1	2	3	4	Σ	Grade

6.0/4.0 VU Formale Methoden der Informatik

185.291 March 25, 2022 Variant B

Kennz. (study id)	Matrikelnummer (student id)	Nachname (surname)	Vorname (first name)

1.) Recall the **HALTING** problem which takes a program and a string as input, and consider the following variant thereof:

> **HALTING-C**
>
> INSTANCE: A program Π that takes a string as input, a string I of even length $2 * n$.
>
> QUESTION: Does Π terminate on one of the two strings resulting from I being cut into two halfs, i.e. does Π halt on $I[1..n]$ or on $I[n+1..2*n]$.

(a) The following function f provides a polynomial-time many-one reduction from **HALTING** to **HALTING-C**: for a program Π and a string I, let $f(\Pi, I) = (\Pi', I')$ with $\Pi' = \Pi$ and $I' = I + I$ (i.e. the concatenation of two copies of string I)

Show that (Π, I) is a yes-instance of **HALTING** $\Longleftrightarrow$ (Π', I') is a yes-instance of **HALTING-C**.

(6 points)

(b) Please answer the following questions and explain your answers

- Is **HALTING-C** decidable?
- Is **HALTING-C** semi-decidable?
- Is the complement of **HALTING-C** semi-decidable?

(9 points)

2.) (a) Consider the following theory $\mathcal{T}_{tree}$ of trees with the signature

$$\Sigma_{tree} = \big\{\{tree, le, ri\},\ \{atom, \doteq\}\big\}.$$

The axioms of $\mathcal{T}_{tree}$ include symmetry, reflexivity and transitivity of equality, functional congruence for *tree*, *le*, *ri*, and predicate congruence for *atom*. In addition we have:

$$\begin{aligned}
&\forall x\,\forall y\ le(tree(x,y)) \doteq x && \text{(left subtree)}\\
&\forall x\,\forall y\ ri(tree(x,y)) \doteq y && \text{(right subtree)}\\
&\forall x\ \big(\neg atom(x) \rightarrow tree(le(x), ri(x)) \doteq x\big) && \text{(construction)}\\
&\forall x\,\forall y\ \neg atom(tree(x,y)) && \text{(atom)}
\end{aligned}$$

We augment theory $\mathcal{T}_{tree}$ by $\mathcal{T}_E$ (with uninterpreted function symbol h) resulting in $\mathcal{T}^E_{tree}$. Clarify the logical status of each of the following formulas. If one is $\mathcal{T}^E_{tree}$-valid or $\mathcal{T}^E_{tree}$-unsatisfiable, then prove it using the semantic argument method. If one is $\mathcal{T}^E_{tree}$-satisfiable but not $\mathcal{T}^E_{tree}$-valid, then present a satisfying and a falsifying interpretation. Argue formally that the formula evaluates to true resp. false under the constructed interpretations.

$$\begin{aligned}
&\varphi_0\colon \neg atom(x) \wedge le(x) \doteq y \wedge ri(x) \doteq z \wedge x \not\doteq tree(y,z)\\
&\varphi_1\colon le(a) \doteq le(b) \wedge ri(a) \doteq ri(b) \wedge \neg atom(a) \wedge \neg atom(b) \rightarrow h(a) \doteq h(b)
\end{aligned}$$

(8 points)

(b) Consider the following clause set $\hat{\delta}(\varphi)$ which has been derived from an (unknown) formula φ by an improved version of Tseitin's translation (atoms have not been labeled and $\overline{z}$ means $\neg z$).

$$\begin{array}{llll}
C_1\colon \overline{\ell_1} \vee x_1 \vee x_2 & C_2\colon \overline{\ell_1} \vee \overline{x_1} \vee \overline{x_2} & C_3\colon \ell_1 \vee \overline{x_1} \vee x_2 & C_4\colon \ell_1 \vee x_1 \vee \overline{x_2}\\
C_5\colon \overline{\ell_2} \vee x_2 \vee x_3 & C_6\colon \overline{\ell_2} \vee \overline{x_2} \vee \overline{x_3} & C_7\colon \ell_2 \vee \overline{x_2} \vee x_3 & C_8\colon \ell_2 \vee x_2 \vee \overline{x_3}\\
C_9\colon \overline{\ell_3} \vee \ell_1 \vee \ell_2 & C_{10}\colon \overline{\ell_3} \vee \overline{\ell_1} \vee \overline{\ell_2} & C_{11}\colon \ell_3 \vee \overline{\ell_1} \vee \ell_2 & C_{12}\colon \ell_3 \vee \ell_1 \vee \overline{\ell_2}\\
C_{13}\colon \overline{\ell_4} \vee x_2 & C_{14}\colon \overline{\ell_4} \vee \ell_3 & C_{15}\colon \ell_4 \vee \overline{x_2} \vee \overline{\ell_3} & \\
C_{16}\colon \overline{\ell_5} \vee x_1 \vee x_3 & C_{17}\colon \ell_5 \vee \overline{x_1} & C_{18}\colon \ell_5 \vee \overline{x_3} & \\
C_{19}\colon \overline{\ell_6} \vee \overline{\ell_4} \vee \ell_5 & C_{20}\colon \ell_6 \vee \ell_4 & C_{21}\colon \ell_6 \vee \overline{\ell_5} &
\end{array}$$

(i) Reconstruct φ from $\hat{\delta}(\varphi)$.

