1.) Provide a reduction from 3-COLORABILITY to 4-COLORABILITY, and prove that your reduction is correct.
   Hint: For the reduction it suffices to introduce one additional vertex to the input graph. 
   (15 points)

2.) (a) Let \( G^E(\varphi^E) \) be the following equality graph for \( \varphi^E \) in NNF:
\[
\begin{array}{c}
x \\
\hline
\end{array}
\]
\[
\begin{array}{c}
y \\
\hline
\end{array}
\]

What are the smallest formulas in NNF represented by \( G^E(\varphi^E) \)? 
(3 points)

(b) We define a syntax variant \( V_1 \) for E-logic as follows:
\[
\text{formula ::= atom} \mid \neg \text{atom} \mid \text{(formula) \mid formula \land formula \mid formula \lor formula}
\]
\[
\text{atom ::= term \equiv term} \mid \text{Boolean variable}
\]
\[
\text{term ::= identifier} \mid \text{constant}
\]

We define a syntax variant \( V_2 \) for E-logic as follows:
\[
\text{formula ::= atom} \mid \neg \text{atom} \mid \text{(formula) \mid formula \land formula \mid formula \lor formula}
\]
\[
\text{atom ::= term}\]
\[
\text{term ::= identifier} \mid \text{constant}
\]

1) Given an E-formula \( \varphi^E_1 \) (according to syntax \( V_1 \)), devise a translation that takes \( \varphi^E_1 \) and results in an E-formula \( \varphi^E_2 \) (according to syntax \( V_2 \)) such that \( \varphi^E_1 \) and \( \varphi^E_2 \) are equi-satisfiable.

2) Prove: If \( \varphi^E_1 \) is E-satisfiable, then \( \varphi^E_2 \) is E-satisfiable. 
(12 points)

3.) Consider the following modified while-rule:
\[
\begin{array}{c}
\{ \text{Inv} \} p \{ \text{Inv} \} \\
\{ \text{Inv} \} \text{ while } e \text{ do } p \text{ od } \{ \text{Inv} \}
\end{array}
\]

(a) Show that this rule is admissible regarding partial correctness. 
(5 points)

(b) Show that the Hoare calculus for partial correctness is no longer complete, if we replace the regular while-rule by the modified one. 
(10 points)

| A rule \( X_1 \cdots X_n \{ F \} p \{ G \} \) is admissible regarding partial correctness, if the conclusion \( \{ F \} p \{ G \} \) is partially correct whenever all premises \( X_1, \ldots, X_n \) are valid formulas/partially correct assertions. |

<table>
<thead>
<tr>
<th>Hoare calculus for partial correctness:</th>
</tr>
</thead>
</table>
| \[
\begin{array}{c}
\{ F \} \text{ skip } \{ F \} \\
\{ F \} \text{ abort } \{ G \} \\
\{ F[v/e] \} v \leftarrow e \{ F \} \\
\{ G \} \{ H \} p \{ G \} \{ F \} \{ G \} q \{ G \} \{ F \} \{ G \} \{ F \} \{ G \} \{ F \} \{ G \} \{ F \} \{ G \} \{ F \} \{ G \} \\
\end{array}
\] |

| \[
\begin{array}{c}
\{ F \} \text{ if } e \text{ then } p \text{ else } q \{ G \} \\
\{ \text{Inv} \land e \} p \{ \text{Inv} \} \\
\{ \text{Inv} \} \text{ while } e \text{ do } p \text{ od } \{ \text{Inv} \land \neg e \} \\
F \Rightarrow F' \{ G' \} \quad G \Rightarrow G' \\
\{ F \} p \{ G \}
\end{array}
\] |
4.) Simulation

Let $M_1 = (S_1, I_1, R_1, L_1)$ and $M_2 = (S_2, I_2, R_2, L_2)$ be two Kripke structures.

**Simulation**

Remember, a relation $H \subseteq S_1 \times S_2$ is a simulation relation if for each $(s, s') \in H$ holds:

- $L_1(s) = L_2(s')$, and
- for each $(s, t) \in R_1$ there is a $(s', t') \in R_2$ such that $(t, t') \in H$.

Further remember, $M_2$ simulates $M_1$, in signs $M_1 \leq M_2$, if there is a simulation relation $H \subseteq S_1 \times S_2$ such that

- for each initial state $s \in I_1$ there is an initial state $s' \in I_2$ with $(s, s') \in H$.

In the following, we say that $H$ witnesses the similarity of $M_1$ and $M_2$ in case $H$ is a simulation relation from $M_1$ to $M_2$ that satisfies the condition stated above.

(a) Show that there is no simulation relation $H$ that witnesses $M_1 \leq M_2$.

(b) Prove the following equivalence of LTL formulae:

$$(Ga) \rightarrow (Fb) \equiv aU(b \lor \neg a)$$

(4 points)

(c) Prove that the following LTL formulae are not equivalent:

$$(Fa) \land (XGa) \not\equiv Fa$$

(2 points)

(d) Consider the following program stated in form of a labeled transition system:

0

\[ x := 1 \]
\[ y := 1 \]

assume($y \geq 100$)

1

\[ x := x + 1 \]
\[ y := y + x \]

assume($x \geq 100$) assume($x < 100$) assume($y < 100$)

2

3

4

ERROR
i. Provide an abstraction for the labeled transition system that uses the predicates $x > 0, y > 0$

ii. Give an error trace in the abstraction

iii. State an additional predicate which can be used to refine the abstraction in order to get rid of the error state. Don’t draw the new abstraction.

(6 points)