1.) We provide next a reduction from **2-COLORABILITY** to **2-SAT**. Let \( G = (V, E) \) be an arbitrary undirected graph (i.e. an arbitrary instance of **2-COLORABILITY**), where \( V = \{v_1, \ldots, v_n\} \). For the reduction we use propositional variables \( x_1, \ldots, x_n \). Then the instance \( \varphi_G \) of **2-SAT** resulting from \( G \) is defined as follows:

\[
\varphi_G = \bigwedge_{[v_i, v_j] \in E} \left( x_i \lor x_j \right) \land \left( \neg x_i \lor \neg x_j \right).
\]

**Task:** Prove the "⇒" direction in the proof of correctness of the reduction, i.e. prove the following statement: if \( G \) is a positive instance of **2-COLORABILITY**, then \( \varphi_G \) is a positive instance of **2-SAT**.

(15 points)

2.) (a) Let \( \varphi^{EUF} \) be the following **EUF**-formula:

\[
\varphi^{EUF} := x = y \land f(x) = g(y) \land z = g(f(y)) \land z \neq g(f(x)) \land P(g(f(y)), x)
\]

where \( f \) and \( g \) denote uninterpreted functions and \( P \) denotes an uninterpreted predicate. Is \( \varphi^{EUF} \) \( E \)-satisfiable or \( E \)-unsatisfiable?

Either provide a concrete \( E \)-interpretation showing that \( \varphi^{EUF} \) is \( E \)-satisfiable or a detailed proof showing that \( \varphi^{EUF} \) is \( E \)-unsatisfiable.

(5 points)

(b) Consider the **EUF**-formula \( \varphi^{EUF} \) from above (Exercise 2a):

\[
\varphi^{EUF} := x = y \land f(x) = g(y) \land z = g(f(y)) \land z \neq g(f(x)) \land P(g(f(y)), x)
\]

Apply Ackermann's reduction to obtain an \( E \)-formula \( \varphi^E \) from \( \varphi^{EUF} \) so that \( \varphi^E \) is \( E \)-satisfiable if and only if \( \varphi^{EUF} \) is \( E \)-satisfiable.

(Attention: here we consider satisfiability-equivalence, and not validity-equivalence.)

(10 points)

3.) Consider the following modified if-rule:

\[
\begin{aligned}
\{ F \land e \} & \ p \{ G \} & \{ F \} & \ q \{ G \} \\
\{ F \} & \text{if } e \text{ then } p \text{ else } q \ fi \{ G \} & \text{if}^{'}
\end{aligned}
\]

(a) Show that this rule is admissible regarding partial correctness.

(4 points)

(b) Show that the Hoare calculus for partial correctness is no longer complete, if we replace the regular if-rule by the modified one.

(7 points)

(c) Characterize all programs \( p \) that satisfy \( \wp(p, \text{true}) = \text{true} \), i.e., specify a condition such that the equation holds exactly when \( p \) satisfies this condition.

(4 points)
A rule \( \{F\} p \{G\} \) is admissible regarding partial correctness, if the conclusion \( \{F\} p \{G\} \) is partially correct whenever all premises \( X_1, \ldots, X_n \) are valid formulas/partially correct assertions.

**Hoare calculus for partial correctness:**

\[
\begin{align*}
\{F\} \text{skip} \{F\} \\
\{F\} \text{abort} \{G\} \\
\{F[v/e]\} v := e \{F\} \\
\{F\} p \{G\} \{G\} q \{H\} \\
\{F\} p; q \{H\} \\
\{F\} \text{if} e \text{ then } p \text{ else } q \{G\} \\
\{\text{Inv} \land e\} p \{\text{Inv}\} \\
\{\text{Inv}\} \text{while} e \text{ do } p \text{ od} \{\text{Inv} \land \neg e\} \\
F \rightarrow F' \{F'\} p \{G'\} G' \rightarrow G \\
\{F\} p \{G\}
\end{align*}
\]

4.) (a) Provide a non-empty simulation relation \( H \) that witnesses \( M_1 \leq M_2 \), where \( M_1 \) and \( M_2 \) are shown below. The initial state of \( M_1 \) is \( s_0 \), the initial state of \( M_2 \) is \( t_0 \):

**Kripke structure \( M_1 \):**

**Kripke structure \( M_2 \):**

(b) Consider the following Kripke structure \( M \):

For each of the following formulae \( \varphi \),
i. check the respective box if the formula is in CTL, LTL, and/or CTL*, and
ii. list the states \( s_i \) on which the formula \( \varphi \) holds; i.e. for which states \( s_i \) do we have \( M, s_i \models \varphi \)?

<table>
<thead>
<tr>
<th>( \varphi )</th>
<th>CTL</th>
<th>LTL</th>
<th>CTL*</th>
<th>States ( s_i )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( F(b \land c) )</td>
<td>□</td>
<td>□</td>
<td>□</td>
<td></td>
</tr>
<tr>
<td>( X(a \land b \land c) )</td>
<td>□</td>
<td>□</td>
<td>□</td>
<td></td>
</tr>
<tr>
<td>( (b) \mathbf{U} (b) )</td>
<td>□</td>
<td>□</td>
<td>□</td>
<td></td>
</tr>
<tr>
<td>( EG(c) )</td>
<td>□</td>
<td>□</td>
<td>□</td>
<td></td>
</tr>
<tr>
<td>( E[(a \land b) \mathbf{U} (a)] )</td>
<td>□</td>
<td>□</td>
<td>□</td>
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</tr>
</tbody>
</table>

(5 points)

(c) **CTL Model Checking Algorithm**

Let \( K = (S, T, L) \) be a Kripke structure and let \( p \) be an atomic proposition. Give an algorithm that computes the set of all states \( s \in S \) that satisfy \( EGp \).

(6 points)