1.) Consider the following problem:

**SMALLER**

**INSTANCE:** A program $\Pi$ such that $\Pi$ takes one string as input and outputs a string. It is guaranteed that $\Pi$ terminates on any input string.

**QUESTION:** Does there exist an input string $I$ for $\Pi$ such that $|\Pi(I)| < |I|$. Here $|J|$ denotes the length of a string $J$, and $\Pi(J)$ is the string returned by $\Pi$ on input string $J$.

Prove that the problem $\text{SMALLER}$ is semi-decidable. For this, describe a procedure that shows the semi-decidability of the problem (i.e. a semi-decision procedure for $\text{SMALLER}$) and argue that it is correct.

Note: we consider only strings that are built from symbols 0 and 1. (15 points)

2.)

(a) Show the following:

\[ \psi_{\text{EUF}} \text{ is satisfiable iff } FC^E(\psi_{\text{EUF}}) \land \text{flat}^E(\psi_{\text{EUF}}) \text{ is satisfiable.} \]

Note: $FC^E(\psi_{\text{EUF}})$ and $\text{flat}^E(\psi_{\text{EUF}})$ are obtained from $\psi_{\text{EUF}}$ by Ackermann’s reduction. (8 points)

(b) Let $\varphi : \forall x \exists y[(s(x) \sim y) \land (y \sim s(x))]$, where $\sim / 2$ is a binary predicate written in infix notation. Let $T$ be a theory which forces $\sim / 2$ to be reflexive. Show by purely semantical means that $T \models \varphi$ holds. (Hint: show that $\text{Mod}(T) \subseteq \text{Mod}(\varphi)$.) (5 points)

(c) Show that the propositional resolution rule is sound. (2 points)

3.) Prove that the following correctness assertion is true regarding total correctness. Use the invariant $2z = y + 5z \land x \geq y$.

Some annotation rules that you might not remember:

\[
\begin{align*}
\text{abort} & \rightarrow \{ \text{false} \} \text{ abort } \{ \text{false} \} \\
\{ F \} &: = e \rightarrow \{ F \} v := e \forall v' (F[v/v'] \land v = e[v/v']) \\
\text{if } e & \text{ then } \{ F \} \ldots \text{ else } \{ G \} \\
\{ F \} & \text{ if } e \text{ then } \ldots \text{ else } \rightarrow \{ F \} \text{ if } e \text{ then } \{ F \} \ldots \text{ else } \{ F \land \neg e \} \\
\text{while } e & \text{ do } \ldots \text{ od } \rightarrow \{ \text{ Inv } \} \text{ while } e \text{ do } \{ \text{ Inv } \land e \land t = t_0 \} \ldots \{ \text{ Inv } \land 0 \leq t < t_0 \} \text{ od } \{ \text{ Inv } \land \neg e \} \\
\text{while } e & \text{ do } \ldots \text{ od } \rightarrow \{ \text{ Inv } \} \text{ while } e \text{ do } \{ \text{ Inv } \land e \land t = t_0 \} \ldots \{ \text{ Inv } \land (e \Rightarrow 0 \leq t < t_0) \} \text{ od } \{ \text{ Inv } \land \neg e \}
\end{align*}
\]

\[
\{ \text{ Pre: } x = 2z \land y = z \land z > 0 \} \\
x := x + y; \\
\text{if } x > 0 \text{ then } \\
\text{ while } x \neq y \text{ do } \\
x := x + 1; \\
y := y + 2 \\
\text{ od } \\
\text{ else } \text{ abort } \\
\text{ fi}
\]

\{ \text{ Post: } x = y \} 

(15 points)
4.) (a) Consider the following labeled transition system:

i. Provide an abstraction for the labeled transition system that uses the predicates lock = 0 and lock = 1. As a shorthand, use p in case predicate lock = 0 holds and ¬p in case it does not hold. Use q in case predicate lock = 1 holds and ¬q otherwise.

ii. Give an error trace in the abstraction.

iii. State a new predicate which can be used to refine the abstraction in order to make the error state unreachable. Only state the predicate; do not draw the refined abstraction.

(5 points)

(b) Consider the following Kripke structure M:

For each of the following formulae
i. determine if the formula is in CTL, LTL, and/or CTL*, and
ii. state on which states s_i the formula \( \varphi \) holds, i.e. \( M, s_i \models \varphi \)

- \( F(a) \)
- \( X(b) \)
- \( A[c \mathbf{U} a] \)
(c) Let $\phi$ and $\psi$ be arbitrary $\text{CTL}^*$ formulae. For each of the following three equivalences, determine whether the equivalence holds. If so, prove correctness. Otherwise, disprove by giving formulae $\phi$ and $\psi$, a Kripke structure $M$ and a state $s$, such that $(M, s)$ satisfies one side of the equivalence but not the other.

i. $E (\phi U \psi) \equiv E (\phi U \psi) \land EF \phi$

ii. $E (\phi U \psi) \equiv E (\phi U \psi) \land EF \psi$

iii. $AXF \phi \equiv AXAF \phi$

(5 points)