\subseteq S, \\ &\textit{and there exists } d \in \delta, \textit{ exists } \delta' \in \mathcal{D}_d^{SF}, \\ &\textit{such that } \delta' \subseteq S. \end{aligned} \tag{A2}$$

By construction, we have that (i) $\mathcal{D}_s^{SF} = \mathcal{D}_s^{CF}$ for all $s \in A'$. Thus it remains to show that A1 and A2 are equivalent. Recall that (ii) for each $\gamma \in \mathcal{CD}_s^{CF}$ it holds that $\gamma \cap S \neq \emptyset$ if and only if there exists $\delta \in \mathcal{D}_s^{CF}$, such that $\delta \subseteq S$.

To show A1 $\Rightarrow$ A2, fix a witness $\gamma \in \mathcal{CD}_s^{CF}$ satisfying A1. We show that for all $\delta \in \mathcal{D}_s^{CF}$, $\delta \not\subseteq S$ and there exists $d \in \delta$ and $\delta' \in \mathcal{D}_d^{SF}$, such that $\delta' \subseteq S$. First note that $\delta \not\subseteq S$ follows immediately from (ii). Furthermore observe that each $\delta \in \mathcal{D}_s^{CF}$ contains some $d \in \gamma$, such that for every $\gamma' \in \mathcal{CD}_d^{CF}$, γ' has non-empty intersection with S. By (ii), the latter implies that there exists some $\delta' \in \mathcal{D}_d^{CF}$ such that $\delta' \subseteq S$.

To show A2 $\Rightarrow$ A1, let $\gamma = \{d \in \delta \mid \delta \in \mathcal{D}_s^{CF} \wedge \exists \delta' \in \mathcal{D}_d^{SF} \textit{ such that } \delta' \subseteq S\}$. We show that $\gamma \cap S = \emptyset$ and for all $d \in \gamma$, for all $\gamma' \in \mathcal{CD}_d^{CF}$ it holds that $\gamma' \cap S \neq \emptyset$. By (ii) and by definition of γ, the latter is satisfied, i.e. $\gamma' \cap S \neq \emptyset$ for each $\gamma' \in \mathcal{CD}_d^{CF}$, for all $d \in \gamma$. Now assume that $\gamma \cap S \neq \emptyset$. Let $c \in S \cap \gamma$. By definition of γ, $c \in \delta$ for some $\delta \in \mathcal{D}_s^{CF}$ and there exists $\delta' \in \mathcal{D}_c^{CF}$ such that $\delta' \subseteq S$. But since $c \in S$, it furthermore holds that $\delta' \not\subseteq S$ by A2, which is a contradiction.

(3) Let $\sigma = com$. We will express completeness of sets in well-formed CAFs and SETAFs in terms of CNF-, respectively, DNF-attack-formulas. Let $S \subseteq claim(A) = A'$. Observe that S is complete iff it is admissible and contains all arguments it defends. For well-formed CAFs, we already know from (2), that S is admissible in CF if for each $s \in S$, A1 is satisfied, i.e. there exists $\gamma \in \mathcal{CD}_s^{CF}$ such that $\gamma \cap S = \emptyset$, and for all $g \in \gamma$, for all $\gamma' \in \mathcal{CD}_g^{CF}$ it holds that $\gamma' \cap S \neq \emptyset$. Now, for complete sets, defense is not only necessary, but also a sufficient criteria for membership: If S defends an argument a, $claim(a) = s$, against any attacker $d \in claim(A)$, then $s \in S$. In terms of CNF-attack-formulas: If there is $\gamma \in \mathcal{CD}_s^{CF}$ such that for all $g \in \gamma$, for all $\gamma' \in \mathcal{CD}_g^{CF}$ it holds that $\gamma' \cap S \neq \emptyset$, then $s \in S$. Combining both implications, we get that $s \in S$ *if and only if* A1 is satisfied. A similar reasoning also applies to complete sets in SETAFs: For any complete set S in SF it holds that $s \in S$ *if and only if* A2 for all $\delta \in \mathcal{D}_s^{CF}$, $\delta \not\subseteq S$ and there exists $d \in \delta, \delta' \in \mathcal{D}_d^{CF}$ such that $\delta' \subseteq S$.

