6.0/4.0 VU Formale Methoden der Informatik
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1.) Consider the following problem:

> **PROB**
>
> INSTANCE: An undirected graph $G = (V, E)$ such that G has a vertex v_0 that is connected to precisely all other vertices of G, i.e. there exists $v_0 \in V$ such that (a) $[v_0, v] \in E$ for all $v \in V \setminus \{v_0\}$, and (b) $[v_0, v_0] \notin E$.
>
> QUESTION: Is it the case that G is 4-colorable? That is, does there exist a function μ from vertices in V to values in $\{1, 2, 3, 4\}$ such that $\mu(v_1) \neq \mu(v_2)$ for any edge $[v_1, v_2] \in E$.

Provide a reduction from **PROB** to 3-**COLORABILITY**, and explain the intuition behind your reduction. **(15 points)**

2.) (a) Show the following:

$$\varphi^{EUF} \text{ is satisfiable iff } FC^E \wedge \mathit{flat}^E \text{ is satisfiable.}$$

FC^E and flat^E are obtained from φ^{EUF} by Ackermann's reduction.
(Hint: FC^E is the same for φ^{EUF} and $\neg\varphi^{EUF}$.) **(10 points)**

(b) Clarify the logical status of each of the following formulas:

i. φ_1^{EUF}: $\quad f(x) \doteq f(y) \wedge x \not\doteq y$

ii. φ_2^{EUF}: $\quad x \doteq y \wedge f(x) \not\doteq f(y)$

If the formula is E-valid or E-unsatisfiable, then give a proof based on E-interpretations and semantics. If the formula is E-satisfiable but not E-valid, then present two E-interpretations, one satisfying the formula and one falsifying it. Argue formally why the formula is true respectively false under the considered E-interpretation.

(5 points)

3.) (a) Show that $\{ F \}\, v := e\, \{ F[v/e] \}$ is not a sound axiom. **(5 points)**

(b) Prove that the following correctness assertion is true regarding total correctness. Use the invariant $2x + y^2 + y = 4z(z + 1) \wedge y \geq 0$.

You may need one of the following annotation rules:

$\{ Inv \}$while e do $\{ Inv \wedge e \wedge t{=}t_0 \} \cdots \{ Inv \wedge 0{\leq}t{<}t_0 \}od\{ Inv \wedge \neg e \}$
$\{ Inv \}$while e do $\{ Inv \wedge e \wedge t{=}t_0 \} \cdots \{ Inv \wedge (e \Rightarrow 0{\leq}t{<}t_0) \}od\{ Inv \wedge \neg e \}$

```
{ x = z ∧ z ≥ 0 }
y := 2x;
while y > 0 do
    x := x + y;
    y := y − 1;
od
{ x ≥ 2z }
```

(10 points)

4.) (a)

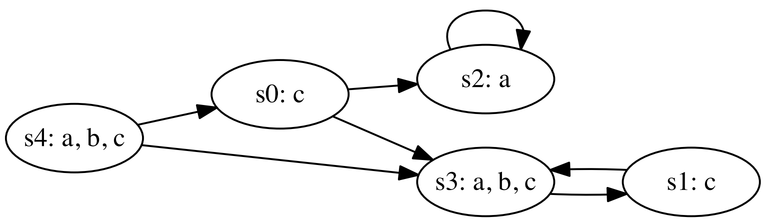
> **DEFINITIONS**
>
> Let $M_1 = (S_1, I_1, R_1, L_1)$ and $M_2 = (S_2, I_2, R_2, L_2)$ be two Kripke structures.
>
> **Simulation**
>
> A relation $H \subseteq S_1 \times S_2$ is a simulation relation if for each $(s, s') \in H$ holds:
> - $L_1(s) = L_2(s')$, and
> - for each $(s, t) \in R_1$ there is a $(s', t') \in R_2$ such that $(t, t') \in H$.
>
> M_2 *simulates* M_1 (denoted as $M_1 \leq M_2$), if there is a simulation relation $H \subseteq S_1 \times S_2$ *such that*
> - for each initial state $s \in I_1$ there is an initial state $s' \in I_2$ with $(s, s') \in H$.
>
> **Trace Inclusion**
>
> Let $M = (S, I, R, L)$ be a Kripke structure. We define the language $\mathcal{L}(M)$ of M to be the set of (infinite) words
>
> $$\mathcal{L}(M) = \{L(s_1)L(s_2)\cdots \mid s_1 s_2 \ldots \text{ is a path of } M \text{ and } s_1 \in I\}$$
>
> We speak of trace inclusion if the language of M_1 is included in the language of M_2, i.e. $\mathcal{L}(M_1) \subseteq \mathcal{L}(M_2)$.

Show that trace inclusion does not entail simulation, i.e.

$$\mathcal{L}(M_1) \subseteq \mathcal{L}(M_2) \qquad \not\Rightarrow \qquad M_1 \leq M_2$$

(5 points)

(b) Consider the following Kripke structure:



For each of the following formulae

i. determine if the formula is in CTL, LTL, and/or CTL*, and

ii. state on which states s_i the formula holds

$\mathbf{G}(a)$
$\mathbf{X}(b \wedge c)$
$\mathbf{AG}(c)$
$\mathbf{EF}(c)$

(5 points)

(c) Use CBMC to solve the *Knapsack problem*:

Given a set of N items $I = \{i_1, i_2, \ldots, i_N\}$, with respective weights $w(i_j)$ and values $v(i_j)$, we select a subset $I' \subseteq I$ to pack into a backpack.

The backpack can carry only up to a maximal weight W that the selected items must not exceed, i.e.

$$\sum_{i \in I'} w(i) \leq W$$

Decide if there is a subset $I' \subseteq I$ such that the value of the selected objects is at least V, i.e.

$$\sum_{i \in I'} v(i) \geq V$$

Write a C program that implements a *guess and check* routine for the Knapsack problem. Ensure that CBMC reports a valid selection of items I' if one exists.

You may assume the following template:

```
int nondet_bool(); // nondeterministacally returns 0 or 1

int N = 10;        // number of items

int values[]  = { 15, 22, 13, 44, 82,
                  12, 32, 41, 33, 14 };  // values of items
int weights[] = { 23, 12, 18, 11, 99,
                  12, 11, 30, 11, 83 };  // weights of items

int V = 200;
int W = 120;

int main() {
  // add your code here:
  // 1. guess a subset
  // 2. put an assertion such that CBMC reports a solution
  //    (if one exists)
}
```

(5 points)