1	2	3	4	Σ	Grade

6.0/4.0 VU Formale Methoden der Informatik

185.291 **May 17, 2024**

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1.) Recall the **HALTING** problem from the lecture:

> **HALTING**
>
> INSTANCE: A program Π that takes a string as input and a string I.
>
> QUESTION: Does Π halt on I?

Consider the following decision problem:

> **NON-EMPTINESS**
>
> INSTANCE: A program Π that takes a string over the alphabet $\Sigma = \{0, 1\}$ as input and outputs *true* or *false*.
>
> QUESTION: Is there an input string for which Π returns *true*?

(a) Provide a many-one reduction from **HALTING** to **NON-EMPTINESS**, i.e., a function that maps every instance (Π, I) of **HALTING** to some instance Π' of **NON-EMPTINESS** such that (Π, I) is a yes-instance of **HALTING** if and only if Π' is a yes-instance of **NON-EMPTINESS**. Argue the correctness of your reduction.

(8 points)

(b) Show that **NON-EMPTINESS** is semi-decidable by providing a semi-decision procedure. For this task, you can assume the existence of the procedure Π_{int} that does the following:

- Π_{int} takes as input a program Π, a string I, and an integer n.
- Π_{int} emulates the first n steps of the run of Π on I. If Π terminates on I within n steps with a return value *true*, then Π_{int} returns *true*. Otherwise, Π_{int} returns *false*.

(7 points)

2.) (a) Show that $b\langle j \triangleleft f\rangle \not\doteq b \rightarrow b[j] \not\doteq f$ is $\mathcal{T}_A^{=}$-valid.

Besides the equality axioms, you have the following ones for the arrays.

$\forall a\, \forall i\, \forall j\, \big(i \doteq j \rightarrow a[i] \doteq a[j]\big)$	(array congruence)
$\forall a\, \forall v\, \forall i\, \forall j\, \big(i \doteq j \rightarrow a\langle i \triangleleft v\rangle[j] \doteq v\big)$	(read-over-write 1)
$\forall a\, \forall v\, \forall i\, \forall j\, \big(i \not\doteq j \rightarrow a\langle i \triangleleft v\rangle[j] \doteq a[j]\big)$	(read-over-write 2)
$\forall a\, \forall b\, \big((\forall j\ a[j] \doteq b[j]) \leftrightarrow a \doteq b\big)$	(extensionality)

Please be precise and justify every proof step.

(12 points)

(b) Consider the clauses $C_0, \ldots, C_6$ in `dimacs` format (in this order from top to bottom, shown in the box) which are given as input to a SAT solver.

```
2 0
-1 4 0
-4 5 0
-2 -4 6 0
-3 -6 7 0
-7 9 0
-5 -6 -7 -9 0
```

- Apply CDCL using the convention that if a variable is assigned as a decision, then it is assigned 'true'. Select variables as decisions in increasing order of their respective integer IDs in the `dimacs` format, starting with variable `1`. Recall that unit clauses require a special treatment.
- When the *first* conflict occurs, draw the complete implication graph, mark the first UIP, give the resolution derivation of the learned asserting clause that corresponds to the first UIP, and stop CDCL. You do not have to solve the formula!

(3 points)

3.) (a) Let p be the following IMP program loop, containing the integer-valued program variables x, y, z, n:

$$
\begin{array}{l}
x := 0; y := 0; z := 0; \\
\textbf{while } x < n \textbf{ do} \\
\quad x := x + 1; \\
\quad y := y + 2; \\
\quad z := z + 3 \\
\textbf{od}
\end{array}
$$

Provide a loop inductive invariant and loop variant and use them to prove the total correctness of the Hoare triple:

$$[n > 1] \;\; p \;\; [n = z - y]$$

(9 points)

(b) Let p be the following IMP program loop, containing the integer-valued program variables x, y:

$$\textbf{if } x < y \ \textbf{ then } x := x + y \textbf{ else } y := y + x$$

Which of the following statements are true?

