1.) Consider the following problem:

**3-COLORABILITY-MIRROR**

INSTANCE: A pair \((G, x)\) with \(G = (V,E)\) an undirected graph and \(x \in V\) a vertex.

QUESTION: Is it true that \(G^* = (V^*,E^*)\) is 3-colorable, where \(G^*\) is defined via vertices \(V^* = V \cup \{v' \mid v \in V\}\) and edges \(E^* = E \cup \{[u',v'] \mid [u,v] \in E\} \cup \{[x,x']\}\)?

By providing a suitable reduction from the standard **3-COLORABILITY** problem, prove that **3-COLORABILITY-MIRROR** is an NP-hard problem. Argue formally that your reduction is correct.

Recall that **3-COLORABILITY** is defined as follows:

**3-COLORABILITY**

INSTANCE: An undirected graph \(G = (V,E)\).

QUESTION: Does there exist a function \(\mu\) from vertices in \(V\) to values in \(\{0,1,2\}\) such that \(\mu(v_1) \neq \mu(v_2)\) for any edge \([v_1,v_2] \in E\).

**Hint:** If there is a valid coloring \(\mu\) for a graph \(G\) then there is also also a valid coloring \(\mu'\) for \(G\) which “swaps” colors (i.e. \(\mu(v) \neq \mu'(v)\) for each vertex \(v\) in \(G\)).

(15 points)

2.) (a) Use a semantic argument to prove the \(T_A\)-validity of the following \(\Sigma_A\)-formula, or provide a counterexample (i.e., a falsifying \(T_A\)-interpretation):

\[
a(i \triangleleft e)[j] = e \rightarrow i = j \vee a[j] = e
\]

(8 points)

(b) Is the following \(\Sigma_A\)-formula \(T_A\)-valid? Justify your answer.

\[
a(i \triangleleft e)[j] = e \rightarrow i = j
\]

(1 point)

(c) Is the following \(\Sigma_A\)-formula \(T_A\)-valid? Justify your answer.

\[
a(i \triangleleft e)[j] = e \rightarrow a[j] = e
\]

(1 point)
(d) Let $\psi$ be a propositional formula in CNF and let $S := \{ C \in \psi \mid \ell \in C \}$ be the set of all clauses $C$ in $\psi$ that contain $\ell$, where $\ell$ is the literal of some arbitrary but fixed variable. Assume that literal $\neg \ell$ does not occur in any clause in $\psi$.

Let $\psi'$ be the CNF obtained from $\psi$ by removing all clauses in $S$: $\psi' := \psi \setminus S$.

Give a detailed proof of the following statement:

$\psi$ is satisfiable if and only if $\psi'$ is satisfiable.

(5 points)

3.) Let $\pi$ be the program $x := x - y; y := x + y; x := y - x$.

(a) Specify a correctness assertion stating that this program swaps the values of the variables $x$ and $y$.

(1 point)

(b) Prove the correctness assertion using weakest preconditions.

(5 points)

(c) Prove the correctness assertion using strongest postconditions.

(9 points)

4.) (a) Provide a non-empty simulation relation $H$ that witnesses $M_1 \leq M_2$, where $M_1$ and $M_2$ are shown below. The initial state of $M_1$ is $s_0$, the initial state of $M_2$ is $t_0$:

Kripke structure $M_1$:

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Kripke structure M2:
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(b) Consider the following Kripke structure $M$:
For each of the following formulae \( \varphi \),

i. check the respective box if the formula is in CTL, LTL, and/or CTL*; and

ii. list the states \( s_i \) on which the formula \( \varphi \) holds; i.e. for which states \( s_i \) do we have \( M, s_i \models \varphi \) ?

<table>
<thead>
<tr>
<th>( \varphi )</th>
<th>CTL</th>
<th>LTL</th>
<th>CTL*</th>
<th>States ( s_i )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( F(b \land c) )</td>
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<tr>
<td>( X(a \land b \land c) )</td>
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<td>( (b) \ U (b) )</td>
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<td>( EG(c) )</td>
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<tr>
<td>( E[(a \land b) \ U (a)] )</td>
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(5 points)

(c) **CTL Model Checking Algorithm**

Let \( K = (S,T,L) \) be a Kripke structure and let \( p \) be an atomic proposition. Give an algorithm that computes the set of all states \( s \in S \) that satisfy \( EGp \).

(6 points)