1.) Consider the following problem:

**HALTING-ON-PAIR**

INSTANCE: A tuple \((\Pi, I_1, I_2)\), where \(I_1, I_2\) are strings and \(\Pi\) is a program that takes two strings as input.

QUESTION: Is it the case that \(\Pi\) halts on the pair \((I_1, I_2)\)?

By providing a reduction from an undecidable problem, prove that **HALTING-ON-PAIR** is undecidable. Argue formally that your reduction is correct. (15 points)

2.) (a) Let \(T^E\) be the theory containing all equality axioms (from \(T^E\)) and the following two axioms.

\[
\forall x \forall y : f(x) = f(y) \rightarrow x = y \\
\forall x : f(x) = f(f(x))
\]

(f-injectivity) (f-idempotency)

Let \(\varphi\) be the formula: \(f(f(f(a))) = f(b) \rightarrow a = b\).

**Prove:** \(\varphi\) is \(T^E\)-valid. (5 points)

(b) Let \(C\) be a satisfiable set of clauses consisting of

\[
C_1: \neg x_1 \lor x_2 \\
C_2: \neg x_2 \lor x_3 \lor x_4 \\
C_3: \neg x_2 \lor \neg x_5 \\
C_4: \neg x_4 \lor x_5 \lor x_6 \\
C_5: \neg x_7 \lor x_8 \\
C_6: \neg x_8 \lor \neg x_9 \\
C_7: x_9 \lor \neg x_{10} \\
C_8: x_3 \lor \neg x_8 \lor x_{10}
\]

Let \(x_1 = 1@1, \ x_3 = 0@2\) and \(x_7 = 1@3\).

i. Draw the implication graph (IG) (don’t forget the decision level and the antecedent!).
ii. Are there UIPs in the IG? If no, why not? If yes, which node is the first UIP and why?
iii. When is a clause asserting?
iv. If the implication graph is a conflict graph, compute the asserting learned clause by resolution (according to the first UIP scheme). Otherwise present a satisfying assignment (constructed from the IG) and argue why it is satisfying.

(10 points)

3.) (a) Let \(F, G,\) and \(H\) be formulas and \(p\) and \(q\) arbitrary programs. Show that the rule

\[
\frac{\{ F \} p; q \{ G \}}{\{ H \land F \} p; q \{ G \land H \}}
\]

is admissible neither for partial nor for total correctness.

Remember that a rule \(\frac{X_1, \ldots, X_n}{\{ F \} p \{ G \}}\) is admissible regarding partial/total correctness, if the conclusion \(\{ F \} p \{ G \}\) is partially/ totalmente correct whenever all premises \(X_1, \ldots, X_n\) are valid formulas or partially/ totally correct assertions. (7 points)
(b) Compute a formula that describes all states for which the following program terminates.

\[
\text{while } 3x \neq 2y \text{ do } x := x - 1; y := y + 2 \text{ od}; x := 0
\]

List three states for which it terminates. 

(8 points)

4.) Model Checking

(a) Provide a non-empty simulation relation \( H \) that witnesses \( M_1 \leq M_2 \), where \( M_1 \) and \( M_2 \) are shown below (\( M_1 \) on the left, \( M_2 \) on the right), the initial state of \( M_1 \) is \( s_0 \), the initial state of \( M_2 \) is \( t_0 \):

**Kripke structure \( M_1 \):**

\[
\begin{align*}
s_0 & : \{b\} \\
& \downarrow \\
& \{a\} \\
& \downarrow \\
& \{c\} \\
& \downarrow \\
& \{b\} \\
& \downarrow \\
& \{a\}
\end{align*}
\]

**Kripke structure \( M_2 \):**

\[
\begin{align*}
t_0 & : \{b\} \\
& \downarrow \\
& \{c\} \\
& \downarrow \\
& \{a\} \\
& \downarrow \\
& \{c\} \\
& \downarrow \\
& \{b\}
\end{align*}
\]

(4 points)

(b) Consider the following Kripke structure \( M \):

\[
\begin{align*}
s_0 & : \{a\} \\
& \downarrow \\
s_4 & : \{b, c\} \\
& \downarrow \\
s_3 & : \{b\} \\
& \downarrow \\
s_1 & : \{a, c\} \\
& \downarrow \\
s_2 & : \{a, b, c\}
\end{align*}
\]

For each of the following formulae

i. determine if the formula is in CTL, LTL, and/or CTL*, and

ii. state on which states \( s_i \) the formula \( \varphi \) holds, i.e. \( M, s_i \models \varphi \)
(5 points)
(6 points)

(c) **Trace Equivalence**

Let $M = (S, I, R, L)$ be a Kripke structure. We define the language $\mathcal{L}(M)$ of $M$ to be the set of (infinite) words

$$\mathcal{L}(M) = \{ L(s_0)L(s_1)\cdots \mid s_0s_1\cdots \text{ is a path of } M \text{ and } s_0 \in I \}$$

We say that two Kripke structures $M_1$ and $M_2$ are trace equivalent iff $\mathcal{L}(M_1) = \mathcal{L}(M_2)$.

Show that there are trace equivalent Kripke Structures which do not satisfy the same CTL formulae. I.e., state two trace equivalent Kripke Structures $M_1$, $M_2$ and a CTL formula $\phi$ s.t. $M_1 \models \phi$ but $M_2 \not\models \phi$.

**Hint:** There are solutions s.t. $M_1$ and $M_2$ have $\leq 5$ states.

(6 points)