6.0/4.0 VU Formale Methoden der Informatik (185.291)

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1.) Consider the following problem:

> **HALTING-NO-INPUT (HNI)**
>
> INSTANCE: A program Π such that Π takes no input.
>
> QUESTION: Does Π terminate?

By providing a reduction from **HALTING** to **HNI**, prove that **HNI** is undecidable. Argue formally that your reduction is correct.

(15 points)

2.) (a) Let φ^E be any E-formula with Boolean variables $b_1, \ldots, b_n$. Construct an E-formula ψ^E without any Boolean variable by replacing each b_i $(i = 1, \ldots, n)$ by an equality e_i of the form $v_i \doteq w_i$, where $v_1, w_1, \ldots, v_n, w_n$ are new distinct term variables (identifiers). Prove: φ^E is E-satisfiable if ψ^E is E-satisfiable. **(12 points)**

(b) Consider the clauses $C_1, \ldots, C_6$ in `dimacs` format (in this order, shown in the box; recall that `0` indicates the end of a clause) which are given as input to a SAT solver.

- Apply CDCL using the convention that if a variable is assigned as a decision, then it is assigned 'true'. Select variables as decisions in *increasing* order of their respective integer IDs in the `dimacs` format, starting with variable `1`.
- When the *first* conflict occurs, draw the complete implication graph, mark the first UIP, give the derivation of the learned asserting clause that corresponds to the first UIP, and stop CDCL. You do *not* have to solve the formula!

```
-1 4 0
-4 5 0
-2 -4 6 0
-3 -6 7 0
-7 9 0
-5 -6 -7 -9 0
```

(3 points)

3.) (a) Show that the scheme $\{\, F \,\}\, v := e \,\{\, F \wedge v = e \,\}$ is not an admissible axiom in general. Remember that an axiom is admissible if all its instances are true correctness assertions.

(5 points)

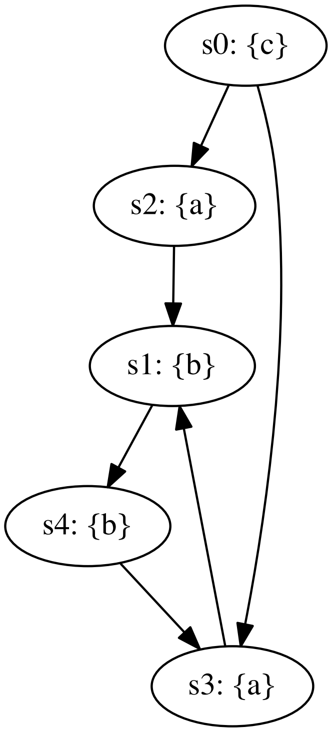
(b) Use weakest preconditions to compute a description of all states from which the following program will terminate.

$y := 3x;$
while $2x \neq y$ do
   $x := x + 2;$
   $y := y + 1$
od

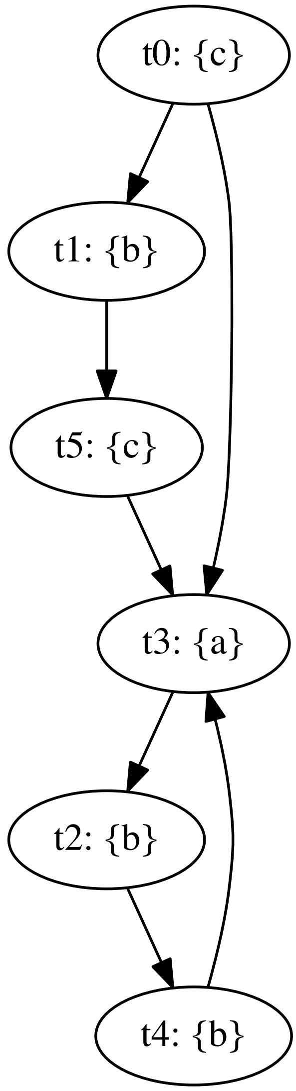
(10 points)

4.) (a) Provide a non-empty simulation relation H that witnesses $M_1 \leq M_2$, where M_1 and M_2 are shown below. The initial state of M_1 is s_0, the initial state of M_2 is t_0:

Kripke structure M_1:

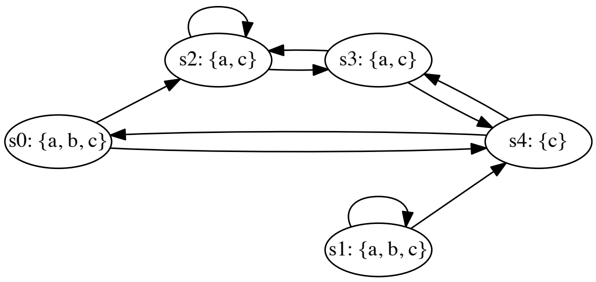


Kripke structure M_2:



(4 points)

(b) Consider the following Kripke structure M:



For each of the following formulae φ,

i. check the respective box if the formula is in CTL, LTL, and/or CTL*, and
ii. list the states s_i on which the formula φ holds; i.e. for which states s_i do we have $M, s_i \models \varphi$?

φ	CTL	LTL	CTL*	States s_i
$\mathbf{F}(a \wedge b \wedge c)$	☐	☐	☐	
$\mathbf{AG}(b)$	☐	☐	☐	
$\mathbf{AX}(a \wedge c)$	☐	☐	☐	
$\mathbf{A}[(a \wedge b)\ \mathbf{U}\ (a)]$	☐	☐	☐	
$\mathbf{E}[(a)\ \mathbf{U}\ (b)]$	☐	☐	☐	

(5 points)

(c) Prove the following equivalence of LTL formulae (4 points):

$$(\mathbf{G}a) \rightarrow (\mathbf{F}b) \equiv a\mathbf{U}(b \vee \neg a)$$

Prove that the following LTL formulae are not equivalent (2 points):

$$(\mathbf{F}a) \wedge (\mathbf{XG}a) \not\equiv \mathbf{F}a$$

(6 points)