1	2	3	4	5	Σ	Grade

6.0/4.0 VU Formale Methoden der Informatik

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1.) Consider the following decision problem:

> **REMOVE VERTEX 3-COL (RV3COL)**
>
> INSTANCE: An undirected graph $G = (V, E)$.
>
> QUESTION: Does there exists a vertex $x \in V$ such that the removal of x from G yields a 3-colorable graph.
> Formally, does there exist $x \in V$ such that the graph $(V \setminus \{x\}, E')$ with $E' = \{(u, v) \in E \mid u \neq x \text{ and } v \neq x\}$ is 3-colorable.

Show NP-hardness of **RV3COL** by providing a polynomial-time many-one reduction from the standard **3-COL** problem. Prove the correctness of your reduction.

Recall that **3-COL** is defined as follows:

> **3-COL**
>
> INSTANCE: An undirected graph $G = (V, E)$.
>
> QUESTION: Does there exist a function μ from vertices in V to values in $\{0, 1, 2\}$ such that $\mu(v_1) \neq \mu(v_2)$ for any edge $[v_1, v_2] \in E$.

2.) In the exercise part, we considered the Ackermann-Péter function which is as follows.

Algorithm 1: The Ackermann-Péter function AP

```
Input: x, y, two non-negative integers
Output: The computed non-negative integer value for x, y
if x == 0 then
    return y + 1;
else if y == 0 then
    return AP(x − 1, 1);
else return AP(x − 1, AP(x, y − 1));
```

Show, using well-founded induction, that

$$\forall x\,\forall y\;\big((x \in \mathbb{N}_0 \;\wedge\; y \in \mathbb{N}_0) \;\rightarrow\; \mathrm{AP}(x,y) > y\big)$$

3.) (a) Let p be the following IMP program:

$$\begin{array}{l}\textbf{while } y < n \textbf{ do}\\ \quad x := x + 4 * y + 2;\\ \quad y := y + 1\\ \textbf{od}\end{array}$$

Prove the total correctness of the following Hoare triple:

$$[n = 10 \land x = 0 \land y = 0]\ p\ [x = 200].$$

(10 points)

(b) Consider the following proof rule:

$$\frac{\{A\}\ x := a_1\ \{B\} \quad \{B \wedge x \leq a_2\}\ p; x := x+1\ \{B\}}{\{A\}\ x := a_1; \textbf{while}\ x \leq a_2\ \textbf{do}\ p; x := x+1\ \textbf{od}\ \{B \wedge x = a_2 + 1\}}$$

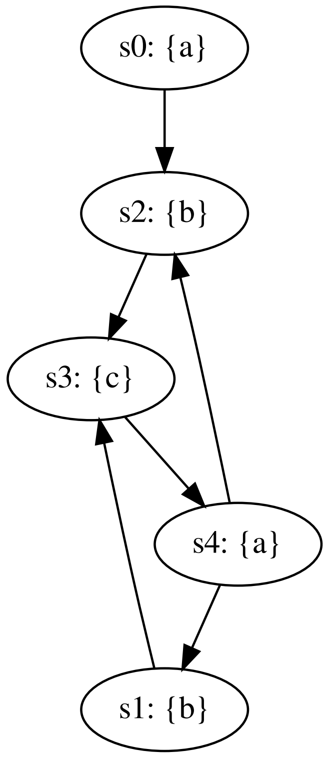
where x is a variable, a_1, a_2 are arithmetic expressions, p is a program, and A, B are assertions.

Is this proof rule sound? If yes, give a formal proof. Otherwise, give a counterexample.

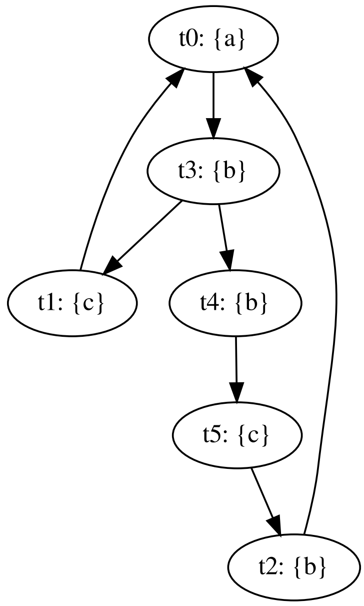
(5 points)

4.) (a) Provide a non-empty simulation relation H that witnesses $M_1 \leq M_2$, where M_1 and M_2 are shown below. The initial state of M_1 is s_0, the initial state of M_2 is t_0:

Kripke structure M_1**:**

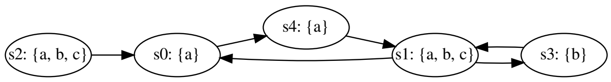


Kripke structure M_2**:**



(4 points)

(b) Consider the following Kripke structure M:



For each of the following formulae φ,

i. check the respective box if the formula is in CTL, LTL, and/or CTL*, and
ii. list the states s_i on which the formula φ holds; i.e. for which states s_i do we have $M, s_i \models \varphi$?

φ	CTL	LTL	CTL*	States s_i
$\mathbf{G}(a)$	☐	☐	☐	
$\mathbf{X}(a \wedge b \wedge c)$	☐	☐	☐	
$\mathbf{AF}(b)$	☐	☐	☐	
$\mathbf{A}[(a \wedge c)\ \mathbf{U}\ (c)]$	☐	☐	☐	
$\mathbf{E}[(b \wedge c)\ \mathbf{U}\ (b)]$	☐	☐	☐	

(5 points)

(c) **LTL tautologies**

Prove that the following formulas are tautologies, i.e., they hold for every Kripke structure M and every path π in M, or find a Kripke structure M and path π in M, for which the formula does not hold and justify your answer.

i. $\mathbf{F}p \Leftrightarrow ((\mathbf{X}\mathbf{G}\neg p) \Rightarrow p)$

ii. $(\mathbf{X}p)\ \mathbf{U}\ q \Leftrightarrow \mathbf{X}(p\ \mathbf{U}\ q)$

(6 points)