6.0/4.0 VU Formale Methoden der Informatik

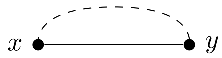
18 October 2013

Kennzahl (study id)	Matrikelnummer (student id)	Familienname (family name)	Vorname (first name)	Gruppe (version)
				A

1.) Provide a reduction from 3-**COLORABILITY** to 4-**COLORABILITY**, and prove that your reduction is correct.

Hint: For the reduction it suffices to introduce one additional vertex to the input graph. **(15 points)**

2.) (a) Let $G^E(\varphi^E)$ be the following equality graph for φ^E in NNF:



What are the smallest formulas in NNF represented by $G^E(\varphi^E)$? **(3 points)**

(b) We define a syntax variant V_1 for E-logic as follows:

$$\begin{aligned} \text{formula} &::= \text{atom} \mid \neg\text{atom} \mid (\text{formula}) \mid \text{formula} \land \text{formula} \mid \text{formula} \lor \text{formula} \\ \text{atom} &::= \text{term} \doteq \text{term} \mid \textit{Boolean variable} \\ \text{term} &::= \text{identifier} \mid \text{constant} \end{aligned}$$

We define a syntax variant V_2 for E-logic as follows:

$$\begin{aligned} \text{formula} &::= \text{atom} \mid \neg\text{atom} \mid (\text{formula}) \mid \text{formula} \land \text{formula} \mid \text{formula} \lor \text{formula} \\ \text{atom} &::= \text{term} \doteq \text{term} \\ \text{term} &::= \text{identifier} \mid \text{constant} \end{aligned}$$

1) Given an E-formula φ_1^E (according to syntax V_1), devise a translation that takes φ_1^E and results in an E-formula φ_2^E (according to syntax V_2) such that φ_1^E and φ_2^E are equi-satisfiable.
2) Prove: If φ_1^E is E-satisfiable, then φ_2^E is E-satisfiable.

(12 points)

3.) Consider the following modified while-rule:

$$\frac{\{\,Inv\,\}\,p\,\{\,Inv\,\}}{\{\,Inv\,\}\ \mathsf{while}\ e\ \mathsf{do}\ p\ \mathsf{od}\,\{\,Inv\,\}}\ \text{mw}$$

(a) Show that this rule is admissible regarding partial correctness. **(5 points)**

(b) Show that the Hoare calculus for partial correctness is no longer complete, if we replace the regular while-rule by the modified one. **(10 points)**

A rule $\frac{X_1 \cdots X_n}{\{\,F\,\}\,p\,\{\,G\,\}}$ is *admissible regarding partial correctness*, if the conclusion $\{\,F\,\}\,p\,\{\,G\,\}$ is partially correct whenever all premises $X_1, \ldots, X_n$ are valid formulas/partially correct assertions.

Hoare calculus for partial correctness:

$$\{\,F\,\}\ \mathsf{skip}\ \{\,F\,\}$$

$$\{\,F\,\}\ \mathsf{abort}\ \{\,G\,\}$$

$$\{\,F[v/e]\,\}\ v \leftarrow e\ \{\,F\,\}$$

$$\frac{\{\,F\,\}\,p\,\{\,G\,\} \quad \{\,G\,\}\,q\,\{\,H\,\}}{\{\,F\,\}\,p;q\,\{\,H\,\}}$$

$$\frac{\{\,F \land e\,\}\,p\,\{\,G\,\} \quad \{\,F \land \neg e\,\}\,q\,\{\,G\,\}}{\{\,F\,\}\ \mathsf{if}\ e\ \mathsf{then}\ p\ \mathsf{else}\ q\ \mathsf{fi}\,\{\,G\,\}}$$

$$\frac{\{\,Inv \land e\,\}\,p\,\{\,Inv\,\}}{\{\,Inv\,\}\ \mathsf{while}\ e\ \mathsf{do}\ p\ \mathsf{od}\,\{\,Inv \land \neg e\,\}}$$

$$\frac{F \Rightarrow F' \quad \{\,F'\,\}\,p\,\{\,G'\,\} \quad G' \Rightarrow G}{\{\,F\,\}\,p\,\{\,G\,\}}$$

4.) Simulation

Let $M_1 = (S_1, I_1, R_1, L_1)$ and $M_2 = (S_2, I_2, R_2, L_2)$ be two Kripke structures.

