1.) Consider the following decision problem:

**HALTING AFTER LINE-FLIP (HALF)**

**INSTANCE:** A tuple \((\Pi, I)\), where \(\Pi\) is a program that takes a string as input; \(I\) a string.

**QUESTION:** Do there exist two consecutive lines of code in \(\Pi\), such that when the two lines are flipped (i.e., the order of the two lines is reversed) in \(\Pi\), the resulting program (a) is syntactically correct and (b) halts on \(I\)?

(1) By providing a suitable many-one reduction from the **HALTING** problem, prove that **HALF** is undecidable.

(2) Is **HALF** semi-decidable? Explain your answer.  

(15 points)
2.) (a) Let $\varphi$ be the first-order formula

$$\forall x \forall y \left[ ((r(x, y) \land p(x)) \rightarrow p(y)) \land (r(x, y) \rightarrow (p(y) \rightarrow p(x))) \right].$$

i. Is $\varphi$ valid? If yes, present a proof. If no, give a counter-example and prove that it falsifies $\varphi$.

ii. Replace $r$ in $\varphi$ by $\approx$ (equality) resulting in $\psi$. Is $\psi$ E-valid? Argue formally!

(5 points)
(b) Show the following:

\[ \varphi^{EUF} \text{ is } E\text{-satisfiable iff } FC^E \land \text{flat}^E \text{ is } E\text{-satisfiable.} \]

\[ FC^E \text{ and flat}^E \text{ are obtained from } \varphi^{EUF} \text{ by Ackermann’s reduction.} \]

(Hint: \( FC^E \) is the same for \( \varphi^{EUF} \) and \( \neg \varphi^{EUF} \).) (10 points)
3.) (a) Let \( p \) be the following program:

\[
\begin{align*}
x &:= 1; \\
y &:= -2; \\
z &:= 2; \\
\textbf{while } x < N \textbf{ do} \\
& \quad x := x - 2 * y - 2 * z + 1; \\
& \quad y := y - 2 \\
& \quad z := z + 2 \\
\textbf{od}
\end{align*}
\]

Provide a formal proof of the partial correctness triple \( \{ N \geq 1 \} \ p \ {z = 2 \ast N} \).

Note: Give an appropriate loop invariant for the \textbf{while} loop in \( p \), to be further used in proving the partial correctness of the above Hoare triple.

\( (10 \text{ points}) \)
(b) Fill in the blank such that ...

... the following Hoare triple is totally correct.

\[
\begin{align*}
\{ & x \geq 2 \} \\
\text{while } & x > 2 \text{ do} \\
\quad \text{if } & x = 2 \times (x/2) \\
\quad \text{then } & x := x/2 \\
\text{else} & \\
\text{od} \\
\{ & x = 2 \}
\end{align*}
\]

... the following Hoare triple is partially but not totally correct.

\[
\begin{align*}
\{ & x \geq 2 \land x = 2 \times (x/2) + 1 \} \\
\text{while } & x > 1 \text{ do} \\
\quad \text{if } & x = 2 \times (x/2) \\
\quad \text{then } & x := x/2 \\
\text{else} & \\
\text{od} \\
\{ & x = 1 \}
\end{align*}
\]

... the following Hoare triple is not partially correct.

\[
\begin{align*}
\{ & x \geq 2 \} \\
\text{while } & x > 1 \text{ do} \\
\quad \text{if } & x = 2 \times (x/2) \\
\quad \text{then } & x := x/2 \\
\text{else} & \\
\text{od} \\
\{ & x = 1 \}
\end{align*}
\]

(5 points)

Note: Recall that / denotes integer division. Give a short informal justifications for your solutions. One or two sentences per example suffice and no formal proof is needed.
4.) (a) Provide a non-empty simulation relation $H$ that witnesses $M_1 \preceq M_2$, where $M_1$ and $M_2$ are shown below. The initial state of $M_1$ is $s_0$, the initial state of $M_2$ is $t_0$:

Kripke structure $M_1$:

Kripke structure $M_2$:

(4 points)
(b) Consider the following Kripke structure \( M \):

For each of the following formulae \( \varphi \),

i. check the respective box if the formula is in CTL, LTL, and/or CTL*, and

ii. list the states \( s_i \) on which the formula \( \varphi \) holds; i.e. for which states \( s_i \) do we have \( M, s_i \models \varphi \)?

<table>
<thead>
<tr>
<th>( \varphi )</th>
<th>CTL</th>
<th>LTL</th>
<th>CTL*</th>
<th>States ( s_i )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \text{G}(a) )</td>
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<td>( \text{X}(a) )</td>
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</table>

(5 points)
(c) **LTL tautologies**

Prove that the following formulas are tautologies, i.e., they hold for every Kripke structure $M$ and every path $\pi$ in $M$, or find a Kripke structure $M$ and path $\pi$ in $M$, for which the formula does not hold and justify your answer.

i. $X(p \mathbf{U} q) \Leftrightarrow (Xp) \mathbf{U} (Xq)$

ii. $\mathbf{FG}Fp \Leftrightarrow \mathbf{GFG}p$  

(6 points)