Since $\mathcal{D}_s^{SF} = \mathcal{D}_s^{CF}$ for each $s \in A'$, and, furthermore, since A1 is equivalent to A2 as shown in (2), we obtain that indeed $S \in com_c(CF)$ iff $S \in com_s(SF)$ for any set $S \subseteq claim(A)$.

(4) Since $grd_c(CF)$ is the subset-minimal claim-based complete extension for any CAF CF by Proposition 4, it follows that $grd_c(CF) = grd_s(SF)$.

(5) We already know that $com_c(CF) = com_s(\mathtt{T}_{cts}(CF))$, by Proposition 1 the set $prf_c(CF)$ is given by the subset-maximal sets of $com_c(CF)$, and by definition $prf_s(\mathtt{T}_{cts}(CF))$ is given by the subset-maximal sets of $com_s(\mathtt{T}_{cts}(CF))$. Hence, we have $prf_c(CF) = prf_s(\mathtt{T}_{cts}(CF))$.

(6) Let $\sigma = stb$. A set S is stable on claim-level in CF if for each $s \in claim(A)$, it holds that $s \in S$ *if and only if* (C1) is satisfied. Similarly, S is stable in $\mathtt{T}_{cts}(CF)$ if for each $s \in A' = claim(A)$, it holds that $s \in S$ if and only if (C2) holds. The statement follows by the equivalence of (C2) and (C1). □

Proofs of Section 4.2

In order to prove Theorem 2 we make use of concepts of Section 6. We start by introducing normalized CAFs which are CAFs without redundant arguments (cf. Definition 11).

Definition 12. *A CAF $CF = (A, R, claim)$ is called* normalized *if there are no redundant arguments in CF.*

Each CAF can be transformed into a normalized CAF without changing the outcome of the reviewed semantics. The following result is by repetitive application of Prop. 3.

Theorem 4. *Any CAF CF can be transformed into an normalized CAF CF', such that $\sigma_c(CF) = \sigma_c(CF')$, for $\sigma \in \{cf, adm, com, grd, stb, prf\}$.*

Next we consider the Translations $\mathtt{T}_{cts}$ and $\mathtt{T}_{stc}$ restricted to the class of all normalized well-formed CAFs, and respectively, SETAF in minimal form[5]. To show the Theorem, it suffices to show that $\mathtt{T}_{cts}$ and $\mathtt{T}_{stc}$ under these restrictions are each others inverse w.r.t. a fixed conversion from CNF- to DNF-formulas and vice versa.

Theorem 2 (restated). Let $\sigma \in \{cf, adm, com, grd, prf, stb\}$. For each SETAF SF in minimal form, $\sigma_s(SF) = \sigma_c(\mathtt{T}_{stc}(SF))$.

Using an appropriate CNF-DNF-conversion, we will show that $\mathtt{T}_{stc}$ and $\mathtt{T}_{cts}$ are each others inverse when restricted to the class of all SETAFs in minimal form and to the class of all normalized CAFs, respectively. Recall that, by Theorem 1, $\sigma_c(CF) = \sigma_s(\mathtt{T}_{cts}(CF))$ for each well-formed CAF and for $\sigma \in \{cf, adm, com, grd, prf, stb\}$, consequently we get that $\sigma_s(SF) = \sigma_s(\mathtt{T}_{cts}(\mathtt{T}_{stc}(SF))) = \sigma_c(\mathtt{T}_{stc}(SF))$.

Let $\mathfrak{C}$ denote the class of all normalized well-formed CAFs and let $\mathfrak{S}$ denote the class of all SETAFs in minimal form. We show that there are CNF-DNF-formula conversions such that $\mathtt{T}_{stc}|_{\mathfrak{S}} = (\mathtt{T}_{cts}|_{\mathfrak{C}})^{-1}$.

To that end, we consider the following conversions.

Definition 13. *Let $X = \{\gamma_0, \dots, \gamma_n\}$ denote a CNF- respectively DNF-formula. We define the corresponding DNF- respectively CNF-formula $con(X)$ as the set of subset-minimal elements of $\{\delta \mid \forall i \leq n : |\delta \cap \gamma_i| \geq 1\}$.*

We call a formula X incomparable if all clauses $\gamma \in X$ are pairwise incomparable, i.e. for all $\gamma, \gamma' \in X$, $\gamma \nsubseteq \gamma'$. Observe that both conversions yield incomparable formulas. We will show that for each incomparable CNF- respectively DNF-formula X, the sequential application of both conversions yield the original formula X, i.e. $con(con(X)) = X$.

