| 6.0/4.0 VU Formale Methoden der Informatik (185.291)<br>December 9, 2016 | | | | |
|---|---|---|---|---|
| Kennzahl (study id) | Matrikelnummer (student id) | Familienname (family name) | Vorname (first name) | Gruppe (version) |
| | | | | |

**1.)** Consider the following problem:

> **PROB**
>
> INSTANCE: A program $\Pi$ such that $\Pi$ takes a string as input, and outputs a string. It is guaranteed that $\Pi$ terminates on any input string.
>
> QUESTION: Do there exist strings $I_1, I_2$ such that $\Pi(I_1) = I_2$, i.e., such that the output of $\Pi$ on the input $I_1$ is equal to $I_2$?

Prove that the problem **PROB** is semi-decidable. For this, describe a procedure that shows the semi-decidability of the problem (i.e. a semi-decision procedure for **PROB** ) and argue that it is correct.

Note: we consider only strings that are built from symbols 0 and 1. **(15 points)**

**2.)** (a) Given the following first-order logic formula $\psi$:

$$\psi : \big[p(f(x,y),u) \land p(x,z)\big] \rightarrow p(f(z,y),u)$$

where $f/2$ is a binary function symbol and $p/2$ is a binary predicate symbol. Let $T$ be a theory which forces $p/2$ to be reflexive, symmetric, and transitive. Additionally, $T$ includes the following axiom related to $p$ and $f$:

$$\forall x_1, x_2, y_1, y_2 : \big[p(x_1,x_2) \land p(y_1,y_2)\big] \rightarrow p(f(x_1,y_1), f(x_2,y_2))$$

Give a detailed proof that $\psi$ is $T$-valid. **(9 points)**

(b) Consider the clauses $C_1, \ldots, C_5$ in `dimacs` format (in this order, shown in the box; recall that `0` indicates the end of a clause) which are given as input to a SAT solver. Apply CDCL to solve the CNF using the convention that if a variable is assigned as a decision, then it is assigned 'false'. Further, select variable `3` as the first decision variable that is assigned.

- Each time when a conflict occurs and after backtracking, draw the implication graph and indicate all UIPs and mark the first UIP. For each UIP, indicate the cut (i.e., a set of edges) and its asserting conflict clause. Learn the asserting conflict clause that corresponds to the first UIP.
- Is the given CNF satisfiable, unsatisfiable, or valid? Can the empty clause be derived from the given CNF during CDCL? Justify your answers to the above questions.

```
-1 -2 -5 0
-1 -2 5 0
2 -4 0
1 3 -4 0
4 0
```

**(6 points)**

**3.)** Let $\pi$ be the program $\quad x := x - y;\ y := x + y;\ x := y - x$ .

(a) Specify a correctness assertion stating that this program swaps that values of the variables $x$ and $y$. **(1 point)**

(b) Prove the correctness assertion using weakest preconditions. **(5 points)**

(c) Prove the correctness assertion using strongest postconditions. **(9 points)**

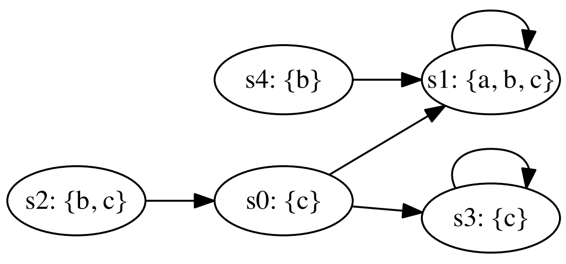
**4.)** (a) Show that simulation is a transitive relation, i.e. given any 3 Kripke structures

$$K_1 = \{S_1, I_1, R_1, L_1\},\ K_2 = \{S_2, I_2, R_2, L_2\} \text{ and } K_3 = \{S_3, I_3, R_3, L_3\}$$

over atomic predicates $AP$, such that $K_1 \leq K_2$ and $K_2 \leq K_3$, show that $K_1 \leq K_3$.

**(5 points)**

(b) Consider the following Kripke structure $M$:




For each of the following formulae $\varphi$,

i. check the respective box if the formula is in CTL, LTL, and/or CTL*, and
ii. list the states $s_i$ on which the formula $\varphi$ holds; i.e. for which states $s_i$ do we have $M, s_i \models \varphi$?

| $\varphi$ | CTL | LTL | CTL* | States $s_i$ |
|---|---|---|---|---|
| $\mathbf{G}(b)$ | ☐ | ☐ | ☐ | |
| $\mathbf{F}(a)$ | ☐ | ☐ | ☐ | |
| $\mathbf{X}(a)$ | ☐ | ☐ | ☐ | |
| $\mathbf{A}[a\ \mathbf{U}\ c]$ | ☐ | ☐ | ☐ | |
| $\mathbf{EF}(a)$ | ☐ | ☐ | ☐ | |

**(5 points)**

(c) The *subset sum problem* is defined as follows: Given a set of $N$ integers $S = \{i_1, i_2, \ldots, i_N\}$, does $S$ have a nonempty subset whose sum is zero?

Write a C program that implements a *guess and check* routine for the subset sum problem and instrument the program with an appropriate CBMC assertion.

You may assume the following template:

```
int nondet_bool(); // non-deterministically returns 0 or 1

// Fixed sample input:
int N = 8;                                          // size of the set
int values[] = { 4, -8, 15, -16, -23, 42, -11, 13 }; // elements in the set

int main() {
  // add code here:
  // 1. guess a solution
  // 2. put an assertion such that CBMC reports if there is a solution
}
```

**(5 points)**