1.) An undirected graph is called a non-2-degree graph if no vertex in the graph has exactly two edges to other vertices.

   Examples: \((\{a, b, c, d\}, \{[a, b], [b, c], [b, d]\})\) is non-2-degree, while \((\{a, b, c, d\}, \{[a, b], [b, c], [c, d]\})\) or \((\{a, b, c\}, \{[a, b], [b, c], [a, c]\})\) are not.

   Consider the following problem:

   **3-COLORABILITY-N2D**

   INSTANCE: A non-2-degree graph \(G = (V, E)\).

   QUESTION: Does there exists a function \(\mu\) from vertices in \(V\) to values in \(\{0, 1, 2\}\) such that \(\mu(v_1) \neq \mu(v_2)\) for any edge \([v_1, v_2] \in E\).

   Use the fact that the standard version of the 3-COLORABILITY problem is NP-complete to prove that 3-COLORABILITY-N2D is NP-complete as well. Give a brief argument for NP-membership and show NP-hardness by providing a many-one reduction from the 3-COLORABILITY problem. Prove the correctness of your reduction.

   Recall that 3-COLORABILITY is defined as follows:

   **3-COLORABILITY**

   INSTANCE: An undirected graph \(G = (V, E)\).

   QUESTION: Does there exists a function \(\mu\) from vertices in \(V\) to values in \(\{0, 1, 2\}\) such that \(\mu(v_1) \neq \mu(v_2)\) for any edge \([v_1, v_2] \in E\).
2.) (a) Consider $\mathcal{T}_{PA}^+$, which is Peano arithmetic with the axioms

$$
\forall x \neg (x + 1 \doteq 0) \quad \text{(zero)}
$$

$$
\forall x \forall y (x + 1 \doteq y + 1 \rightarrow x \doteq y) \quad \text{(successor)}
$$

$$
F[0] \land (\forall x (F[x] \rightarrow F[x + 1])) \rightarrow \forall x F[x] \quad \text{(induction)}
$$

$$
\forall x (x + 0 \doteq x) \quad \text{(plus zero)}
$$

$$
\forall x \forall y (x + (y + 1) \doteq (x + y) + 1) \quad \text{(plus successor)}
$$

$$
\forall x (x \cdot 0 \doteq 0) \quad \text{(times zero)}
$$

$$
\forall x \forall y (x \cdot (y + 1) \doteq (x \cdot y) + x) \quad \text{(time successor)}
$$

together with the following additional axioms:

$$
\forall x (x^0 \doteq 1) \quad \text{(exp zero)}
$$

$$
\forall x \forall y (x^{y+1} \doteq x^y \cdot x) \quad \text{(exp succ)}
$$

$$
\forall x \forall z (\exp_3(x, 0, z) \doteq z) \quad \text{(exp}_3^\text{zero)}
$$

$$
\forall x \forall y \forall z (\exp_3(x, y + 1, z) \doteq \exp_3(x, y, x \cdot z)) \quad \text{(exp}_3^\text{succ)}
$$

Show that $\forall x \forall y \forall z (\exp_3(x, y, z) \doteq x^y \cdot z)$ is $\mathcal{T}_{PA}^+$-valid. Perform an induction proof and use the semantic argument method from the lecture to formally prove the formula in the base case and in the step case. In order to simplify the proof, you may use the formulas $(L): \forall x (1 \cdot x \doteq x)$ and $(A): \forall x \forall y \forall z (x \cdot (y \cdot z) \doteq (x \cdot y) \cdot z)$ as additional lemmas.

Please be precise and indicate exactly why proof lines follow from some other(s). Moreover, recall that equality handling is performed using equality axioms.

(12 points)
(b) Show the soundness of the following variant of the resolution rule.

\[
\frac{C \lor p \lor q \quad D \lor \neg p \quad E \lor \neg q}{C \lor D \lor E}
\]  

(3 points)
3.) Note that all programs within this exercise are programs over the integers, that is, every program variable can only take integer values.

(a) Show that the Hoare triple \( \{ x = 6 \land y = 9 \} \ p \ { y = 0 } \) is valid with respect to partial correctness where \( p \) is the following program:

\[
\text{while } x > 0 \text{ do }
  \quad x := x - 2;
  \quad y := y - 3;
\text{ od }
\]

(10 points)
(b) Let $p$ be the program below. Is the Hoare triple $[n > 10 \land 2 < a < 10] \ p [n \leq 0]$ valid with respect to total correctness? If yes, prove its validity using the Hoare calculus. Otherwise, provide a counterexample – that is, a state that does not satisfy the correctness assertion – and argue why it is a counterexample.

\[
\text{while } n > 0 \text{ do } \\
\quad n := n - a; \\
\quad \text{if } n = 0 \text{ then } \\
\quad \quad \text{abort} \\
\quad \text{fi} \\
\text{od} 
\]

(2 points)
(c) Is the following alternative rule for assignment sound/admissible? If yes, provide a proof. Otherwise, provide a counterexample and argue why it is a counterexample.

\[
\vdash \{true\} x := e \{x = e\}
\]

(3 points)
4.) (a) Consider $M_1$ shown below with initial state $t_0$.

Provide a Kripke structure $M_2$ such that

- $M_1 \leq M_2$ and
- $M_2$ has at most 3 states.

Furthermore provide a non-empty simulation relation $H$ that witnesses $M_1 \leq M_2$. 

(5 points)
Consider the following Kripke structure $M$:

For each of the following formulae $\varphi$,

i. check the respective box if the formula is in CTL, LTL, and/or CTL*, and

ii. list the states $s_i$ on which the formula $\varphi$ holds; i.e. for which states $s_i$ do we have $M, s_i \models \varphi$?

<table>
<thead>
<tr>
<th>$\varphi$</th>
<th>CTL</th>
<th>LTL</th>
<th>CTL*</th>
<th>States $s_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\text{X}(c)$</td>
<td>☐</td>
<td>☐</td>
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<tr>
<td>$(\langle a \rangle \text{ U } (c))$</td>
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<tr>
<td>$\text{AG}(b \land c)$</td>
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<td>☐</td>
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</tr>
<tr>
<td>$\text{EX}(c)$</td>
<td>☐</td>
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<tr>
<td>$\text{E}[\langle a \land c \rangle \text{ U } (c)]$</td>
<td>☐</td>
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</tr>
</tbody>
</table>
(c) LTL tautologies

Prove or disprove (e.g. by providing a counter-example) the following CTL formula:

\[(AGp \land EFq) \Rightarrow (AF(p \Rightarrow q))\]

(5 points)