[5] A SETAF $SF = (A, R)$ is in minimal form if it has no attacks $(A, a), (B, a) \in R$ such that $A \subset B$.

Lemma 2. *Let $X = \{\gamma_0, \dots, \gamma_n\}$ be incomparable then $con(con(X)) = X$.*

Proof. Let $Y = \{\delta_0, \dots, \delta_m\}$ denote the subset-minimal elements of $\{\delta \mid \forall i \leq n : |\delta \cap \gamma_i| \geq 1\}$, and let $L = \{\zeta \mid \forall j \leq m : |\zeta \cap \delta_j| \geq 1\}$. We show that the set of subset-minimal elements $\min_{\subseteq}(L)$ of L equals X.

$X \subseteq \min_{\subseteq}(L)$: First note that $\gamma \in L$ for all $\gamma \in X$, since $|\delta_j \cap \gamma| \geq 1$ for all $j \leq m$. For each $\gamma \in X$, for each $a \in \gamma$, there is some δ st $\delta \cap \gamma = \{a\}$. Take all γ_i such that $a \notin \gamma_i$, then there is some $\delta \subseteq \{b \mid b \in \gamma_i \setminus \gamma\} \cup \{a\}$ and $|\delta \cap \gamma| = 1$. Now, assume that there is some $\gamma' \subset \gamma$, $\gamma' \in \min_{\subseteq}(L)$. Let $a \in \gamma \setminus \gamma'$. Using the construction above, we get that there exists $\delta \in Y$ such that $\delta \cap \gamma = \{a\}$, consequently, $\gamma' \cap \delta = \emptyset$. It follows that $\gamma \in \min_{\subseteq}(L)$ for all $\gamma \in X$.

$\min_{\subseteq}(L) \subseteq X$: Towards a contradiction, let $\zeta \in \min_{\subseteq}(L) \setminus X$, and let $\delta \subseteq \bigcup_{i \leq n} \gamma_i \setminus \zeta$. Such a δ exists, otherwise $\zeta \supseteq \gamma$ for some $\gamma \in \min_{\subseteq}(L)$. But then $\delta \cap \zeta = \emptyset$. □

We are now ready to prove Theorem 2.

Proof of Theorem 2. Observe that (i) a well-formed CAF $CF = (A, R, claim)$ is normalized if and only if for each claim $c \in claim(A)$ it holds that $\mathcal{CD}_c^{CF}$ is incomparable. Similarly, (ii) a SETAF $SF = (A', R')$ is in minimal form if and only if for every argument $a \in A'$ it holds that $\mathcal{D}_a^{SF}$ is incomparable. Furthermore note that (iii) the output of both translations $\mathtt{T}_{cts}$ and $\mathtt{T}_{stc}$ solely depends on the choice of the CNF-DNF-conversion. Let $CF = (A, R, claim)$ and let $\mathtt{T}_{cts}$ and $\mathtt{T}'_{cts}$ denote translations using fixed CNF-DNF-conversions $\mathcal{D}_c^{CF}$ and respectively, $\mathcal{D}_c'^{CF}$ for each $c \in claim(A)$. Then $\mathtt{T}_{cts}(CF) = \mathtt{T}'_{cts}(CF)$ iff $\mathcal{D}_c^{CF} = \mathcal{D}_c'^{CF}$ for all $c \in claim(A)$. Similarly, for every SETAF $SF = (A', R')$, $\mathtt{T}_{stc}(SF) = \mathtt{T}'_{stc}(SF)$ iff $\mathcal{CD}_a^{SF} = \mathcal{CD}_a'^{SF}$ for all $a \in A'$. Since $con(con(X)) = X$ for each incomparable CNF- respectively DNF-formula X, we have that $\mathtt{T}_{stc}(\mathtt{T}_{cts}(CF)) = CF$ for each normalized well-formed CAF and, similar, $\mathtt{T}_{cts}(\mathtt{T}_{stc}(SF)) = SF$ for each SETAF in minimal form. It follows that $\mathtt{T}_{stc}|_{\mathfrak{S}} = (\mathtt{T}_{cts}|_{\mathfrak{C}})^{-1}$.