(i) $x < 0 \implies \text{wlp}(p, x < 0)$;

(ii) $x < 0 \implies \text{wlp}(p, x > 0)$;

(iii) $x < 0 \implies \text{wlp}(p, x < y)$.

In each of the cases above, justify your answer.

(6 points)

4.) (a) Consider the Kripke structures M_1, M_2 below, with initial states s_0 and t_0, respectively.

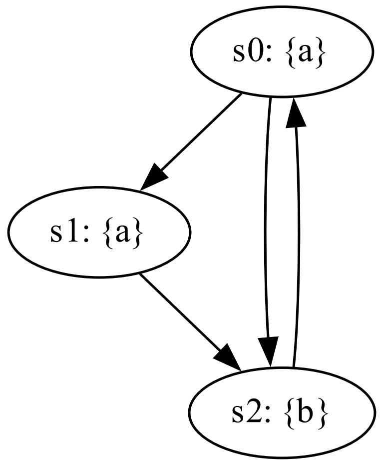


Figure 1: M_1

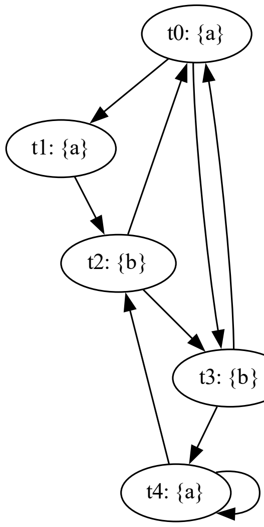
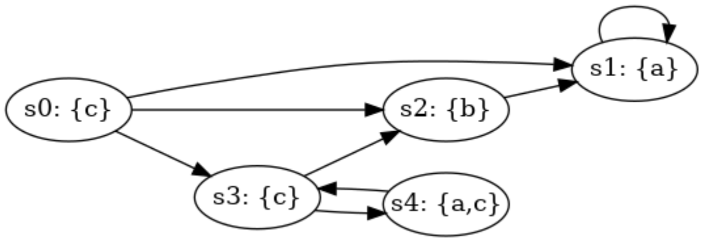


Figure 2: M_2

Provide two distinct simulation relations witnessing $M_1 \preceq M_2$. Are there any other simulation relations witnessing $M_1 \preceq M_2$? Justify your answer.

(4 points)

(b) Consider the following Kripke structure M.



For each of the following formulae φ,

i. indicate whether the formula is in CTL, LTL, and/or CTL*, and

ii. List the states s_i on which the formula φ holds; i.e., for which states do we have $M, s_i \models \varphi$?

Note: If φ is a path formula, list the states s_i on which $M, s_i \models \mathbf{A}\varphi$.

φ	CTL	LTL	CTL*	states s_i
$\mathbf{FG}\, a$	☐	☐	☐	
$\mathbf{GF}\, a$	☐	☐	☐	
$\neg\mathbf{AG}\,\neg b$	☐	☐	☐	
$\mathbf{E}\,(\,\mathbf{X}\, b\ \wedge\ \mathbf{FG}\, a\,)$	☐	☐	☐	
$c\ \mathbf{U}\ b$	☐	☐	☐	

(5 points)

(c) An LTL formula is a *tautology* if it holds for every Kripke structure M and every path π in M. For each of the following formulas, prove that it is a tautology, or find a Kripke structure M and a path π in M for which the formula does not hold and justify your answer.

i. $a \ \mathbf{U} \ (\neg a \wedge \mathbf{XG}a) \Rightarrow \mathbf{G}(a \vee \mathbf{X}a)$

ii. $\mathbf{G}(a \vee \mathbf{X}a) \Rightarrow a \ \mathbf{U} \ (\neg a \wedge \mathbf{XG}a)$

(6 points)

Grading scheme: 0–29 nicht genügend, 30–35 genügend, 36–41 befriedigend, 42–47 gut, 48–60 sehr gut