6.0/4.0 VO Formale Methoden der Informatik (WS2010)

May 6, 2011

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1.) We provide next a reduction from **2-COLORABILITY** to **2-SAT**. Let $G = (V, E)$ be an arbitrary undirected graph (i.e. an arbitrary instance of **2-COLORABILITY**), where $V = \{v_1, \ldots, v_n\}$. For the reduction we use propositional variables $x_1, \ldots, x_n$. Then the instance φ_G of **2-SAT** resulting from G is defined as follows:

$$\varphi_G = \bigwedge_{[v_i, v_j] \in E} (x_i \vee x_j) \wedge (\neg x_i \vee \neg x_j).$$

Task: Prove the "⇐" direction in the proof of correctness of the reduction, i.e. prove the following statement: if φ_G is a positive instance of **2-SAT**, then G is a positive instance of **2-COLORABILITY**.

Note: For any property that you use in your proof, make it perfectly clear why this property holds (e.g., "by the problem reduction", "by the assumption X", "by the definition X", etc.) **(15 points)**

2.) (a) Prove the formula $\varphi\colon ((x \wedge y) \rightarrow z) \rightarrow (x \rightarrow (y \rightarrow z))$ in the following steps:

(i) Use the Tseitin translation and compute $\hat{\delta}(\varphi)$. (Hint: It is allowed to avoid the labels for atoms; use the atoms instead. Moreover, use ℓ_φ as the label for φ.)

(ii) Show the validity of

$$\bigwedge_{D \in \hat{\delta}(\varphi)} D \rightarrow \ell_\varphi$$

by resolution. If the formula is not valid, then provide a counter-example. **(5 points)**

(b) Given the following clauses, draw an implication graph starting with $A@1$ followed by $\neg F@2$ (if necessary).

$$\begin{array}{llll} C_1\colon\ \neg a \vee b & C_2\colon\ \neg a \vee c \vee e & C_3\colon\ \neg b \vee d & C_4\colon\ \neg c \vee \neg d \\ C_5\colon\ a \vee e \vee \neg b & C_6\colon\ b \vee c & C_7\colon\ b \vee \neg c & C_8\colon\ f \vee \neg e \end{array}$$

Is the clause set unsatisfiable? If yes, then give a proof; if not, then provide a model. (Hint: If you reach a conflict within your implication graph, you do not have to apply backtracking. Instead, you may start to use other arguments to show the satisfiability or unsatisfiability of the clause set.) **(5 points)**

(c) Let $\varphi\colon \exists x \exists y \exists z \big[p(x, y) \wedge p(y, z)\big]$ where p is a binary predicate symbol and let $R\colon \forall x\, p(x, x)$ be the first-order reflexivity axiom. Choose an arbitray model $M = (I, U)$ from $Mod(R)$ and show that M is also a model of φ. **(5 points)**

3.) (a) Show that the following modified if-rule is partially (or totally) correct.

$$\frac{\{\,F\,\}\ p\ \{\,G\,\} \quad \{\,F\,\}\ q\ \{\,G\,\}}{\{\,F\,\}\ \mathsf{if}\ e\ \mathsf{then}\ p\ \mathsf{else}\ q\ \mathsf{fi}\ \{\,G\,\}}\ (\text{if}'')$$

Hint: It is not necessary to use the semantics of correctness assertions; use the rules of the Hoare calculus. **(3 points)**

(b) Show that the Hoare calculus is not complete, if the regular if-rules are replaced by the rule if″ of the previous exercise.
Hint: Find a correctness assertion that is correct (argue why it is!) but that cannot be derived in the modified calculus (explain why it can't!). **(5 points)**

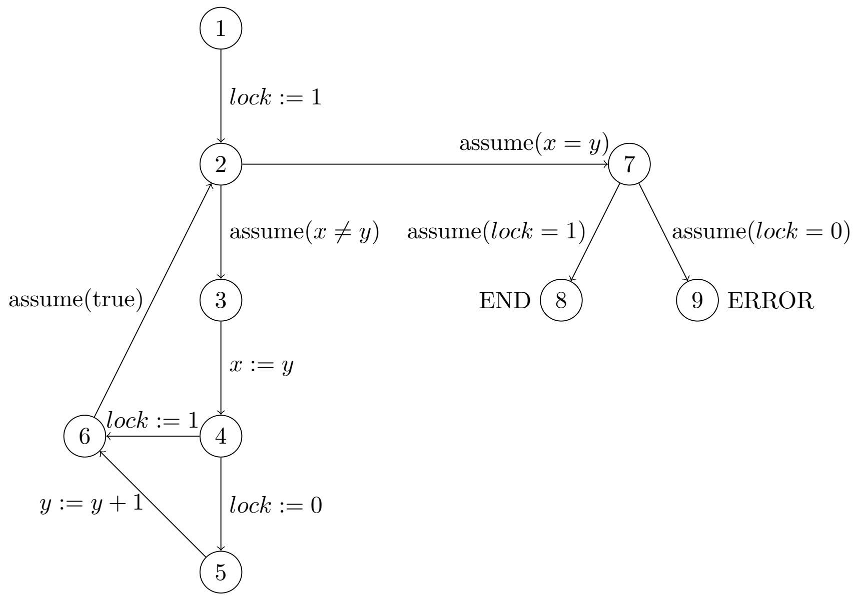
4.) Prove the partial correctness of the assertion below using the invariant $Inv \equiv x + y = x_0 + y_0$. Give a bound function and explain, how the precondition and the invariant have to be modified to prove total correctness. (It is not necessary to give a formal termination proof.)

$\{\, 1\colon x = x_0 \wedge y = y_0 \,\}$
while $x \neq y$ do
  $x \leftarrow x - 1;$
  $y \leftarrow y + 1$
od
$\{\, 2\colon y = (x_0 + y_0)/2 \,\}$

(7 points)

5.) (a) Find a Kripke structure K with initial state s_0 that has the properties $\mathbf{AG}(q \Rightarrow (\neg p \wedge \mathbf{AX}(\neg p \wedge \mathbf{A}[(\neg q)\mathbf{U}p])))$ and $\mathbf{AG}(p \Rightarrow (\neg q \wedge \mathbf{AX}(\neg q \wedge \mathbf{A}[(\neg p)\mathbf{U}q])))$, but not $\mathbf{EF}((\mathbf{G}\neg p) \vee (\mathbf{G}\neg q))$. Justify your choice. **(5 points)**

(b) Consider the following labeled transition:



i. Provide an abstraction for the labeled transition system that uses the predicate $lock = 1$.
ii. Give an error trace in the abstraction.
iii. Introduce a new predicate to refine the abstraction to get rid of the error state. Give the new abstraction.

(5 points)

(c) We define the *release operator* R by $\phi \mathsf{R} \psi := \psi \mathsf{U} \phi \vee \mathsf{G}\psi$. Intuitively ψ releases ϕ, if ψ is true until the first position in which ϕ is true (or forever if such a position does not exist).

Let $K = (S, T, L)$ be a Kripke structure and p, q be a atomic propositions. Give a graph-theoretic algorithm for computing the set of states where $\mathsf{A}[p\mathsf{R}q]$ holds.

(5 points)