6.0/4.0 VU Formale Methoden der Informatik
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1.) Consider the following problem:

> **DIFFERENT (DIFF)**
>
> INSTANCE: A pair (Π_1, Π_2) of programs that take one string as input and return a string as output. It is guaranteed that Π_1 and Π_2 terminate on any input string.
>
> QUESTION: Does there exist a string I on which the programs Π_1 and Π_2 produce a different output, i.e. $\Pi_1(I) \neq \Pi_2(I)$?

Note: we consider only strings that are built from symbols 0 and 1.

Prove that the problem **DIFF** is semi-decidable. For this, describe a procedure that shows the semi-decidability of the problem (i.e. a semi-decision procedure for **DIFF**) and argue that it is correct.

(15 points)

2.) (a) When is a theory $\mathcal{T} = (\Sigma, \mathcal{A})$ complete? Name a complete and an incomplete theory. **(2 points)**

(b) Translate the following formula

$$\varphi^E: \quad a \doteq b \land a \mathrel{\dot{\neq}} c \land (a \mathrel{\dot{\neq}} d \lor e \doteq f \lor g \doteq h) \land g \doteq i \land h \doteq j \land (b \doteq c \lor g \mathrel{\dot{\neq}} i \lor i = j)$$

into a propositional formula φ^p such that φ^E is E-satisfiable if and only if φ^p is satisfiable.
Hint: First simplify φ^E before you construct the propositional skeleton and the transitivity constraints. In the simplifications steps, indicate the simple contradictory cycles and the pure literals. **(13 points)**

3.) Use weakest preconditions to show that the correctness assertion

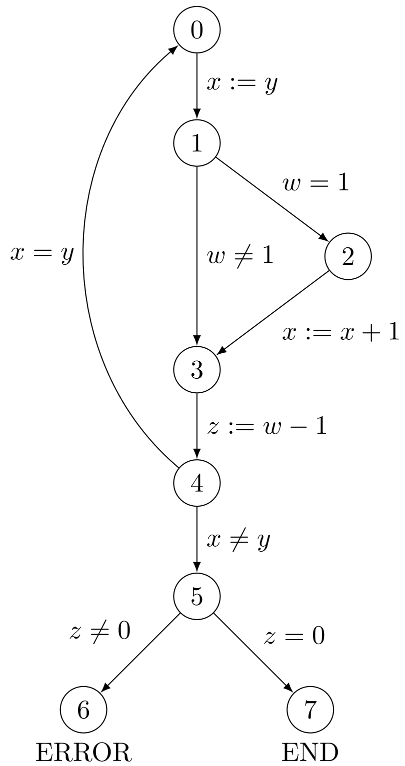
$$\{\, x = x_0 \,\}\; y := x;\ \textsf{if } y \geq 0 \textsf{ then } P \textsf{ else } y := y - 3x \textsf{ fi} \,\{\, y = -2x_0 \,\}$$

is true regarding total correctness, where P is the program

$$\textsf{while } x > 0 \textsf{ do } x := x - 1;\ y := y - 3 \textsf{ od} \ .$$

Remember the weakest precondition of conditional statements and loops:
$\mathrm{wp}(\textsf{if } e \textsf{ then } p \textsf{ else } q \textsf{ fi}, G) = (e \land \mathrm{wp}(p, G)) \lor (\neg e \land \mathrm{wp}(q, G))$
$\mathrm{wp}(\textsf{while } e \textsf{ do } p \textsf{ od}, G) = \exists i\,(i \geq 0 \land F_i) = F_0 \lor \exists i\,(i \geq 1 \land F_i)$,
where $F_0 = \neg e \land G$ and $F_{i+1} = e \land \mathrm{wp}(p, F_i)$. **(15 points)**

4.) (a) Consider the following labeled transition system:



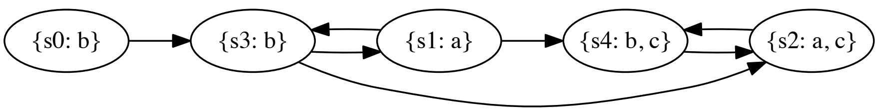
i. Provide an abstraction for the labeled transition system that uses the predicates $x = y$ and $z = 0$. As a shorthand, use p in case predicate $x = y$ holds and $\bar{p}$ in case it does not hold. Use q in case predicate $z = 0$ holds and $\bar{q}$ otherwise.

ii. Give an error trace in the abstraction.

iii. State a new predicate which can be used to refine the abstraction in order to get rid of the error state. Note, you just have to give the predicate, you don't have to draw the new abstraction.

(5 points)

(b) Consider the following Kripke structure M:



For each of the following formulae

i. determine if the formula is in CTL, LTL, and/or CTL*, and

ii. state on which states s_i the formula φ holds, i.e. $M, s_i \models \varphi$

- $\mathbf{G}(c)$
- $\mathbf{X}(a \wedge c)$
- $\mathbf{AG}(a)$
- $\mathbf{EF}(a)$

(5 points)

(c) Consider an arbitrary Kripke structure M and an atomic proposition $p \in AP$. Answer the following questions about CTL*:

i. Show that $M \models \mathbf{AG}\,\mathbf{AF}\,p$ implies $M \models \mathbf{G}\,\mathbf{F}\,p$.

ii. Show that $M \models \mathbf{G}\,\mathbf{F}\,p$ implies $M \models \mathbf{AG}\,\mathbf{AF}\,p$.

iii. Does $M \models \mathbf{F}\,\mathbf{G}\,p$ imply $M \models \mathbf{AF}\,\mathbf{AG}\,p$? If not, give a Kripke structure as an example. Otherwise, give a proof.

(5 points)