6.0/4.0 VU Formale Methoden der Informatik

185.291 **December 12, 2017**

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1.) Consider the following problem:

> **3-COLORABILITY-MIRROR**
>
> INSTANCE: A pair (G, x) with $G = (V, E)$ an undirected graph and $x \in V$ a vertex.
>
> QUESTION: Is it true that $G^* = (V^*, E^*)$ is 3-colorable, where G^* is defined via vertices $V^* = V \cup \{v' \mid v \in V\}$ and edges $E^* = E \cup \{[u', v'] \mid [u, v] \in E\} \cup \{[x, x']\}$?

By providing a suitable reduction from the standard **3-COLORABILITY** problem, prove that **3-COLORABILITY-MIRROR** is an NP-hard problem. Argue formally that your reduction is correct.

Recall that **3-COLORABILITY** is defined as follows:

> **3-COLORABILITY**
>
> INSTANCE: An undirected graph $G = (V, E)$.
>
> QUESTION: Does there exists a function μ from vertices in V to values in $\{0, 1, 2\}$ such that $\mu(v_1) \neq \mu(v_2)$ for any edge $[v_1, v_2] \in E$.

Hint: If there is a valid coloring μ for a graph G then there is also also a valid coloring μ' for G which "swaps" colors (i.e. $\mu(v) \neq \mu'(v)$ for each vertex v in G).

(15 points)

2.) (a) Use a semantic argument to prove the $\mathcal{T}_A$-validity of the following Σ_A-formula, or provide a counterexample (i.e., a falsifying $\mathcal{T}_A$-interpretation):

$$a\langle i \triangleleft e\rangle[j] = e \rightarrow i = j \vee a[j] = e$$

(8 points)

(b) Is the following Σ_A-formula $\mathcal{T}_A$-valid? Justify your answer.

$$a\langle i \triangleleft e\rangle[j] = e \rightarrow i = j$$

(1 point)

(c) Is the following Σ_A-formula $\mathcal{T}_A$-valid? Justify your answer.

$$a\langle i \triangleleft e\rangle[j] = e \rightarrow a[j] = e$$

(1 point)

(d) Let ψ be a propositional formula in CNF and let $S := \{C \in \psi \mid \ell \in C\}$ be the set of all clauses C in ψ that contain ℓ, where ℓ is the literal of some arbitrary but fixed variable. Assume that literal $\neg\ell$ does not occur in any clause in ψ.

Let ψ' be the CNF obtained from ψ by removing all clauses in S: $\psi' := \psi \setminus S$.

Give a detailed proof of the following statement:

ψ is satisfiable if and only if ψ' is satisfiable.

(5 points)

3.) Let π be the program $x := x - y;\ y := x + y;\ x := y - x$.

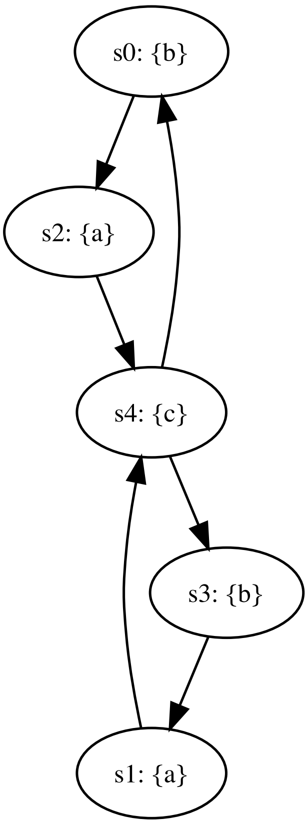
(a) Specify a correctness assertion stating that this program swaps that values of the variables x and y. **(1 point)**

(b) Prove the correctness assertion using weakest preconditions. **(5 points)**

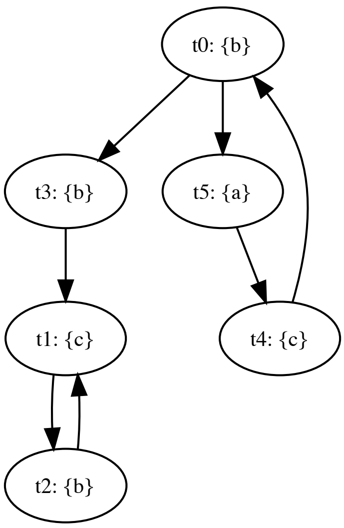
(c) Prove the correctness assertion using strongest postconditions. **(9 points)**

4.) (a) Provide a non-empty simulation relation H that witnesses $M_1 \leq M_2$, where M_1 and M_2 are shown below. The initial state of M_1 is s_0, the initial state of M_2 is t_0:

Kripke structure M_1:

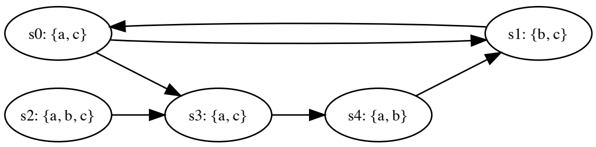


Kripke structure M_2:



(4 points)

(b) Consider the following Kripke structure M:



For each of the following formulae φ,

i. check the respective box if the formula is in CTL, LTL, and/or CTL*, and
ii. list the states s_i on which the formula φ holds; i.e. for which states s_i do we have $M, s_i \models \varphi$?

φ	CTL	LTL	CTL*	States s_i
$\mathbf{F}(b \land c)$	☐	☐	☐	
$\mathbf{X}(a \land b \land c)$	☐	☐	☐	
$((b)\ \mathbf{U}\ (b))$	☐	☐	☐	
$\mathbf{EG}(c)$	☐	☐	☐	
$\mathbf{E}[(a \land b)\ \mathbf{U}\ (a)]$	☐	☐	☐	

(5 points)

(c) **CTL Model Checking Algorithm**

Let $K = (S, T, L)$ be a Kripke structure and let p be an atomic proposition. Give an algorithm that computes the set of all states $s \in S$ that satisfy $\mathbf{EG}p$.

(6 points)