1.) Consider the following problem:

**DIFFERENT (DIFF)**

**INSTANCE:** A pair \((\Pi_1, \Pi_2)\) of programs that take one string as input and return a string as output. It is guaranteed that \(\Pi_1\) and \(\Pi_2\) terminate on any input string.

**QUESTION:** Does there exist a string \(I\) on which the programs \(\Pi_1\) and \(\Pi_2\) produce a different output, i.e. \(\Pi_1(I) \neq \Pi_2(I)\)?

Note: we consider only strings that are built from symbols 0 and 1.

Prove that the problem DIFF is semi-decidable. For this, describe a procedure that shows the semi-decidability of the problem (i.e. a semi-decision procedure for DIFF) and argue that it is correct.

(15 points)

2.)

(a) When is a theory \(T = (\Sigma, A)\) complete? Name a complete and an incomplete theory.

(b) Translate the following formula \(\varphi^E\):

\[
a \equiv b \land a \neq c \land (a \neq d \lor e \equiv f \lor g \equiv h) \land g \equiv i \land h \equiv j \land (b \equiv c \lor g \neq i \lor i = j)
\]

into a propositional formula \(\varphi^p\) such that \(\varphi^E\) is E-satisfiable if and only if \(\varphi^p\) is satisfiable.

Hint: First simplify \(\varphi^E\) before you construct the propositional skeleton and the transitivity constraints. In the simplifications steps, indicate the simple contradictory cycles and the pure literals.

(13 points)

3.) Use weakest preconditions to show that the correctness assertion

\[
\{ x = x_0 \} \ y := x; \text{ if } y \geq 0 \text{ then } P \text{ else } y := y - 3x \} \{ y = -2x_0 \}
\]

is true regarding total correctness, where \(P\) is the program

while \(x > 0\) do \(x := x - 1; \ y := y - 3\) od.

Remember the weakest precondition of conditional statements and loops:

\[
wp(\text{if } e \text{ then } p \text{ else } q \text{ fi}, G) = (e \land wp(p, G)) \lor (\neg e \land wp(q, G))
\]

\[
wp(\text{while } e \text{ do } p \text{ od}, G) = \exists i (i \geq 0 \land F_i) = F_0 \lor \exists i (i \geq 1 \land F_i),
\]

where \(F_0 = \neg e \land G\) and \(F_{i+1} = e \land wp(p, F_i)\).

(15 points)

4.)

(a) Consider the following labeled transition system:
(b) Consider the following Kripke structure $M$:

For each of the following formulae

i. determine if the formula is in CTL, LTL, and/or CTL*, and

ii. state on which states $s_i$ the formula $\varphi$ holds, i.e. $M, s_i \models \varphi$

- $\mathbf{G}(c)$
- $\mathbf{X}(a \land c)$
- $\mathbf{AG}(a)$
- $\mathbf{EF}(a)$

(c) Consider an arbitrary Kripke structure $M$ and an atomic proposition $p \in AP$.

Answer the following questions about CTL*:

i. Show that $M \models \mathbf{AG} \mathbf{AF} p$ implies $M \models \mathbf{GF} p$.

ii. Show that $M \models \mathbf{GF} p$ implies $M \models \mathbf{AG} \mathbf{AF} p$. 


iii. Does $M \models FGp$ imply $M \models AFAGp$? If not, give a Kripke structure as an example. Otherwise, give a proof.

(5 points)