By Theorem 1, we can conclude that $\sigma_s(SF) = \sigma_s(\mathtt{T}_{cts}(\mathtt{T}_{stc}(SF))) = \sigma_c(\mathtt{T}_{stc}(SF))$ for each SETAF SF in minimal form. □

Proofs of Section 5

In order to prove Lemma 1 and Theorem 3, we show further properties of att-unitary CAFs. To this end we recall the definition of the characteristic function from [16].

Definition 14. *For any AF $F = (A, R)$, the* characteristic function *$\mathcal{F}_F : 2^A \to 2^A$ of F is defined as $\mathcal{F}_F(S) = \{x \in A \mid x$ is defended by $S\}$. For any CAF $CF = (A, R, claim)$, for $E \subseteq A$, we use $\mathcal{F}_{CF}(E)$ to abbreviate $\mathcal{F}_{(A,R)}(E)$.*

We first show that in att-unitary CAFs if a set of arguments defends one argument with a specific claim then it also defends all the other arguments with that claim and thus there is a one-to-one mapping between complete claim-set and complete extensions.

Lemma 3. *Let $CF = (A, R, claim)$ be att-unitary and let $E \subseteq A$. Then*

1. $c \in claim(\mathcal{F}_{CF}(E))$ *iff* $x \in \mathcal{F}_{CF}(E)$ *for all* $x \in A$ *such that* $claim(x) = c$*; and*
2. $|com_c(CF)| = |com((A, R))|$.

Proof. To prove (1), let $c \in claim(\mathcal{F}_{CF}(E))$, then there is an argument $x \in \mathcal{F}_{CF}(E)$, such that $claim(x) = c$. Let $y \in A$, $claim(y) = c$. Since $y^- = x^-$, we can conclude that E defends y, hence the statement follows. By definition of complete semantics, $E \in com((A, R))$ iff $\mathcal{F}_{CF}(E) = E$, consequently $c \in claim(E)$ iff $x \in E$ for all $x \in A$, $claim(x) = c$. Thus (2) follows. □

We next exploit Lemma 3 to show Lemma 1.

Lemma 1 (restated). Let $CF = (A, R, claim)$ be att-unitary and let $S \in \sigma_c(CF)$ for $\sigma \in \{adm, com, grd, prf, stb\}$. Then $E_S^{\max} = \{x \mid claim(x) \in S\} \in \sigma((A, R))$.

Proof. $S \in \sigma_c(CF)$ implies the existence of some set of arguments $E \subseteq A$, $claim(E) = S$, such that $E \in \sigma((A, R))$. Due to att-unitaryness, $E^- = (E_S^{\max})^-$. The statement follows for $\sigma = adm$ since $(E_S^{\max})^- = E^- \subseteq E^+ \subseteq (E_S^{\max})^+$. Let $\sigma = com$, then $\mathcal{F}_{CF}(E) = E$ by definition. By Lemma 3, $c \in S$ iff $x \in E$ for each argument x such that $claim(c) = x$. It follows that $E = E_S^{\max}$. Therefore, since $\sigma((A, R)) \subseteq com((A, R))$, it also holds that $E_S^{\max} \in \sigma((A, R))$ for $\sigma \in \{grd, prf, stb\}$. □

We are now prepared to prove Theorem 3.

Theorem 3 (restated). Let $CF = (A, R, claim)$ be att-unitary. Then $\sigma_c(CF) = \sigma(\mathtt{T}_{cta}(CF))$ for $\sigma \in \{adm, com, grd, prf, stb\}$.

Proof. Let $F = \mathtt{T}_{cta}(CF) = (A', R')$. For a set $S \subseteq claim(A) = A'$, we denote by $E_S^{\max} = \{x \mid claim(x) \in S\}$ the maximal representative of S in CF.

(1) Let $\sigma = adm$ and let $S \in adm_c(CF)$, then $E_S^{max} \in adm((A, R))$ by Lemma 1. We show that $S \in adm(F)$. First note that S is conflict-free since $(x, y) \notin R$ for all $x, y \in E_S^{max}$. To show that S defends itself, let $b \in S$ and let $(a, b) \in R'$ for some $a \in A'$. Then by definition of R', there are arguments $x, y \in A$, $claim(x) = a$, $claim(y) = b$, such that $(x, y) \in R$. Since $y \in E_S^{\max}$ and $E_S^{\max} \in adm((A, R))$, y is defended by some $z \in E_S^{\max}$, i.e. $(z, x) \in R$ for some $z \in A$ such that $claim(z) \in S$. Consequently, $(claim(z), b) \in R'$ which shows that S defends itself.

