6.0/4.0 VU Formale Methoden der Informatik (185.291)

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Kennzahl (study id)	Matrikelnummer (student id)	Familienname (family name)	Vorname (first name)	Gruppe (version)

1.) Consider the following problem:

> **PROB**
>
> INSTANCE: A triple (P, K, m), where
>
> - $P = \{p_1, \ldots, p_n\}$ is a set of elements (intuitively, a set of persons),
> - $K \subseteq P \times P$ is a binary relation over P (intuitively, $(p, p') \in K$ means that p knows p'),
> - m is an integer (intuitively, the number of seats at a dinner table).
>
> QUESTION: Is it possible to sit m persons at the dinner table so that none of the persons knows another person at the table? That is, does there exist $S \subseteq P$ such that $|S| = m$ and $(p, p') \notin K$ for all $p \in S$ and $p' \in S$ with $p \neq p'$?

Provide a polynomial time reduction from **PROB** to **CLIQUE**, and prove the correctness of your reduction.

Recall that **CLIQUE** is defined as follows:

> **CLIQUE**
>
> INSTANCE: A pair (G, k), where $G = (V, E)$ is an undirected graph and k is an integer.
>
> QUESTION: Does there exist a set $C \subseteq V$ such that (i) $|C| \geq k$ and (ii) for all $v, v' \in C$ with $v \neq v'$ we have $[v, v'] \in E$.

(15 points)

2.) (a) Suppose `a`, `b`, `c` are (unsigned) integers in a programming language like C and `a` $\leq$ `b`. Which problem can occur with a C statement `c=(a+b)/2`? What is a simple solution to the problem? **(3 points)**

(b) Translate the following formula

$$\varphi^E : \neg\big(a \doteq b \wedge a \not\doteq c \rightarrow (a \doteq d \wedge e \not\doteq f \wedge g \not\doteq h) \vee g \not\doteq i \vee h \not\doteq j \vee (b \not\doteq c \wedge g \doteq i \wedge i \not\doteq j)\big)$$

into a propositional formula φ^p such that φ^E is E-satisfiable if and only if φ^p is satisfiable.

Hint: Simplify your formula before you construct the propositional skeleton and the transitivity constraints. In the simplifications steps, indicate the simple contradictory cycles and the pure literals. **(12 points)**

3.) Consider the following modified while-rule:

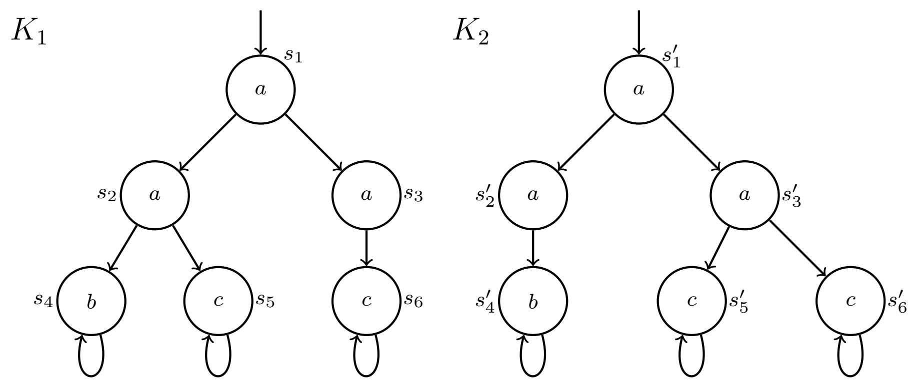
$$\frac{\{\, Inv \wedge e \,\}\, p \,\{\, Inv \,\}}{\{\, Inv \wedge e \,\}\, \texttt{while } e \texttt{ do } p \texttt{ od} \,\{\, Inv \wedge \neg e \,\}}\ \text{mw}$$

(a) Show that this rule is admissible regarding partial correctness. **(5 points)**

(b) Show that the Hoare calculus for partial correctness is no longer complete, if we replace the regular while-rule by the modified one. **(10 points)**

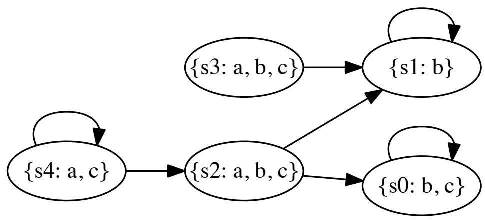
4.) Model Checking

(a) Let K_1 and K_2 be the two Kripke structures given below. Check which of the relations $K_1 \leq K_2$, $K_1 \geq K_2$, $K_1 \equiv K_2$ hold on K_1 and K_2. Justify your answer.



(5 points)

(b) Consider the following Kripke structure M:



For each of the following formulae

i. determine if the formula is in CTL, LTL, and/or CTL*, and
ii. state on which states s_i the formula φ holds, i.e. $M, s_i \models \varphi$

- $\mathbf{G}(b)$
- $\mathbf{X}(c)$
- $\mathbf{AX}(a \wedge b \wedge c)$
- $\mathbf{EF}(a \wedge b \wedge c)$

(5 points)

(c) Consider an arbitrary Kripke structure M and an atomic proposition $p \in AP$. Answer the following questions about CTL*:

i. Show that $M \models \mathbf{AX}\,\mathbf{AF}\,p$ implies $M \models \mathbf{X}\,\mathbf{F}\,p$ (i.e., give a proof).
ii. Show that $M \models \mathbf{AF}\,\mathbf{AF}\,p$ implies $M \models \mathbf{F}\,p$ (i.e., give a proof).
iii. Does $M \models \mathbf{X}\,\mathbf{F}\,\mathbf{X}\,p$ imply $M \models \mathbf{X}\,\mathbf{X}\,\mathbf{X}\,\mathbf{F}\,p$? If not, give a Kripke structure as an example. Otherwise, give a proof.

(5 points)