6.0/4.0 VU Formale Methoden der Informatik (185.291)
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Kennzahl (study id)	Matrikelnummer (student id)	Familienname (family name)	Vorname (first name)	Gruppe (version)

1.) Consider the following problem:

> **SMALLER**
>
> INSTANCE: A program Π such that Π takes one string as input and outputs a string. It is guaranteed that Π terminates on any input string.
>
> QUESTION: Does there exists an input string I for Π such that $|\Pi(I)| < |I|$. Here $|J|$ denotes the length of a string J, and $\Pi(J)$ is the string returned by Π on input string J.

Prove that the problem **SMALLER** is semi-decidable. For this, describe a procedure that shows the semi-decidability of the problem (i.e. a semi-decision procedure for **SMALLER**) and argue that it is correct.

Note: we consider only strings that are built from symbols 0 and 1. **(15 points)**

2.) (a) Show the following:

$$\psi^{EUF} \text{ is satisfiable iff } FC^E(\psi^{EUF}) \wedge \mathit{flat}^E(\psi^{EUF}) \text{ is satisfiable.}$$

Note: $FC^E(\psi^{EUF})$ and $\mathit{flat}^E(\psi^{EUF})$ are obtained from ψ^{EUF} by Ackermann's reduction. **(8 points)**

(b) Let $\varphi : \forall x \exists y[(s(x) \sim y) \wedge (y \sim s(x))]$, where $\sim/2$ is a binary predicate written in infix notation. Let T be a theory which forces $\sim/2$ to be reflexive. Show by purely semantical means that $T \models \varphi$ holds. (Hint: show that $\mathit{Mod}(T) \subseteq \mathit{Mod}(\varphi)$.) **(5 points)**

(c) Show that the propositional resolution rule is sound. **(2 points)**

3.) Prove that the following correctness assertion is true regarding total correctness. Use the invariant $2x = y + 5z \wedge x \geq y$.

Some annotation rules that you might not remember:

abort $\mapsto$ $\{\text{false}\}$ abort $\{\text{false}\}$ $\qquad \{F\}v := e \mapsto \{F\}v := e\{\exists v'(F[v/v'] \wedge v = e[v/v'])\}$
if e then $\{F\} \cdots$ else $\{G\} \mapsto \{(e \Rightarrow F) \wedge (\neg e \Rightarrow G)\}$if e then $\{F\} \cdots$ else $\{G\}$
$\{F\}$if e then $\cdots$ else $\mapsto \{F\}$if e then $\{F \wedge e\} \cdots$ else $\{F \wedge \neg e\}$
while e do $\cdots$ od $\mapsto \{\mathit{Inv}\}$while e do $\{\mathit{Inv} \wedge e \wedge t = t_0\} \cdots \{\mathit{Inv} \wedge 0 \leq t < t_0\}od\{\mathit{Inv} \wedge \neg e\}$
while e do $\cdots$ od $\mapsto \{\mathit{Inv}\}$while e do $\{\mathit{Inv} \wedge e \wedge t = t_0\} \cdots \{\mathit{Inv} \wedge (e \Rightarrow 0 \leq t < t_0)\}od\{\mathit{Inv} \wedge \neg e\}$

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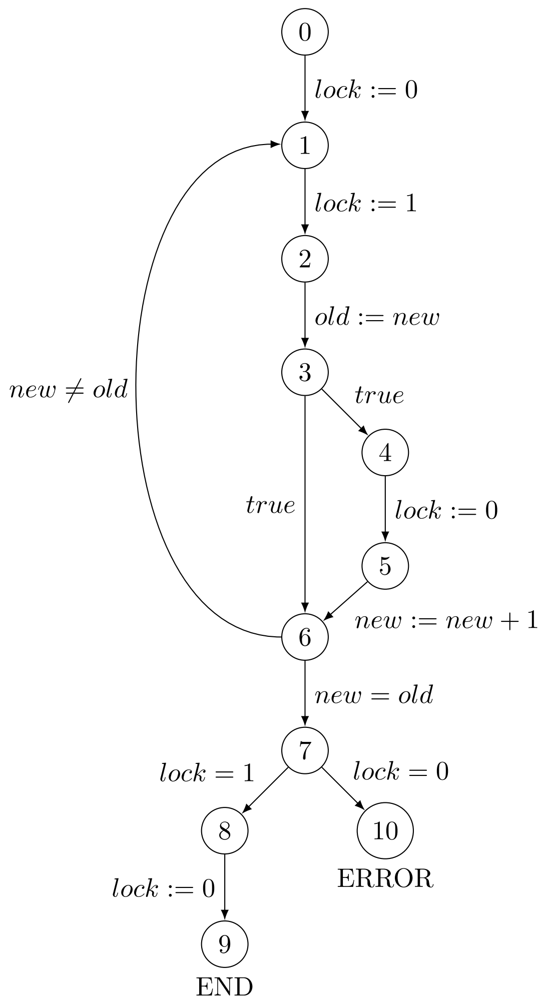
{ Pre: x = 2z ∧ y = z ∧ z > 0 }
x := x + y;
if x > 0 then
    while x ≠ y do
        x := x + 1;
        y := y + 2
    od
else
    abort
fi
{ Post: x = y }
```

(15 points)

4.) (a) Consider the following labeled transition system:



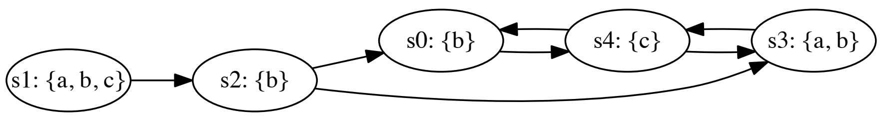
i. Provide an abstraction for the labeled transition system that uses the predicates $lock = 0$ and $lock = 1$. As a shorthand, use p in case predicate $lock = 0$ holds and $\bar{p}$ in case it does not hold. Use q in case predicate $lock = 1$ holds and $\bar{q}$ otherwise.

ii. Give an error trace in the abstraction.

iii. State a new predicate which can be used to refine the abstraction in order to make the error state unreachable. Only state the predicate; do *not* draw the refined abstraction.

(5 points)

(b) Consider the following Kripke structure M:



For each of the following formulae

i. determine if the formula is in CTL, LTL, and/or CTL*, and

ii. state on which states s_i the formula φ holds, i.e. $M, s_i \models \varphi$

- $\mathbf{F}(a)$
- $\mathbf{X}(b)$
- $\mathbf{A}[c\ \mathbf{U}\ a]$

- $\mathbf{EX}(c)$

(5 points)

(c) Let ϕ and ψ be arbitrary CTL* formulae. For each of the following three equivalences, determine whether the equivalence holds. If so, prove correctness. Otherwise, disprove by giving formulae ϕ and ψ, a Kripke structure M and a state s, such that (M, s) satisfies one side of the equivalence but not the other.

i. $\mathbf{E}\,(\phi\,\mathbf{U}\,\psi) \equiv \mathbf{E}\,(\phi\,\mathbf{U}\,\psi) \wedge \mathbf{EF}\,\phi$

ii. $\mathbf{E}\,(\phi\,\mathbf{U}\,\psi) \equiv \mathbf{E}\,(\phi\,\mathbf{U}\,\psi) \wedge \mathbf{EF}\,\psi$

iii. $\mathbf{AXF}\,\phi \equiv \mathbf{AXAF}\,\phi$

(5 points)