To show the other direction, let $S \in adm(F)$. We show that E_S^{max} is admissible in (A, R): By definition of R', E_S^{max} does not contain any conflicts. Now, let $y \in E_S^{\max}$, $claim(y) = b$, and let $(x, y) \in R$ for some $x \in A$, $claim(x) = a$. By definition of R', we have that $(a, b) \in R'$. Since S defends itself, there is some $c \in S$ such that $(c, a) \in R'$, therefore there exist $z, x' \in A$, $claim(z) = c$, $claim(x') = a$, such that $(z, x') \in R$. By att-unitaryness, $x^- = x'^-$, thus $(z, x) \in R$. Note that $z \in E_S^{max}$ by definition, hence E_S^{max} defends itself.

(2) Let $\sigma = com$ and let $S \in com_c(CF)$. Then $E_S^{\max}$ is complete in (A, R) by Lemma 1. By (1), $S \in adm(F)$. We show that S contains all arguments it defends: Let $a \in A'$ be defended by S, i.e. for all $b \in A'$ such that $(b, a) \in R'$, there is some $c \in S$ such that $(c, b) \in R'$. By definition of Translation 3, it holds that for all $b \in A'$ such that $(b, a) \in R'$, (i) there is $y \in A$, $claim(y) = b$, such that $(y, x) \in R$, for some $x \in A$, $claim(x) = a$; and (ii) there are $y', z \in A$, $claim(y') = b$, $claim(z) = c$ for $c \in S$, such that $(z, y') \in R$. Since CF is att-unitary, $y^- = y'^-$, thus x is defended against y by $z \in E_S^{\max}$. Consequently, $x \in E_S^{\max}$ and therefore $a = claim(x) \in S$. in CF.

To show the other direction, let $S \in com(F)$. We show that $E_S^{\max} \in com((A, R))$. By (1), $E_S^{max} \in adm((A, R))$. To show that E_S^{max} contains all arguments it defends, let $x \in \mathcal{F}_{CF}(E_S^{\max})$, $claim(x) = a$. For each $y \in A$ such that $(y, x) \in R$, there is some $z \in E_S^{\max}$ such that $(z, y) \in R$. By construction of R', we have that $(claim(y), a), (claim(z), claim(y)) \in R'$ and $claim(z) \in S$, thus S defends a. Since $S \in com(F)$, we conclude that $a \in S$. By definition of $E_S^{\max}$ we have that $x \in E_S^{\max}$.

(3) Let $\sigma = grd$. By (2) and since $grd_c(CF)$ is the subset-minimal complete extension by Proposition 4, it follows that $grd_c(CF) = grd(F)$.

(4) Let $\sigma = prf$. Recall that $prf_c(CF) \subseteq com_c(CF)$. Since each $S \in com_c(CF)$ is realized by $E_S^{\max}$ and, by Lemma 3, $|com_c(CF)| = |com((A, R))|$, we have that for each $S, S' \in com_c(CF)$, $S \subseteq S'$ iff $E_S^{\max} \subseteq E_{S'}^{\max}$. By (2), $S \in com_c(CF)$ iff $S \in com(F)$, thus the statement follows.

(5) Let $\sigma = stb$ and $S \in stb_c(CF)$. By Lemma 1, $E_S^{max} \in stb((A, R))$. Furthermore S is conflict-free in $\mathtt{T}_{cta}(CF)$ by definition of R'. For each $x \in A \setminus E_S^{max}$, there is $y \in E_S^{max}$ such that $(y, x) \in R$, consequently $(claim(y), claim(x)) \in R'$, hence S attacks all arguments $a \in A' \setminus S$.

