1.) Consider the following problem:

**SOME-HALTS**

**INSTANCE:** A triple \((\Pi_1, \Pi_2, I)\), where \(I\) is string and \(\Pi_1, \Pi_2\) are programs that take a string as input.

**QUESTION:** Is it true that \(\Pi_1\) halts on \(I\) or \(\Pi_2\) halts on \(I\)?

By providing a reduction from **HALTING** to **SOME-HALTS**, prove that **SOME-HALTS** is undecidable. Argue formally that your reduction is correct.

(15 points)

2.) (a) First define the concept of a \(T\)-interpretation. Then use it to define the following:

i. the \(T\)-satisfiability of a formula;

ii. the \(T\)-validity of a formula.

Additionally define the completeness of a theory \(T\) and give an example for a complete theory and an incomplete theory.

(5 points)

(b) Prove that the following formula \(\varphi\) is \(T_{cons}^{E}\)-valid:

\[
\varphi : \quad cons(a, b) \equiv cons(c, d) \rightarrow a \equiv c \land b \equiv d
\]

Hints: Please be precise! Recall the axioms of left and right projection in \(T_{cons}^{E}\):

- \(\text{car}(cons(x, y)) \equiv x\) (left projection)
- \(\text{cdr}(cons(x, y)) \equiv y\) (right projection)

(10 points)

3.) Show that the following correctness assertion is totally correct. Describe the function computed by the program if we consider \(k\) as its input and \(m\) as its output.

Hints: Use the formula \(l = (m + 1)^3 \land 0 \leq m^3 \leq k\) as loop invariant. Depending on how you choose the variant, use one of the following annotation rules:

- while \(e\) do \(
  \begin{align*}
    \{ \text{Inv} \} & \quad e \{ \text{Inv} \land e \land t = t_0 \} \cdots \{ \text{Inv} \land 0 \leq t < t_0 \} \od \{ \text{Inv} \land \neg e \} \\
    \{ k \geq 0 \} & \quad l := 1; \\
    m := 0; \\
    \text{while } l \leq k \text{ do} \\
    \quad n := 3 \cdot m + 6; \\
    \quad m := m + 1; \\
    \quad l := l + m \cdot n + 1 \\
    \od
  \end{align*}
\)

(15 points)
4.) Simulation and Bisimulation

(a) Let $K_1$ and $K_2$ be the two Kripke structures given below. Check which of the relations $K_1 \leq K_2$, $K_1 \geq K_2$, $K_1 \equiv K_2$ hold on $K_1$ and $K_2$. Justify your answer.

(b) Show that simulation is a transitive relation: Given any 3 Kripke structures $K_1 = \{S_1, R_1, L_1\}$, $K_2 = \{S_2, R_2, L_2\}$ and $K_3 = \{S_3, R_3, L_3\}$ such that $K_1 \leq K_2$ and $K_2 \leq K_3$, it holds that $K_1 \leq K_3$.

(c) Consider the following Kripke Structure:

Determine on which states the LTL-formula $\mathcal{G}(a \mathcal{U} b)$ holds.