1	2	3	4	Σ	Grade

6.0/4.0 VU Formale Methoden der Informatik

185.291 May 19, 2023

Kennzahl (study id)	Matrikelnummer (student id)	Familienname (family name)	Vorname (first name)	Gruppe (version)

1.) A *triangle-graph* is any undirected graph (V, E) that contains edges $(a, b), (b, c), (c, a)$ for some $\{a, b, c\} \subseteq V$. Recall the **DOMINATING SET** problem. Consider the following variant thereof.

> **DS-TRIANGLE**
>
> INSTANCE: A *triangle-graph* $G = (V, E)$, and an integer k.
>
> QUESTION: Does there exist a dominating set of vertices of size at most k, i.e., is there a set $S \subseteq V$ with $|S| \leq k$ such that for every vertex $v \in V$ it either holds $v \in S$ or there is some $w \in S$ such that $(v, w) \in E$.

Recall that the (standard) **DOMINATING SET** problem is defined over *arbitrary* undirected graphs (together with an integer k) and has the same question.

(a) The following function g provides a polynomial-time many-one reduction from the problem **DOMINATING SET** to **DS-TRIANGLE**:

$$g((G, k)) = (f(G), k + 1).$$

We define $f(G) = (V', E')$ as follows. For a graph $G = (V, E)$, let $\{a, b, c\}$ be a set of fresh vertices. Moreover we define:

$$V' = V \cup \{a, b, c\},$$
$$E' = E \cup \{(a, b), (b, c), (c, a)\}.$$

Show the correctness of the reduction, i.e., show that (G, k) is a positive instance of **DOMINATING SET** if and only if $g((G, k))$ is a positive instance of **DS-TRIANGLE**.

(9 points)

(b) Check which statements are true/false. 1 point for each correct answer, -1 for each incorrect answer, 0 for no answer. Negative points do not carry over to other exercises.

You may use the fact that **DOMINATING SET** is NP-complete. Recall also that **SATISFIABILITY** is NP-complete.

true	false	
○	○	The correctness of the reduction in (a) proves NP-hardness of **DS-TRIANGLE**.
○	○	The correctness of the reduction in (a) proves NP-hardness of the complement of **DS-TRIANGLE**.
○	○	The correctness of the reduction in (a) proves that **DS-TRIANGLE** is at least as hard as **SATISFIABILITY**.
○	○	If an exponential time algorithm for **DS-TRIANGLE** exists then this proves $P \neq NP$.
○	○	The fact that **DS-TRIANGLE** is a special case of **DOMINATING SET** proves NP-membership of **DS-TRIANGLE**.
○	○	The correctness of the reduction in (a) proves that there is a polynomial-time many-one reduction from **SATISFIABILITY** to **DS-TRIANGLE**.

(6 points)

2.) (a) Let φ^E be the following formula of E-logic:

$$(x_5 = x_6 \vee x_4 \neq x_5) \wedge x_4 \neq x_6 \wedge x_4 = x_2 \wedge x_2 = x_3 \wedge (x_3 \neq x_1 \vee x_4 = x_1)$$

Apply the *Sparse Method* and present an equisatisfiable propositional formula. **(6 points)**

(b) Let $\Sigma = (\{a/0,\ b/0,\ f/2\}, \{p/2,\ \approx/2\})$ and let $\mathcal{T}$ be a first-order theory containing the following axioms:

$$\forall x \forall y \Big(x \approx y \to y \approx x \Big) \tag{sy}$$

$$\forall x \forall y \Big(p(x, y) \to \big(p(x, f(x, y)) \land p(f(x, y), y) \big) \Big) \tag{pd}$$

$$\forall x \forall y \Big(p(x, y) \to x \not\approx y \Big) \tag{pi}$$

i. Use the semantic argument method to prove the following statement:
Let $\mathcal{I}$ be a $\mathcal{T}$-interpretation with $\mathcal{I} \models p(a, b)$, then it holds that

$$\mathcal{I} \models f(a, b) \not\approx a \land f(a, b) \not\approx b \land a \not\approx b.$$

ii. Is φ: $P(f(a, b), f(b, a))$ $\mathcal{T}$-valid? If yes, then give a proof in the semantic argument method. If no, then present a counterexample and show that it falsifies φ.

(9 points)

3.)

(a) Let p be the following IMP program loop, containing the integer-valued program variables x, y, z:

$x := 0; y := 0; z := 5;$
while $y < n$ **do**
  $x := x - y;$
  $y := y + 1;$
  $z := z - 5;$
od

Give an inductive invariant for **while** loop in p and prove the validity of the partial correctness triple $\{n = 11\}p\{x + 55 = 0\}$.

(9 points)

(b) Let p be the following IMP program:

$$\textbf{if } x \neq y \quad \textbf{then} \quad x := x * y; \textbf{skip} \quad \textbf{else} \quad y := y * x; \textbf{abort}$$

where x, y are integer-valued program variables.