Now, let $S \in stb(F)$ and let $b \in A' \setminus S$. Then there is some argument $a \in S$ such that $(a, b) \in R'$. By definition of R', there are arguments $x, y \in A$, $claim(x) = a$, $claim(y) = b$, such that $(x, y) \in R$. By att-unitaryness, $(x, y') \in R$ for each $y' \in A$ such that $claim(y') = b$. Hence each argument $y \in A \setminus E_S^{max}$ is attacked by E_S^{max}. □

Proofs of Section 6

Proposition 3 (restated). Let $CF = (A, R, claim)$ be a CAF, with $a \in A$ redundant in CF w.r.t. some $b \in A$, and let $CF' = (A', R', claim)$ with $A' = A \setminus \{a\}$, and $R' = R \cap (A' \times A')$. Then, for $\sigma \in \{cf, adm, com, grd, stb, prf\}$, $\sigma_c(CF) = \sigma_c(CF')$.

Proof. We show the result step-by-step for the different semantics.

(1) Let $S \in cf_c(CF)$ and let E be a cf-realization of S in CF. If $a \notin E$, then E is a cf-realization of S in CF' as well and thus $S \in cf_c(CF')$. If $a \in E$, consider $E' = (E \setminus \{a\}) \cup \{b\}$; by definition $claim(E') = claim(E)$, thus it remains to show that $E' \in cf((A', R'))$. First, we have $(b, b) \notin R$ as otherwise $(b, a) \in R$ (since $a_R^- \supseteq b_R^-$) and also $(a, a) \in R$ (since $a_R^+ = b_R^+$). By $a \in E \in cf((A, R)$ we have $a_R^- \cap E = emptyset$ and $a_R^+ \cap E = emptyset$ and, since $a_R^- \supseteq b_R^-$ and $a_R^+ = b_R^+$, we obtain $E' \in cf((A, R)$ and thus $E' \in cf((A', R')$. For the other direction, let $S \in cf_c(CF')$ and E' a cf-realization of S in CF'. Clearly, E' remains conflict-free in (A, R) and thus $S \in cf_c(CF)$.

(2) Let $S \in adm_c(CF)$ and let E be an adm-realization of S in CF. If $a \notin E$, then E is conflict-free in CF'. Furthermore, since $E_R^- \subseteq E_R^+$, we have that $E_{R'}^- = E_R^- \setminus \{a\} \subseteq E_R^+ \setminus \{a\} = E_{R'}^+$, hence E defends itself in (A', R') and thus adm-realizes S in CF'. If $a \in E$, define $E' = (E \setminus \{a\}) \cup \{b\}$. We already know that E' is a cf-realization of S in CF', thus it remains to show that E' defends itself in (A', R'). First observe that $E_R'^- \subseteq E_R^-$, since $b_R^- \subseteq a_R^-$, and $E_R^+ = E_R'^+$, since $b_R^+ = a_R^+$. Second, $E_{R'}'^- = E_R'^-$: otherwise $(a, c) \in R$ for some $c \in E'$, but since $E \in cf((A, R))$, this implies $c = b$ which would yield $(a, a) \in R$ via $a_R^+ = b_R^+$. Finally, $E_R'^+ =$

$E'^{+}_{R'}$: otherwise $(c, a) \in R$ for some $c \in E'$; this implies $(b, a) \in R$ (since $E \in cf((A, R))$) and $(a, a) \in R$ via $a^-_R \supseteq b^-_R$. Together with $E^-_R \subseteq E^+_R$, we thus obtain $E'^{-}_{R'} = E'^{-}_R \subseteq E^-_R \subseteq E^+_R = E'^{+}_R = E'^{+}_{R'}$ and hence E' defends itself in (A', R'). To show the other direction, let $S \in adm_c(CF')$. Then there is a set E' which adm-realizes S in CF'. We show that E' is an adm-realization of S in CF: Clearly, E' is conflict-free in (A, R). It remains to show that E' defends itself in (A, R). Suppose this is not the case. Then $a \notin E'^{+}_R$ and $(a, c) \in R$ for some $c \in E'$. It follows that $b \notin E'$ since $b^+_R = a^+_R$ and $E' \in cf(A', R')$. Since $E' \in adm((A', R'))$, it then also follows that $b \in E'^{+}_{R'} = E'^{+}_R$. Because of $a^-_R \supseteq b^-_R$, we arrive at $a \in E'^{+}_R$, a contradiction.

