1.) Consider the following problem:

**IMPERFECT-COMPRESSION (IC)**

**INSTANCE:** A program (i.e. a source code) II such that II takes one string as input and outputs a string. It is guaranteed that II terminates on any input string.

**QUESTION:** Does there exists an input string $I$ for II such that $|\Pi(I)| > |I|$. Here $|J|$ denotes the length of a string $J$, and $\Pi(J)$ is the string returned by II on input string $J$.

Prove that the problem IC is semi-decidable. For this, describe a procedure that shows the semi-decidability of the problem (i.e. a semi-decision procedure for IC) and argue that it is correct. (15 points)

2.) Let $\varphi$ be the formula $(x \rightarrow (y \rightarrow z)) \rightarrow ((x \land y) \rightarrow z)$.

(a) Use the Tseitin translation and compute $\hat{\delta}(\varphi)$. (Hint: It is allowed to avoid the labels for atoms; use the atoms instead. Moreover, use $\ell_{\varphi}$ as the label for $\varphi$.) (5 points)

(b) Try to derive the empty clause $\square$ from

$$\left( \bigwedge_{D \in \hat{\delta}(\varphi)} D \right) \land \neg \ell_{\varphi}$$

by resolution. (4 points)

(c) Answer the following questions and explain in detail.

i. Is $\bigwedge_{D \in \hat{\delta}(\varphi)} D$ satisfiable? If so then provide a model.

ii. Is $\bigwedge_{D \in \hat{\delta}(\varphi)} D$ valid? If not then provide a counterexample.

iii. Is $\left( \bigwedge_{D \in \hat{\delta}(\varphi)} D \right) \rightarrow \ell_{\varphi}$ valid?

iv. Is $\varphi$ valid? (6 points)

3.) Show that the following correctness assertion is totally correct. Describe the function computed by the program if we consider $a$ as its input and $c$ as its output.

```plaintext
{ 1: a \geq 0 }
     b \leftarrow 1;
     c \leftarrow 0;
{ Inv: b = (c + 1)^3 \land 0 \leq c^3 \leq a }
while b \leq a do
    d \leftarrow 3 * c + 6;
    c \leftarrow c + 1;
    b \leftarrow b + c * d + 1
od
{ 2: c^3 \leq a < (c + 1)^3 }
```

(15 points)
4.) Simulation and Bisimulation

(a) Let $K_1$ and $K_2$ be the two Kripke structures given below. Check which of the relations $K_1 \leq K_2$, $K_1 \geq K_2$, $K_1 \equiv K_2$ hold on $K_1$ and $K_2$. Justify your answer.

(b) Show that simulation is a transitive relation: Given any 3 Kripke structures $K_1 = \{S_1, R_1, L_1\}$, $K_2 = \{S_2, R_2, L_2\}$ and $K_3 = \{S_3, R_3, L_3\}$ such that $K_1 \leq K_2$ and $K_2 \leq K_3$, it holds that $K_1 \leq K_3$.

(c) Given the Kripke structures $K_1 = (S_1, R_1, L_1)$, $K_2 = (S_2, R_2, L_2)$, $K_3 = (S_3, R_3, L_3)$, the simulation relation $H_1 \subseteq S_1 \times S_2$ from $K_1$ to $K_2$ and the simulation relation $H_2 \subseteq S_2 \times S_3$ from $K_2$ to $K_3$, state an algorithm which computes a simulation $H_3$ from $K_1$ to $K_3$. 

(5 points)