(ii) Prove the validity of φ by resolution (no additional translation to normal form is allowed!). You are allowed to add a single unit clause (i.e., a clause containing exactly one literal). Please explain your approach!

(7 points)

3.) (a) Let p be the following IMP program loop, containing the integer-valued program variables x, y:

$$\begin{array}{l} \textbf{while } x = y \textbf{ do} \\ \quad x := 2 * x + y; \\ \quad y := y - 2 * x \\ \textbf{od} \end{array}$$

Which of the following program assertions are inductive loop invariants of p?

- $I_1 : \quad x = 0 \wedge y = 0$
- $I_2 : \quad x - y = 0$
- $I_3 : \quad x = 0 \wedge y = 1$

Give formal details justifying your answer. That is, if an assertion is an inductive loop invariant, provide a formal proof of it based on Hoare logic or using weakest liberal preconditions. If an assertion is not an inductive loop invariant, give a counterexample.

Note: You need to use the definition of an assertion being an inductive invariant.

(9 points)

(b) Let A be an arbitrary post-condition. Which of the following Hoare triples are valid total correctness assertions?

- $[true]$ **skip** $[A]$
- $[false]$ **skip** $[A]$

Give formal details justifying your answer. That is, if a triple is valid, provide a formal proof of it based on Hoare logic. If an assertion is not valid, give a counterexample (that is, an instance of A for which the triple does not hold).

(4 points)

(c) Consider the Hoare triple $[A]p[B]$, where p is an arbitrary IMP program and A, B are arbitrary program assertions. Assume there is a state σ that satisfies A and there is a state σ' such that $< p, \sigma > \rightarrow \sigma'$ and σ' satisfies B.

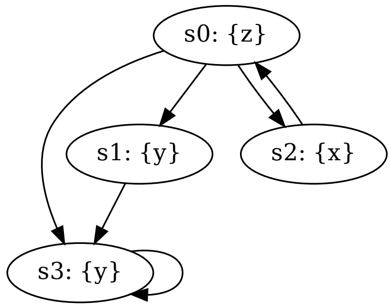
Given this information, is $[A]p[B]$ totally correct?

Answer the question with either a Yes or a No answer, and provide a short justification for your answer.

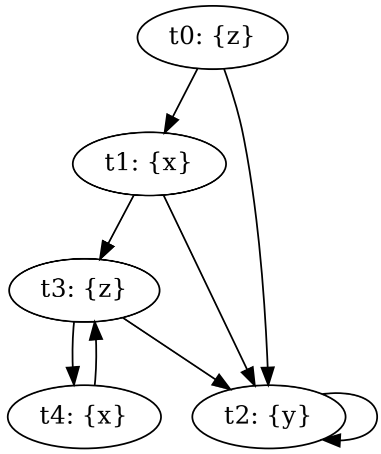
(2 points)

4.) (a) Consider the Kripke structures M_1 and M_2. The initial state of M_1 is s_0, the initial state of M_2 is t_0.

Kripke structure M_1:



Kripke structure M_2:

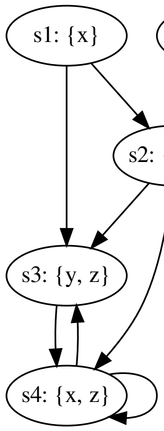


i. Check whether M_2 simulates M_1, i.e., provide a simulation relation that witnesses $M_1 \preceq M_2$, or briefly explain why M_2 does not simulate M_1.

ii. Check whether M_1 simulates M_2, i.e., provide a simulation relation that witnesses $M_2 \preceq M_1$, or briefly explain why M_1 does not simulate M_2.

(4 points)

(b) Consider the following Kripke structure M:



For each of the following formulae φ,

i. indicate whether the formula is in CTL, LTL, and/or CTL*, and

ii. list the states s_i on which the formula φ holds; i.e. for which states s_i do we have $M, s_i \models \varphi$?
(If φ is a path formula, list the states s_i such that $M, s_i \models \mathbf{A}\varphi$.)

φ	CTL	LTL	CTL*	States s_i
$\mathbf{G}(z)$	☐	☐	☐	
$\mathbf{EGF}(y)$	☐	☐	☐	
$\mathbf{A}[(x)\ \mathbf{U}\ (z)]$	☐	☐	☐	
$\mathbf{EX}(y)$	☐	☐	☐	
$\mathbf{FG}(y)$	☐	☐	☐	

(5 points)

(c) **LTL tautologies**

An LTL formula is a *tautology* if it holds for every Kripke structure M and every path π in M. For each of the following formulas, prove that it is a tautology, or find a Kripke structure M and path π in M for which the formula does not hold and justify your answer.

i. $\mathbf{G}(y \Rightarrow \mathbf{F}x) \Rightarrow (y \ \mathbf{U} \ \mathbf{G}(x \wedge \neg y))$
ii. $(y \ \mathbf{U} \ \mathbf{G}(x \wedge \neg y)) \Rightarrow \mathbf{G}(y \Rightarrow \mathbf{F}x)$

(6 points)

Grading scheme: 0–29 nicht genügend, 30–35 genügend, 36–41 befriedigend, 42–47 gut, 48–60 sehr gut