1	2	3	4	Σ

6.0/4.0 VU Formale Methoden der Informatik (185.291)

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1.) An undirected graph (V, E) is called a *degree-restricted graph* if for each vertex $v \in V$ it holds that the degree of v is odd (i.e. 1, 3, 5,...).

Examples: $(\{a, b, c, d\}, \{[a, b], [a, c], [a, d]\})$ is degree-restricted since vertex a has degree 3 and vertices $\{b, c, d\}$ have all degree 1.

$(\{a, b, c, d\}, \{[a, b], [b, c], [c, d]\})$ is not degree-restricted since vertices b and c have an even degree 2.

Consider the following variant of the 3-coloring problem:

> **3-COLORABILITY-DEGREE-RESTRICTED (3COLD)**
>
> INSTANCE: A degree-restricted graph $G = (V, E)$.
>
> QUESTION: Does there exist a function μ from vertices in V to values in $\{0, 1, 2\}$ such that $\mu(v_1) \neq \mu(v_2)$ for any edge $[v_1, v_2] \in E$.

(a) The following function f provides a polynomial-time many-one reduction from **3COL** to **3COLD**: for an undirected graph $G = (\{v_1, \dots, v_n\}, E)$, add for each vertex v_i with even degree

- a new vertex x_i
- an edge $[v_i, x_i]$

to G, and let $f(G)$ be the resulting degree-restricted graph.

Show that G is a yes-instance of **3COL** $\iff$ $f(G)$ is a yes-instance of **3COLD**.

(9 points)

(b) In what follows assume the reduction from **3COL** to **3COLD** is correct, and recall that **3COL** is NP-complete.

Tick the correct statements (for ticking a correct statement a certain number of points is given; ticking an incorrect statement results in a substraction of the same amount; you cannot go below 0 points):

- ○ Since **3COL** is NP-complete, our reduction shows that **3COLD** is NP-hard.
- ○ Since **3COL** is NP-hard, our reduction shows that **3COLD** is NP-hard.
- ○ Since **3COL** is in NP, our reduction shows that **3COLD** is NP-hard.
- ○ Since **3COLD** is a special case of **3COL**, it follows that **3COLD** is contained in NP.
- ○ Since **3COLD** is a special case of **3COL**, it follows that **3COLD** is NP-hard.
- ○ Since **3COLD** is a special case of **3COL**, it follows that **3COLD** is NP-complete (even without the above reduction).

(6 points)

2.) (a) Consider the following clause set $\hat{\delta}(\varphi)$ which has been derived from an (unknown) formula φ by an improved version of Tseitin's translation (atoms have not been labeled).

C_1:	$\ell_1 \vee \neg x \vee \neg y$	C_2:	$\neg \ell_1 \vee x$	C_3:	$\neg \ell_1 \vee y$
C_4:	$\neg \ell_2 \vee \neg y \vee z$	C_5:	$\ell_2 \vee y$	C_6:	$\ell_2 \vee \neg z$
C_7:	$\neg \ell_3 \vee \neg \ell_1 \vee z$	C_8:	$\ell_3 \vee \ell_1$	C_9:	$\ell_3 \vee \neg z$
C_{10}:	$\neg \ell_4 \vee \neg x \vee \ell_2$	C_{11}:	$\ell_4 \vee x$	C_{12}:	$\ell_4 \vee \neg \ell_2$
C_{13}:	$\neg \ell_5 \vee \neg \ell_4 \vee \ell_3$	C_{14}:	$\ell_5 \vee \ell_4$	C_{15}:	$\ell_5 \vee \neg \ell_3$

(i) Reconstruct φ from $\hat{\delta}(\varphi)$.

(ii) Prove the correctness of the propositional resolution rule.

(iii) Prove the validity of φ by resolution (no additional translation to normal form is allowed!). You are allowed to add a single unit clause (i.e., a clause containing exactly one literal). Please explain your approach!

(7 points)

(b) Clarify the logical status of each of the following formulas. If one is $\mathcal{T}^E_{cons}$-valid or $\mathcal{T}^E_{cons}$-unsatisfiable, then prove it using the semantic argument method. If one is $\mathcal{T}^E_{cons}$-satisfiable but not $\mathcal{T}^E_{cons}$-valid, then present a satisfying and a falsifying interpretation. Argue formally that the formula evaluates to true resp. false under the constructed interpretations.

$$\varphi_0 \colon \mathit{cons}(\mathit{car}(x), \mathit{cdr}(x)) \doteq \mathit{cons}(y, z) \wedge \mathit{cons}(\mathit{car}(x), \mathit{cdr}(x)) \not\doteq x$$
$$\rightarrow x \not\doteq \mathit{cons}(y, z)$$
$$\varphi_1 \colon \neg \mathit{atom}(x) \wedge \mathit{car}(x) \doteq y \wedge \mathit{cdr}(x) \doteq z \wedge x \not\doteq \mathit{cons}(y, z)$$

Besides the equality axioms, the following axioms of $\mathcal{T}^E_{cons}$ may be helpful.

