6.0/4.0 VU Formale Methoden der Informatik (185.291) March 17, 2017				
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1.) Provide a reduction from 2-**COLORABILITY** to 3-**COLORABILITY**, and prove that your reduction is correct. **(15 points)**

Hint: For the reduction it suffices to suitably introduce one additional vertex to the input graph.

We recall the definitions of **2-COLORABILITY** and **3-COLORABILITY**:

> **2-COLORABILITY**
>
> INSTANCE: An undirected graph $G = (V, E)$.
>
> QUESTION: Does there exist a function μ from vertices in V to values in $\{1, 2\}$ such that $\mu(v_1) \neq \mu(v_2)$ for any edge $[v_1, v_2] \in E$.

> **3-COLORABILITY**
>
> INSTANCE: An undirected graph $G = (V, E)$.
>
> QUESTION: Does there exist a function μ from vertices in V to values in $\{1, 2, 3\}$ such that $\mu(v_1) \neq \mu(v_2)$ for any edge $[v_1, v_2] \in E$.

2.) (a) Clarify the logical status of each of the following formulas. If one is $\mathcal{T}^E_{cons}$-valid or $\mathcal{T}^E_{cons}$-unsatisfiable, then prove it using semantics. If one is $\mathcal{T}^E_{cons}$-satisfiable but not $\mathcal{T}^E_{cons}$-valid, then present a satisfying and a falsifying interpretation. Argue formally that the formula evaluates to true resp. false under the constructed interpretations.

i. φ_0: $cons(car(x), cdr(x)) \doteq cons(y, z) \land cons(car(x), cdr(x)) \not\doteq x \rightarrow x \not\doteq cons(y, z)$

ii. φ_1: $\neg atom(x) \land car(x) \doteq y \land cdr(x) \doteq z \land x \not\doteq cons(y, z)$

iii. φ_2: $car(x) \doteq y \land cdr(x) \doteq z \land x \not\doteq cons(y, z)$

Besides the equality axioms, the following axioms of $\mathcal{T}^E_{cons}$ may be helpful.

- $\forall x, y\, car(cons(x, y)) \doteq x$ (left projection)
- $\forall x, y\, cdr(cons(x, y)) \doteq y$ (right projection)
- $\forall x\, \neg atom(x) \rightarrow cons(car(x), cdr(x)) \doteq x$ (construction)
- $\forall x, y\, \neg atom(cons(x, y))$ (atom)

(12 points)

(b) Show that the propositional resolution rule is sound. **(3 points)**

3.) Verify that the following program doubles the value of x, i.e., that x contains two times its initial value when the program terminates. For which inputs does it terminate? Choose appropriate pre- and postconditions and show that the assertion is totally correct.
Hint: Use $y = 2x_0 + x$ as a starting point for the invariant, where x_0 denotes the initial value of x. You may have to extend the formula to prove termination.

Remember the annotation rule
while e do $\cdots$ od $\mapsto$ $\{\, Inv \,\}$while e do $\{\, Inv \land e \land t = t_0 \,\} \cdots \{\, Inv \land (e \Rightarrow 0 \leq t < t_0) \,\}od\{\, Inv \land \neg e \,\}$

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y := 3x;
while 2x ≠ y do
    x := x + 1;
    y := y + 1;
od
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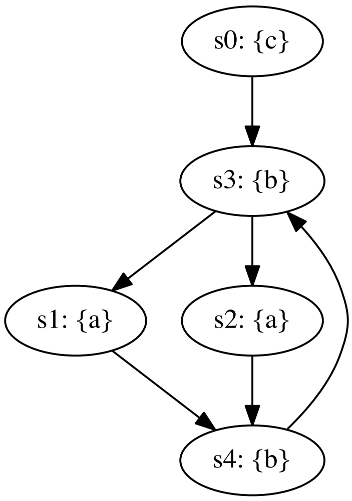
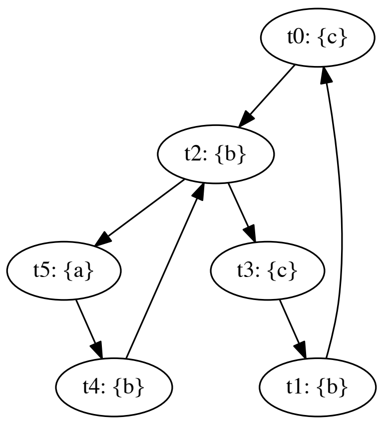
(15 points)

4.) (a) Provide a non-empty simulation relation H that witnesses $M_1 \leq M_2$, where M_1 and M_2 are shown below. The initial state of M_1 is s_0, the initial state of M_2 is t_0:

Kripke structure M_1:

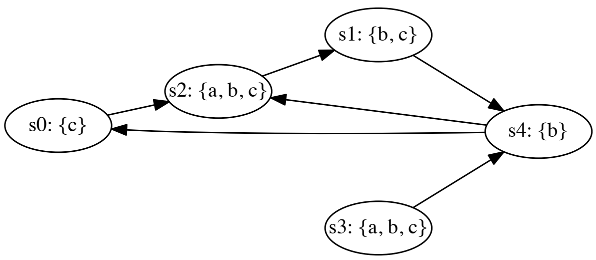


Kripke structure M_2:



(4 points)

(b) Consider the following Kripke structure M:



For each of the following formulae φ,

i. check the respective box if the formula is in CTL, LTL, and/or CTL*, and

ii. list the states s_i on which the formula φ holds; i.e. for which states s_i do we have $M, s_i \models \varphi$?

φ	CTL	LTL	CTL*	States s_i
$((b \wedge c)\ \mathbf{U}\ (a))$	☐	☐	☐	
$\mathbf{AX}(a)$	☐	☐	☐	
$\mathbf{EG}(b)$	☐	☐	☐	
$\mathbf{EF}(a)$	☐	☐	☐	
$\mathbf{E}[(a)\ \mathbf{U}\ (a)]$	☐	☐	☐	

(5 points)

(c) **CTL Model Checking Algorithm**

Let $K = (S, T, L)$ be a Kripke structure and let p, q be a atomic propositions. Give an algorithm that computes the set of all states $s \in S$ that satisfy $\mathbf{A}[p\mathbf{U}q]$.

(6 points)