1	2	3	4	Σ	Grade

6.0/4.0 VU Formale Methoden der Informatik

185.291 **May, 3 2019**

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1.) An undirected graph is called a *non-2-degree graph* if no vertex in the graph has exactly two edges to other vertices.

Examples: $(\{a,b,c,d\},\{[a,b],[b,c],[b,d]\})$ is non-2-degree, while $(\{a,b,c,d\},\{[a,b],[b,c],[c,d]\})$ or $(\{a,b,c\},\{[a,b],[b,c],[a,c]\})$ are not.

Consider the following problem:

> **3-COLORABILITY-N2D**
>
> INSTANCE: A non-2-degree graph $G = (V,E)$.
>
> QUESTION: Does there exists a function μ from vertices in V to values in $\{0,1,2\}$ such that $\mu(v_1) \neq \mu(v_2)$ for any edge $[v_1, v_2] \in E$.

Use the fact that the standard version of the **3-COLORABILITY** problem is NP-complete to prove that **3-COLORABILITY-N2D** is NP-complete as well. Give a brief argument for NP-membership and show NP-hardness by providing a many-one reduction from the **3-COLORABILITY** problem. Prove the correctness of your reduction.

Recall that **3-COLORABILITY** is defined as follows:

> **3-COLORABILITY**
>
> INSTANCE: An undirected graph $G = (V,E)$.
>
> QUESTION: Does there exists a function μ from vertices in V to values in $\{0,1,2\}$ such that $\mu(v_1) \neq \mu(v_2)$ for any edge $[v_1, v_2] \in E$.

(15 points)

2.) (a) Consider $\mathcal{T}_{PA}^{+}$, which is Peano arithmetic with the axioms

$\forall x\ \neg(x + 1 \doteq 0)$ (zero)

$\forall x\ \forall y\ (x + 1 \doteq y + 1 \rightarrow x \doteq y)$ (successor)

$F[0] \wedge (\forall x\ (F[x] \rightarrow F[x + 1])) \rightarrow \forall x\ F[x]$ (induction)

$\forall x\ (x + 0 \doteq x)$ (plus zero)

$\forall x\ \forall y\ (x + (y + 1) \doteq (x + y) + 1)$ (plus successor)

$\forall x\ (x \cdot 0 \doteq 0)$ (times zero)

$\forall x\ \forall y\ (x \cdot (y + 1) \doteq (x \cdot y) + x)$ (time successor)

together with the following additional axioms:

$\forall x\ (x^0 \doteq 1)$ (exp zero)

$\forall x\ \forall y\ (x^{y+1} \doteq x^y \cdot x)$ (exp succ)

$\forall x\ \forall z\ (exp_3(x, 0, z) \doteq z)$ (exp_3 zero)

$\forall x\ \forall y\ \forall z\ (exp_3(x, y + 1, z) \doteq exp_3(x, y, x \cdot z))$ (exp_3 succ)

Show that $\forall x \forall y \forall z\ (exp_3(x, y, z) \doteq x^y \cdot z)$ is $\mathcal{T}_{PA}^{+}$-valid. Perform an induction proof and use the semantic argument method from the lecture to formally prove the formula in the base case and in the step case. In order to simplify the proof, you may use the formulas (L): $\forall x\ (1 \cdot x \doteq x)$ and (A): $\forall x \forall y \forall z\ (x \cdot (y \cdot z) \doteq (x \cdot y) \cdot z)$ as additional lemmas.

Please be precise and indicate exactly why proof lines follow from some other(s). Moreover, recall that equality handling is performed using equality axioms.

(12 points)

(b) Show the soundness of the following variant of the resolution rule.

$$\frac{C \vee p \vee q \quad D \vee \neg p \quad E \vee \neg q}{C \vee D \vee E}$$

(3 points)

3.) Note that all programs within this exercise are programs over the integers, that is, every program variable can only take integer values.

(a) Show that the Hoare triple $\{x = 6 \wedge y = 9\}\ p\ \{y = 0\}$ is valid with respect to partial correctness where p is the following program:

$$
\begin{array}{l}
\textbf{while } x > 0 \textbf{ do} \\
\quad x := x - 2; \\
\quad y := y - 3; \\
\textbf{od}
\end{array}
$$

(10 points)

(b) Let p be the program below. Is the Hoare triple $[n > 10 \wedge 2 < a < 10]\ p\ [n \leq 0]$ valid with respect to total correctness? If yes, prove its validity using the Hoare calculus. Otherwise, provide a counterexample – that is, a state that does not satisfy the correctness assertion – and argue why it is a counterexample.

```
while n > 0 do
  n := n − a;
  if n = 0 then
    abort
  fi
od
```

(2 points)

(c) Is the following alternative rule for assignment sound/admissible? If yes, provide a proof. Otherwise, provide a counterexample and argue why it is a counterexample.

$$\vdash \{true\}\ x := e\ \{x = e\}$$

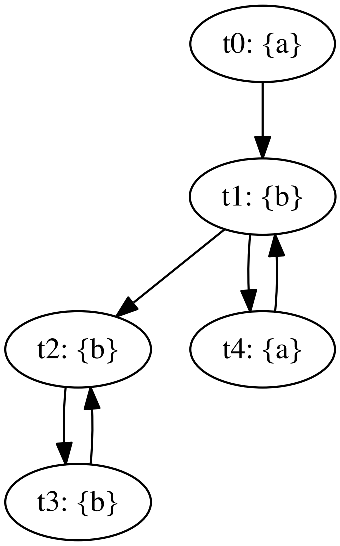
(3 points)

4.) (a) Consider M_1 shown below with initial state t_0.

Provide a Kripke structure M_2 such that

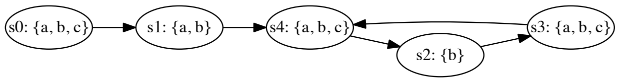
- $M_1 \leq M_2$ and
- M_2 has at most 3 states.

Furthermore provide a non-empty simulation relation H that witnesses $M_1 \leq M_2$.



(5 points)

(b) Consider the following Kripke structure M:



For each of the following formulae φ,

i. check the respective box if the formula is in CTL, LTL, and/or CTL*, and
ii. list the states s_i on which the formula φ holds; i.e. for which states s_i do we have $M, s_i \models \varphi$?

φ	CTL	LTL	CTL*	States s_i
$\mathbf{X}(c)$	☐	☐	☐	
$((a)\ \mathbf{U}\ (c))$	☐	☐	☐	
$\mathbf{AG}(b \wedge c)$	☐	☐	☐	
$\mathbf{EX}(c)$	☐	☐	☐	
$\mathbf{E}[(a \wedge c)\ \mathbf{U}\ (c)]$	☐	☐	☐	

(5 points)

(c) **LTL tautologies**

Prove or disprove (e.g. by providing a counter-example) the following CTL formula:

$$\big(\mathbf{AG}p \wedge \mathbf{EF}q\big) \Rightarrow \big(\mathbf{AF}(p \Rightarrow q)\big)$$

(5 points)