- $\forall x, y\ \mathit{car}(\mathit{cons}(x, y)) \doteq x$ (left projection)
- $\forall x, y\ \mathit{cdr}(\mathit{cons}(x, y)) \doteq y$ (right projection)
- $\forall x\ \neg \mathit{atom}(x) \rightarrow \mathit{cons}(\mathit{car}(x), \mathit{cdr}(x)) \doteq x$ (construction)
- $\forall x, y\ \neg \mathit{atom}(\mathit{cons}(x, y))$ (atom)

(8 points)

(15 points)

3.) (a) Let p be the following IMP program loop, containing the integer-valued program variables x, i, b:

while $x < b$ **do**
  $i := i + 1;$
  $x := i * i;$
od

Which of the following program assertions are inductive loop invariants of p?

- I_1 : $x - i * i \neq 0$
- I_2 : $x \leq b$
- I_3 : $true$

Give formal details justifying your answer. That is, if an assertion is an inductive loop invariant, provide a formal proof of it based on Hoare logic. If an assertion is not an inductive loop invariant, give a counterexample.

Note: You need to use the definition of an assertion being inductive invariant.

(6 points)

(b) Let p be the following IMP program loop, containing the integer-valued program variables i, x, y:

while $i < 10$ **do**
  $x := x - 1;$
  $i := i + 1;$
  $y := y + x + i;$
od

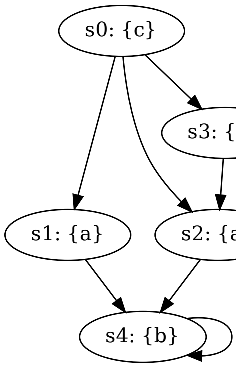
Give a loop invariant for the **while** loop in p and prove the validity of the partial correctness triple $\{i = 0 \wedge x = 10 \wedge y = 0\}\ p\ \{y = 100\}$.

Note: Make sure that your invariant expresses equalities among i, x, y as well equalities among i, x.

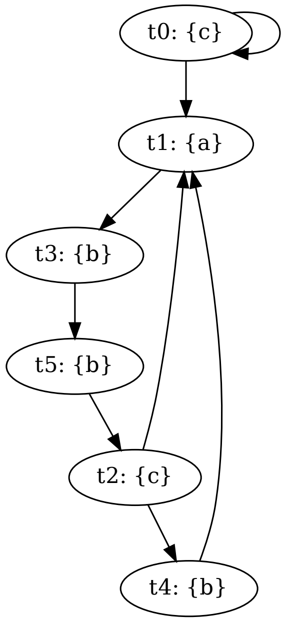
(9 points)

4.) (a) Consider the Kripke structures M_1 and M_2. The initial state of M_1 is s_0, the initial state of M_2 is t_0.

Kripke structure M_1:



Kripke structure M_2:

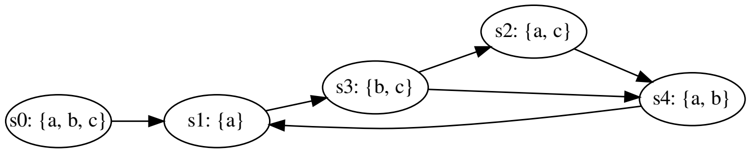


i. Briefly explain why M_2 does not simulate M_1.

ii. Add a minimal set of edges to M_2 such that the extended Kripke structure M_2' simulates M_1. Provide the additional edges and a non-empty simulation relation H that witnesses $M_1 \preceq M_2'$.

(4 points)

(b) Consider the following Kripke structure M:



For each of the following formulae φ,

i. indicate whether the formula is in CTL, LTL, and/or CTL*, and
ii. list the states s_i on which the formula φ holds; i.e. for which states s_i do we have $M, s_i \models \varphi$?
(If φ is a path formula, list the states s_i such that $M, s_i \models \mathbf{A}\varphi$.)

φ	CTL	LTL	CTL*	States s_i
$\mathbf{EF}(c)$	☐	☐	☐	
$c\ \mathbf{U}\ b$	☐	☐	☐	
$\mathbf{AX}(c)$	☐	☐	☐	
$\mathbf{G}(a)$	☐	☐	☐	
$\mathbf{AF}(a \wedge c)$	☐	☐	☐	

(5 points)

(c) **LTL tautologies**

Prove that the following formulas are tautologies, i.e., they hold for every Kripke structure M and every path π in M, or find a Kripke structure M and path π in M, for which the formula does not hold and justify your answer.

i. $\neg\mathbf{G}p \wedge \mathbf{F}q \Rightarrow p\ \mathbf{U}\ (q \vee \neg p)$

ii. $p\ \mathbf{U}\ (q \vee \neg p) \Rightarrow \neg\mathbf{G}p \wedge \mathbf{F}q$

(6 points)

Grading scheme: 0–29 nicht genügend, 30–35 genügend, 36–41 befriedigend, 42–47 gut, 48–60 sehr gut