6.0/4.0 VU Formale Methoden der Informatik 185.291 May 4, 2018			
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1.) An undirected graph is called a *non-terminal graph* if each vertex in the graph has at least two edges to other vertices. Examples: $(\{a,b,c\},\{[a,b],[b,c],[a,c]\})$ is non-terminal, while $(\{a,b,c\},\{[a,b],[b,c]\})$ or $(\{a,b,c,d\},\{[a,b],[b,c],[a,c]\})$ are not.

Consider the following problem:

> **3-COLORABILITY-NT**
>
> INSTANCE: A non-terminal graph $G = (V, E)$.
>
> QUESTION: Does there exists a function μ from vertices in V to values in $\{0, 1, 2\}$ such that $\mu(v_1) \neq \mu(v_2)$ for any edge $[v_1, v_2] \in E$.

Use the fact that the standard version of the **3-COLORABILITY** problem is NP-complete, to prove that **3-COLORABILITY-NT** is NP-complete as well. Give a brief argument for NP-membership and show NP-hardness by a reduction from **3-COLORABILITY**.

Recall that **3-COLORABILITY** is defined as follows:

> **3-COLORABILITY**
>
> INSTANCE: An undirected graph $G = (V, E)$.
>
> QUESTION: Does there exists a function μ from vertices in V to values in $\{0, 1, 2\}$ such that $\mu(v_1) \neq \mu(v_2)$ for any edge $[v_1, v_2] \in E$.

Hint: When reducing from **3-COLORABILITY** to **3-COLORABILITY-NT**, replace (certain) vertices by a triangle.

(15 points)

2.) (a) Recall that arrays are represented functionally. For instance, $write(a, i, e)$ is denoted by $a\langle i \triangleleft e\rangle$. Similarly, $read(a, k)$ is denoted by $a[k]$. Show that the following formula $a\langle i \triangleleft e\rangle\langle j \triangleleft f\rangle[k] \doteq g \land j \not\doteq k \land i \doteq j \rightarrow a[k] \doteq g$ is $\mathcal{T}_A$-valid. Please justify any step in your proof in detail.

Besides the equality axioms, you have the following ones for the arrays.

i. $\forall a, i, j \ \big(i \doteq j \ \rightarrow \ a[i] \doteq a[j]\big)$ (array congruence)

ii. $\forall a, v, i, j \ \big(i \doteq j \ \rightarrow \ a\langle i \triangleleft v\rangle[j] \doteq v\big)$ (read-over-write 1)

iii. $\forall a, v, i, j \ \big(i \not\doteq j \ \rightarrow \ a\langle i \triangleleft v\rangle[j] \doteq a[j]\big)$ (read-over-write 2)

(12 points)

(b) Show that the propositional resolution rule is sound. **(3 points)**

3.) Consider the following axioms of Hoare calculus:

$$\{\, G[v/e] \,\}\ v := e\ \{\, G \,\} \quad \text{(as)} \qquad \{\, F \,\}\ v := e\ \{\, F[v/e] \,\} \quad \text{(xx)}$$

$$\{\, F \,\}\ v := e\ \{\, \exists v'\, (F[v/v'] \land v = e[v/v']) \,\} \quad \text{(as}'\text{)} \qquad \{\, F \,\}\ v := e\ \{\, F \land v = e \,\} \quad \text{(as}''\text{)}$$

provided v' does not occur in F and e — provided v does not occur in F and e

(a) Show that the axioms (as) and (as′) are equivalent, i.e., that a complete calculus needs only one of the axioms. **(6 points)**

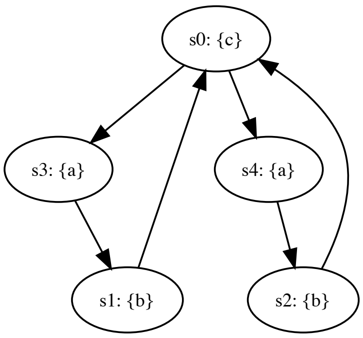
(b) Show that the axiom (as″) is sound, i.e., that each instance of it is a true correctness assertion. You may assume that (as) and (as′) are sound. **(3 points)**

(c) Show that the axiom (xx) is not sound. **(3 points)**

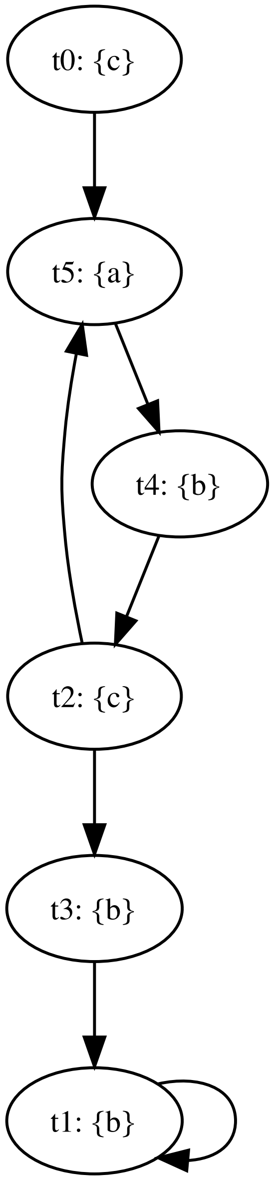
(d) Show that the Hoare calculus is not complete if it contains axiom (as″) but neither (as) nor (as′). **(3 points)**

4.) (a) Provide a non-empty simulation relation H that witnesses $M_1 \leq M_2$, where M_1 and M_2 are shown below. The initial state of M_1 is s_0, the initial state of M_2 is t_0:

Kripke structure M_1:

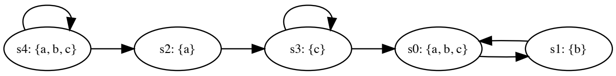


Kripke structure M_2:



(4 points)

(b) Consider the following Kripke structure M:



For each of the following formulae φ,

i. check the respective box if the formula is in CTL, LTL, and/or CTL*, and
ii. list the states s_i on which the formula φ holds; i.e. for which states s_i do we have $M, s_i \models \varphi$?

φ	CTL	LTL	CTL*	States s_i
$\mathbf{G}(b)$	☐	☐	☐	
$\mathbf{AF}(c)$	☐	☐	☐	
$((c)\ \mathbf{U}\ (a))$	☐	☐	☐	
$\mathbf{EG}(c)$	☐	☐	☐	
$\mathbf{E}[(a \wedge b)\ \mathbf{U}\ (c)]$	☐	☐	☐	

(5 points)

(c) **LTL tautologies**

Prove or disprove (e.g. by providing a counter-example) the following LTL formulas:

i.
$$(p\ \mathbf{U}\ \neg q) \rightarrow (\neg \mathbf{G} q)$$

ii.
$$(\mathbf{FG}p) \rightarrow (\mathbf{GF}p)$$

iii.
$$\mathbf{FX}p \rightarrow \mathbf{XG}p$$

(6 points)