1.) Consider the following problem:

**HALTING-NO-INPUT (HNI)**

**INSTANCE:** A program II such that II takes no input.

**QUESTION:** Does II terminate?

By providing a reduction from HALTING to HNI, prove that HNI is undecidable. Argue formally that your reduction is correct.

(15 points)

2.)

(a) Let $\varphi^E$ be any $E$-formula with Boolean variables $b_1, \ldots, b_n$. Construct an $E$-formula $\psi^E$ without any Boolean variable by replacing each $b_i$ ($i = 1, \ldots, n$) by an equality $e_i$ of the form $v_i = w_i$, where $v_1, w_1, \ldots, v_n, w_n$ are new distinct term variables (identifiers).

Prove: $\varphi^E$ is $E$-satisfiable if $\psi^E$ is $E$-satisfiable.

(12 points)

(b) Consider the clauses $C_1, \ldots, C_6$ in dimacs format (in this order, shown in the box; recall that 0 indicates the end of a clause) which are given as input to a SAT solver.

- Apply CDCL using the convention that if a variable is assigned as a decision, then it is assigned ‘true’. Select variables as decisions in increasing order of their respective integer IDs in the dimacs format, starting with variable 1.
- When the first conflict occurs, draw the complete implication graph, mark the first UIP, give the derivation of the learned asserting clause that corresponds to the first UIP, and stop CDCL. You do not have to solve the formula!

(3 points)

3.)

(a) Show that the scheme $\{ F \} v := e \{ F \land v = e \}$ is not an admissible axiom in general. Remember that an axiom is admissible if all its instances are true correctness assertions.

(5 points)

(b) Use weakest preconditions to compute a description of all states from which the following program will terminate.

```
y := 3x;
while 2x ≠ y do
  x := x + 2;
y := y + 1
od
```

(10 points)
4.) (a) Provide a non-empty simulation relation $H$ that witnesses $M_1 \leq M_2$, where $M_1$ and $M_2$ are shown below. The initial state of $M_1$ is $s_0$, the initial state of $M_2$ is $t_0$:

**Kripke structure $M_1$:**

**Kripke structure $M_2$:**

(b) Consider the following Kripke structure $M$: 
For each of the following formulae $\varphi$,

i. check the respective box if the formula is in CTL, LTL, and/or CTL*, and

ii. list the states $s_i$ on which the formula $\varphi$ holds; i.e. for which states $s_i$ do we have $M, s_i \models \varphi$?

<table>
<thead>
<tr>
<th>$\varphi$</th>
<th>CTL</th>
<th>LTL</th>
<th>CTL*</th>
<th>States $s_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$F(a \land b \land c)$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$AG(b)$</td>
<td></td>
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<tr>
<td>$AX(a \land b)$</td>
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</tr>
<tr>
<td>$A[(a \land b) U (a)]$</td>
<td></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>$E[(a) U (b)]$</td>
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</tbody>
</table>

(5 points)

c) Prove the following equivalence of LTL formulae (4 points):

$$(Ga) \rightarrow (Fb) \equiv aU(b \lor \neg a)$$

Prove that the following LTL formulae are not equivalent (2 points):

$$(Fa) \land (XGa) \nleq Fa$$

(6 points)