(3) Let $S \in com_c(CF)$ and let E be a com-realization of S in CF. If $a \notin E$, we know from above that E is an adm-realization of S in CF'. It remains to show that E contains all arguments it defends in (A', R'). Let $d \in A'$ be defended by E in (A', R'); we have that $d^-_{R'} \subseteq E^+_{R'} = E^+_R \setminus \{a\}$. If $(a, d) \notin R$, then $d^-_R = d^-_{R'}$, consequently $d^-_R \subseteq E^+_R$, and therefore $d \in E$. If $(a, d) \in R$, then $(b, d) \in R$ (since $a^+_R = b^+_R$), and thus $(b, d) \in R'$. Since $a^-_R \supseteq b^-_R$, we conclude that d is defended by E in (A, R) and therefore $d \in E$. Now, if $a \in E$, then $b \in E$, since $a^-_R \supseteq b^-_R$. Let $E' = E \setminus \{a\}$. We know from above that E' is an adm-realization of S in CF'. Moreover, for each $d \in A'$ which is defended by E' in (A', R'), it holds that $d^-_R = d^-_{R'} \subseteq E'^{+}_{R'} = E'^{+}_R = E^+_R$, therefore $d \in E'$. Thus $E' \in com((A', R'))$. To show the other direction, let $S \in com_c(CF')$ and let E' be a com-realization of S in CF'. We already have that E' is admissible in (A, R). If $b \notin E'$, then $E' \in com((A, R))$: Consider $d \in A$ such that $d^-_R \subseteq E'^{+}_R$. First note that $d \neq a$, otherwise $b \in E'$ since $a^-_R \supseteq b^-_R$. Furthermore, $d^-_{R'} = d^-_R \setminus \{a\} \subseteq E'^{+}_R \setminus \{a\} = E'^{+}_{R'}$, thus $d \in E'$. If $b \in E'$, then either E' or $E = E' \cup \{a\}$ is complete in (A, R). For each argument $c \in A \setminus \{a\}$, if $c^-_R \subseteq E'^{+}_R$, then $c^-_{R'} \subseteq E'^{+}_{R'}$, i.e. E' contains all arguments from $A \setminus \{a\}$ it defends in CF. Hence it remains to check whether a is defended by E'. If this is the case, i.e. if $a^-_R \subseteq E'^{+}_R$, then E is conflict-free in CF: Towards a contradiction, assume that a is in conflict with some $c \in E$. If $(a, c) \in R$, then also $(b, c) \in R$ by $a^+_R = b^+_R$. Since $E \in cf((A, R))$, we have that $c = a$, i.e. $(b, a) \in R$. But since E' defends a, we get that $(e, b) \in R$ for some $e \in E'$, contradiction. If $(c, a) \in R$, then there is an argument $d \in E'$ such that $(d, c) \in R$. Since $E \in cf((A, R))$ it follows that $c = a$, i.e. $(a, a) \in R$ and, therefore, $(b, a) \in R$, which leads to the same contradiction. It follows that E is admissible in (A, R). Furthermore, $E \in com((A, R))$, since E and E' defend the same arguments. If $a^-_R \not\subseteq E'^{+}_R$, then $E' \in com((A, R))$. In both cases, S has a com-realization in CF. We thus have shown that $com_c(CF) = com_c(CF')$.

(4) From $com_c(CF) = com_c(CF')$ and Proposition 4, it follows that $grd_c(CF) = grd_c(CF')$.

(5) Let $S \in prf_c(CF)$ and let E be a subset-maximal admissible set in (A, R) such that $claim(E) = S$. First consider the case $a \notin E$. From (2), we know that E is admissible in (A', R'). It remains to show there is no admissible set $F \subseteq A'$ in (A', R') such that $E \subset F$. Towards a contradiction, assume such F exists. Again, using the argument from (2), F is admissible in (A, R), a contradiction. If $a \in E$, then $b \in E$, since $a^-_R \supseteq b^-_R$ and each preferred set is also complete. Let $E' = E \setminus \{a\}$; in (3) we have shown that $E' \in com((A', R'))$ and likewise that for a complete set F of (A', R') with $b \in F$, $F \cup \{a\}$ is complete for (A, R) in case $a^-_R \subseteq F^+_R$. It follows that for each such F, $E \not\subset F$. Now, let $S \in prf_c(CF')$ and E' a prf-realization of S in CF'. Then E' is admissible in CF. We show that either E' or $E = E' \cup \{a\}$ is a maximal admissible set in (A, R): Towards a contradiction, assume that there is an admissible set $F \in A$ in (A, R) such that $F \supset E'$. If $a \notin F$, then F is admissible in CF', contradiction to the maximality of E'. If $a \in F$, then $F' = (F \setminus \{a\}) \cup \{b\} \in adm((A', R'))$. Since E' is maximal in (A', R'), we conclude that $E' = F'$.

