6.0/4.0 VU Formale Methoden der Informatik
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1.) Consider the following problem:

> **HALTING-ON-PAIR**
>
> INSTANCE: A tuple (Π, I_1, I_2), where I_1, I_2 are strings and Π is a program that takes two strings as input.
>
> QUESTION: Is it the case that Π halts on the pair (I_1, I_2)?

By providing a reduction from an undecidable problem, prove that **HALTING-ON-PAIR** is undecidable. Argue formally that your reduction is correct. **(15 points)**

2.) (a) Let $\mathcal{T}_f^E$ be the theory containing all equality axioms (from $\mathcal{T}^E$) and the following two axioms.

$$\forall x \forall y : f(x) \doteq f(y) \rightarrow x \doteq y \qquad \text{(f-injectivity)}$$

$$\forall x : f(x) \doteq f(f(x)) \qquad \text{(f-idempotency)}$$

Let φ be the formula: $f(f(f(a))) \doteq f(b) \rightarrow a \doteq b$.

Prove: φ is $\mathcal{T}_f^E$-valid. **(5 points)**

(b) Let $\mathcal{C}$ be a satisfiable set of clauses consisting of

$C_1 : \neg x_1 \vee x_2$ $\quad C_2 : \neg x_2 \vee x_3 \vee x_4$ $\quad C_3 : \neg x_2 \vee \neg x_5$ $\quad C_4 : \neg x_4 \vee x_5 \vee x_6$
$C_5 : \neg x_7 \vee x_8$ $\quad C_6 : \neg x_8 \vee \neg x_9$ $\quad C_7 : x_9 \vee \neg x_{10}$ $\quad C_8 : x_3 \vee \neg x_8 \vee x_{10}$

Let $x_1 = 1@1$, $x_3 = 0@2$ and $x_7 = 1@3$.

i. Draw the implication graph (IG) (don't forget the decision level and the antecedent!).
ii. Are there UIPs in the IG? If no, why not? If yes, which node is the first UIP and why?
iii. When is a clause asserting?
iv. If the implication graph is a conflict graph, compute the asserting learned clause by resolution (according to the first UIP scheme). Otherwise present a satisfying assignment (constructed from the IG) and argue why it is satisfying.

(10 points)

3.) (a) Let F, G, and H be formulas and p and q arbitrary programs. Show that the rule

$$\frac{\{\, F \,\}\, p; q \,\{\, G \,\}}{\{\, H \wedge F \,\}\, p; q \,\{\, G \wedge H \,\}}$$

is admissible neither for partial nor for total correctness.

Remember that a rule $\frac{X_1 \cdots X_n}{\{\, F \,\}\, p \,\{\, G \,\}}$ is *admissible regarding partial/total correctness*, if the conclusion $\{\, F \,\}\, p \,\{\, G \,\}$ is partially/totally correct whenever all premises $X_1, \ldots, X_n$ are valid formulas or partially/totally correct assertions. **(7 points)**

(b) Compute a formula that describes all states for which the following program terminates.

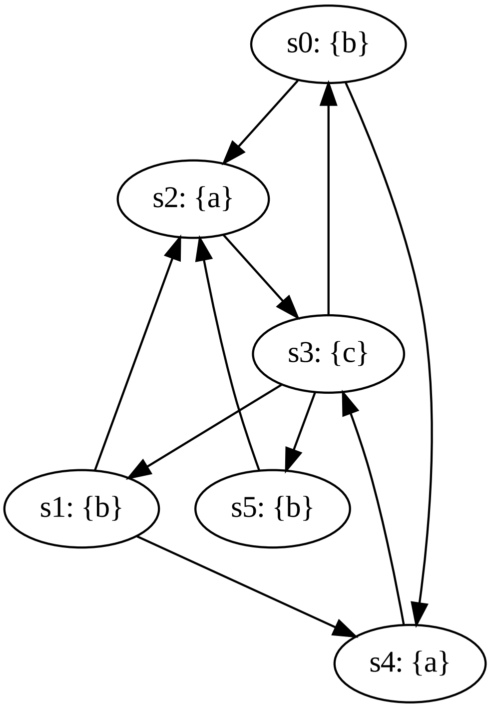
$$\text{while } 3x \neq 2y \text{ do } x := x - 1; y := y + 2 \text{ od}; x := 0$$

List three states for which it terminates. **(8 points)**

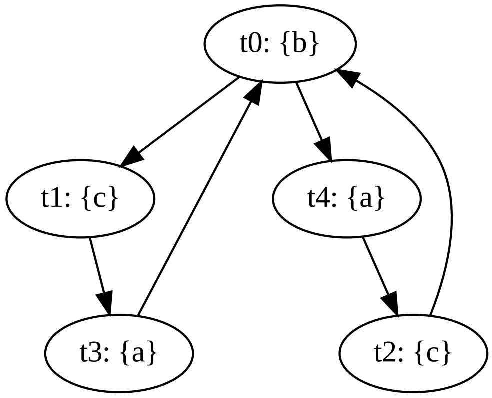
4.) Model Checking

(a) Provide a non-empty simulation relation H that witnesses $M_1 \leq M_2$, where M_1 and M_2 are shown below (M_1 on the left, M_2 on the right), the initial state of M_1 is s_0, the initial state of M_2 is t_0:

Kripke structure M_1:

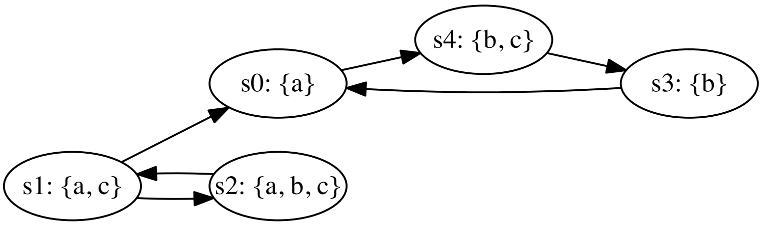


Kripke structure M_2:



(4 points)

(b) Consider the following Kripke structure M:



For each of the following formulae

i. determine if the formula is in CTL, LTL, and/or CTL*, and

ii. state on which states s_i the formula φ holds, i.e. $M, s_i \models \varphi$

$\mathbf{EG}(a)$
$\mathbf{EX}(a \wedge b)$
$\mathbf{EF}(a \wedge b)$ **(5 points)**
$\mathbf{X}(b \wedge c)$
$\mathbf{F}(a \wedge b)$

(c) **Trace Equivalence**

Let $M = (S, I, R, L)$ be a Kripke structure. We define the language $\mathcal{L}(M)$ of M to be the set of (infinite) words

$$\mathcal{L}(M) = \{L(s_0)L(s_1)\cdots \mid s_0 s_1 \ldots \text{ is a path of } M \text{ and } s_0 \in I\}$$

We say that two Kripke structures M_1 and M_2 are *trace equivalent* iff $\mathcal{L}(M_1) = \mathcal{L}(M_2)$.

Show that there are trace equivalent Kripke Structures which do not satisfy the same CTL formulae. I.e., state two trace equivalent Kripke Structures M_1, M_2 and a CTL formula ϕ s.t. $M_1 \models \phi$ but $M_2 \not\models \phi$.
Hint: There are solutions s.t. M_1 and M_2 have ≤ 5 states.

(6 points)