Simulation

Remember, a relation $H \subseteq S_1 \times S_2$ is a simulation relation if for each $(s, s') \in H$ holds:

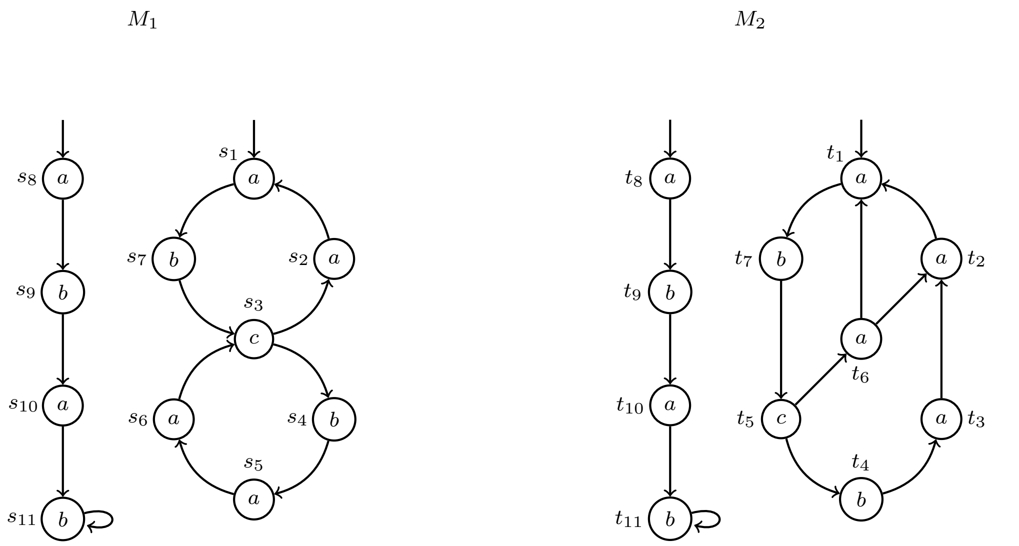
- $L_1(s) = L_2(s')$, and
- for each $(s, t) \in R_1$ there is a $(s', t') \in R_2$ such that $(t, t') \in H$.

Further remember, M_2 *simulates* M_1, in signs $M_1 \leq M_2$, if there is a simulation relation $H \subseteq S_1 \times S_2$ *such that*

- for each initial state $s \in I_1$ there is an initial state $s' \in I_2$ with $(s, s') \in H$.

In the following, we say that H *witnesses the similarity of* M_1 *and* M_2 in case H is a simulation relation from M_1 to M_2 that satisfies the condition stated above.

(a) Show that there is no simulation relation H that witnesses $M_1 \leq M_2$.



(3 points)

(b) Prove the following equivalence of LTL formulae:

$$(\mathbf{G}a) \rightarrow (\mathbf{F}b) \equiv a\mathbf{U}(b \vee \neg a)$$

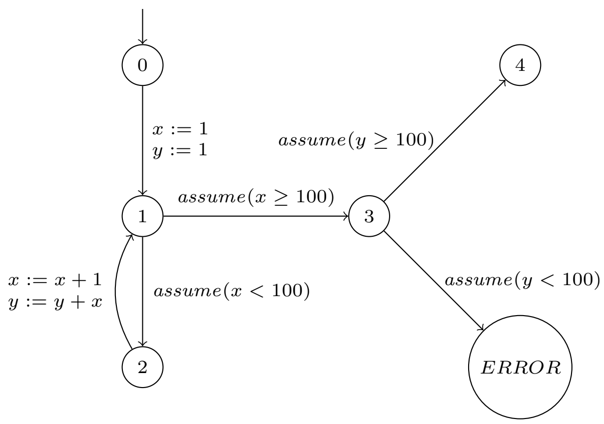
(4 points)

(c) Prove that the following LTL formulae are not equivalent:

$$(\mathbf{F}a) \wedge (\mathbf{XG}a) \not\equiv \mathbf{F}a$$

(2 points)

(d) Consider the following program stated in form of a *labeled transition system*:



i. Provide an abstraction for the labeled transition system that uses the predicates $x > 0$, $y > 0$
ii. Give an error trace in the abstraction
iii. State an additional predicate which can be used to refine the abstraction in order to get rid of the error state. Don't draw the new abstraction.

(6 points)