6.0/4.0 VU Formale Methoden der Informatik 185.291 WS 2013 09 May 2014				
Kennzahl (study id)	Matrikelnummer (student id)	Familienname (family name)	Vorname (first name)	Gruppe (version)
				A

1.) Consider the following problem:

> **SOME-HALTS**
>
> INSTANCE: A triple (Π_1, Π_2, I), where I is string and Π_1, Π_2 are programs that take a string as input.
>
> QUESTION: Is it true that Π_1 halts on I or Π_2 halts on I?

By providing a reduction from **HALTING** to **SOME-HALTS**, prove that **SOME-HALTS** is undecidable. Argue formally that your reduction is correct.

(15 points)

2.) (a) First define the concept of a $\mathcal{T}$-interpretation. Then use it to define the following:

i. the $\mathcal{T}$-satisfiability of a formula;
ii. the $\mathcal{T}$-validity of a formula.

Additionally define the completeness of a theory $\mathcal{T}$ and give an example for a complete theory and an incomplete theory. **(5 points)**

(b) Prove that the following formula φ is $\mathcal{T}^E_{cons}$-valid:

$$\varphi: \qquad cons(a,b) \doteq cons(c,d) \rightarrow a \doteq c \wedge b \doteq d$$

Hints: Please be precise! Recall the axioms of left and right projection in $\mathcal{T}^E_{cons}$:

$$car(cons(x,y)) \doteq x \qquad \text{(left projection)}$$
$$cdr(cons(x,y)) \doteq y \qquad \text{(right projection)}$$

(10 points)

3.) Show that the following correctness assertion is totally correct. Describe the function computed by the program if we consider k as its input and m as its output.

Hints: Use the formula $l = (m+1)^3 \wedge 0 \leq m^3 \leq k$ as loop invariant. Depending on how you choose the variant, use one of the following annotation rules:

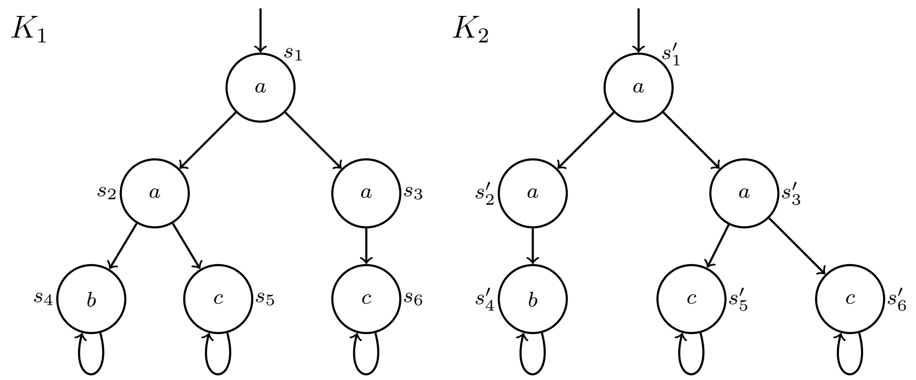
while e do $\cdots$ od $\mapsto$ $\{ Inv \}$while e do $\{ Inv \wedge e \wedge t = t_0 \} \cdots \{ Inv \wedge 0 \leq t < t_0 \}od\{ Inv \wedge \neg e \}$
while e do $\cdots$ od $\mapsto$ $\{ Inv \}$while e do $\{ Inv \wedge e \wedge t = t_0 \} \cdots \{ Inv \wedge (e \rightarrow 0 \leq t < t_0) \}od\{ Inv \wedge \neg e \}$

```
{ k ≥ 0 }
l := 1;
m := 0;
while l ≤ k do
    n := 3 * m + 6;
    m := m + 1;
    l := l + m * n + 1
od
{ m^3 ≤ k < (m + 1)^3 }
```

(15 points)

4.) Simulation and Bisimulation

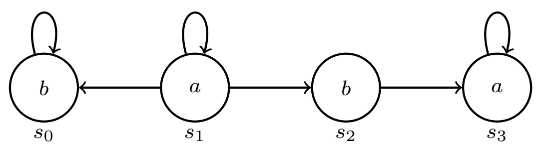
(a) Let K_1 and K_2 be the two Kripke structures given below. Check which of the relations $K_1 \leq K_2$, $K_1 \geq K_2$, $K_1 \equiv K_2$ hold on K_1 and K_2. Justify your answer.



(5 points)

(b) Show that simulation is a transitive relation: Given any 3 Kripke structures $K_1 = \{S_1, R_1, L_1\}$, $K_2 = \{S_2, R_2, L_2\}$ and $K_3 = \{S_3, R_3, L_3\}$ such that $K_1 \leq K_2$ and $K_2 \leq K_3$, it holds that $K_1 \leq K_3$. **(8 points)**

(c) Consider the following Kripke Structure:



Determine on which states the LTL-formula $\mathtt{G}(a\ \mathtt{U}\ b)$ holds. **(2 points)**