(6) Let $S \in stb_c(CF)$, let E be a stb-realization of S in CF, i.e. $E^+_R = A \setminus E$ and E is conflict-free. In case $a \notin E$, it is easy to see that $E \in cf((A', R'))$ and $E^+_{R'} = A' \setminus E$; thus E is a stb-realization of S in CF'. In case $a \in E$, we observe that $b \in E$ holds, too: otherwise, $b \in E^+_R$ and therefore, by $a^-_R \supseteq b^-_R$, also $a \in E^+_R$. A contradiction to $E \in cf((A, R))$. We show that $E' = E \setminus \{a\}$ is a stb-realization of S in CF'. Clearly, $claim(E') = claim(E) = S$. Moreover, $E' \in cf((A', R'))$, and $E'^{+}_{R'} = A' \setminus E' = A \setminus E$. Hence, $E' \in stb((A', R'))$. For the other direction, let $S \in stb_c(CF')$ and E a stb-realization of S in CF'. Clearly $E \in cf((A, R))$. If $a \in E^+_R$, we immediately get $E \in stb((A, R))$. So suppose $a \notin E^+_R$. Since $a^-_R \supseteq b^-_R$, $b \notin E^+_R$, and it follows that $b \in E$, since E is stable in (A', R'). We conclude that $(b, a) \notin R$ and thus, since $a^+_R = b^+_R$, $(a, a) \notin R$. Moreover, as $b \notin E^-_R$ we have $a \notin E^-_R$ as well. That is, we have $(a, a) \notin R$, $a \notin E^+_R$, $a \notin E^-_R$, and thus $(E \cup \{a\}) \in cf((A, R))$. Moreover, in (A, R) the set $E \cup \{a\}$ attacks each argument $c \in A \setminus E$ and, since $b \in E$, $claim(E \cup \{a\}) = claim(E) = S$. Hence, $E \cup \{a\}$ is a stb-realization of S in CF. □

The following result has been mentioned inline in Section 6.

Proposition 5. *For every $\sigma \in \{cf, adm, com, grd, stb, prf\}$, for every well-formed CAF $CF = (A, R, claim)$, if there exists an att-unitary CAF $CF' = (A, R', claim)$ with $|R \triangle R'| = 1$, then there is an AF F such that $\sigma_c(CF) = \sigma(F)$.*

Proof. We show that CF can be represented using exactly one argument per claim. Let $a, b \in A$ with $claim(a) = claim(b)$. We show that a is redundant w.r.t. b in CF or vice versa. By well-formedness, $a^+_R = b^+_R$. Moreover, CF is almost att-unitary, i.e. att-unitaryness is satisfied by adding or removing a single attack. By $|R \triangle R'| = 1$, there are $c, d \in A$ such that either (a) $R \setminus R' = \{(c, d)\}$ or (b) $R' \setminus R = \{(c, d)\}$. In the case $a \neq d$ and $b \neq d$ we have $a^-_R = a^-_{R'}$ and $b^-_R = b^-_{R'}$. By att-unitaryness of CF', $a^-_R = b^-_R$ follows. W.l.o.g. we assume $a = d$. In the case (a), att-unitaryness is retained by removing the attack (c, a). We have $a^-_{R'} = a^-_R \setminus \{c\}$ and therefore $a^-_R \supseteq b^-_R$ (since $b^-_{R'} = a^-_{R'}$ by att-unitaryness of CF'). In case (b), att-unitaryness is satisfied after adding the attack (c, a). It follows that $b^-_R \supseteq a^-_R$ since $b^-_R = b^-_{R'} = a^-_{R'} = a^-_R \cup \{c\}$. In both cases, a^-_R, b^-_R are comparable and thus we have shown that for every two arguments a, b having the same claim, either a is redundant w.r.t. b in CF or vice versa. □