Provide a non-trivial pre-condition A and a non-trivial post-condition B, such that:

(a) $\{A\}\ p\ \{B\}$ is not valid;

(b) $\{A\}\ p\ \{B\}$ is valid but $[A]\ p\ [B]$ is not valid;

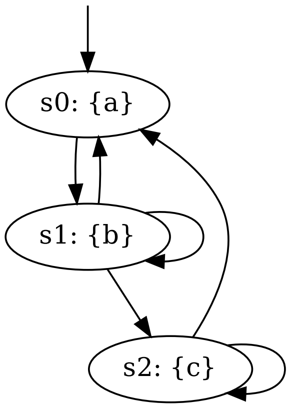
(c) $[A]\ p\ [B]$ is valid.

Trivial pre-condition/post-condition means equivalent to `true` or `false`, so your pre-conditions A and postconditions B should not be equivalent to `true` or `false`.

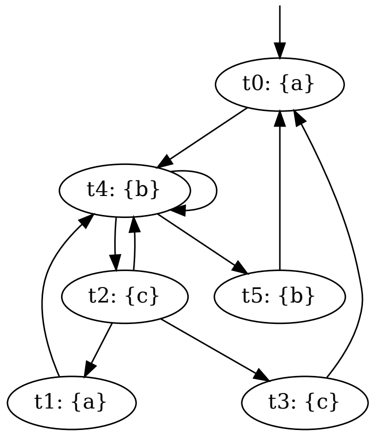
(6 points)

4.) (a) Consider the Kripke structures M_1 and M_2. The initial state of M_1 is s_0, the initial state of M_2 is t_0.

Kripke structure M_1:



Kripke structure M_2:

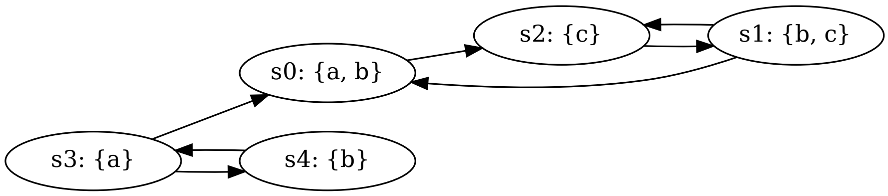


i. Check whether M_2 simulates M_1, i.e., provide a simulation relation that witnesses $M_1 \preceq M_2$, or briefly explain why M_2 does not simulate M_1.

ii. Check whether M_1 simulates M_2, i.e., provide a simulation relation that witnesses $M_2 \preceq M_1$, or briefly explain why M_1 does not simulate M_2.

(4 points)

(b) Consider the following Kripke structure M:



For each of the following formulae φ,

i. indicate whether the formula is in CTL, LTL, and/or CTL*, and
ii. list the states s_i on which the formula φ holds; i.e. for which states s_i do we have $M, s_i \models \varphi$?
(If φ is a path formula, list the states s_i such that $M, s_i \models \mathbf{A}\varphi$.)

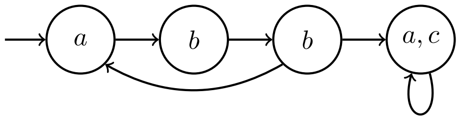
φ	CTL	LTL	CTL*	States s_i
$\mathbf{AX}(b)$	☐	☐	☐	
$((c)\ \mathbf{U}\ (b))$	☐	☐	☐	
$\mathbf{F}(b \wedge c)$	☐	☐	☐	
$\mathbf{E}[(b \wedge c)\ \mathbf{U}\ (a)]$	☐	☐	☐	

(4 points)

(c) Let $M = (S, S_0, R, AP, L)$ be a Kripke structure over a set of propositional symbols AP. We define $M' = (S', S'_0, R', AP', L')$ as follows:

- $AP' \subseteq AP$,
- $S' = S$, $S'_0 = S_0$, $R' = R$, and
- $L'(s) = L(s) \cap AP'$, where $s \in S$.

i. Consider the concrete instance M over $AP = \{a, b, c\}$ below. Draw M' with $AP' = \{a, c\}$ according to the definition above.



ii. Given any M and M' according to the definitions above, prove that for any ACTL formula φ over propositions from AP' the following holds:

$$M \models \varphi \text{ if and only if } M' \models \varphi$$

Hint: Use the semantics of ACTL and use induction on the structure of the formula (structural induction).

Hint: Recall the definition of ACTL formulae over AP:

- p and $\neg p$ are ACTL formulae for $p \in AP$,
- if φ is an ACTL formula, then $\mathbf{AX}\,\varphi$, $\mathbf{AF}\,\varphi$, and $\mathbf{AG}\,\varphi$ are ACTL formulae,
- if φ and ψ are ACTL formulae, then $\varphi \wedge \psi$, $\varphi \vee \psi$, and $\mathbf{A}\,[\varphi\,\mathbf{U}\,\psi]$ are ACTL formulae.

(7 points)