Existence Conditions for Extensions in Abstract Argumentation Frameworks[1]

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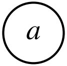
Research Seminar, November 26, 2015

[1] This research has been supported by FWF (project I1102).

Fact Check I



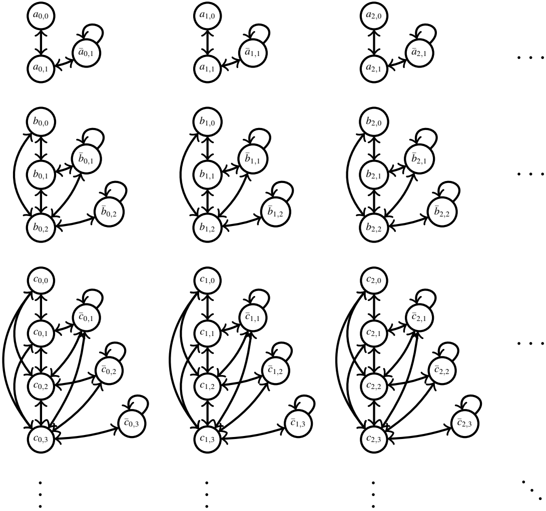
some argument

Fact Check II

$$\longrightarrow$$

some attack

Fact Check III



some infinite argumentation framework

Naive and Preferred Semantics

Definition

Maximal conflict-free sets are called *naive* extensions. *Admissibility* is the concept of self-defense. Maximal admissible sets are called *preferred* extensions.

Theorem ([Spanring, 2014])

Existence of naive/preferred extensions is equivalent to the axiom of choice (AC).

Definition (Axiom of Choice (variant))

For any given set of sets Σ there exists a choice function $\delta : \Sigma \to \bigcup \Sigma$ with $\delta(\sigma) \in \sigma$ for each $\sigma \in \Sigma$.

(AC)$\Rightarrow prf(F) \neq \emptyset$

Definition (Zorn's Lemma)

If any chain of a non-empty partially ordered set has an upper bound then there is at least one maximal element.

Definition (Partial Order)

A *partial order* $(P, \leq)$ is a set P with a binary relation $\leq$ that fulfills

- reflexivity: $a \leq a$,
- antisymmetry: $a \leq b \wedge b \leq a \Rightarrow a = b$,
- transitivity: $a \leq b \wedge b \leq c \Rightarrow a \leq c$.

Definition (Axiom of Union)

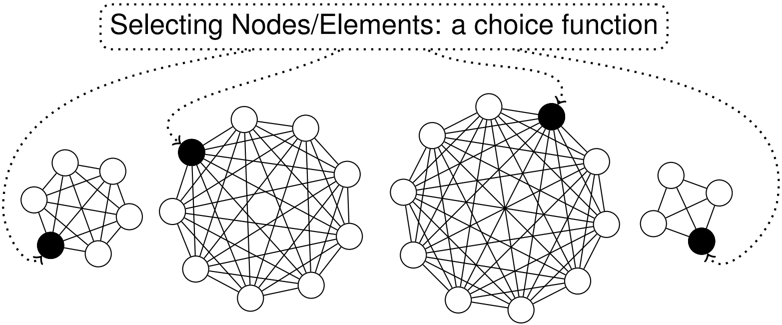
The union over the elements of a set is a set.

$$\forall z \exists y \forall x \forall u (x \in z \wedge u \in x) \Leftrightarrow u \in y$$

$(\forall F prf(F) \neq \emptyset) \Rightarrow$ **(AC)**

Definition (ZF-Axioms)

- Comprehension: we can construct formalizable subsets of sets.
- Union: the union over the elements of a set is a set.
- Replacement: definable functions deliver images of sets.
- Power Set: we can construct the power set of any set.



Definitions

Definition ([Dung, 1995])

An argumentation framework is a pair $F = (A, R)$ of arguments A and attacks $R \subseteq A \times A$. The range of a set of arguments S is given as $S^+ = S \cup \{a \in A, S \rightarrowtail a\}$.

Definition ([Verheij, 2003, Caminada and Verheij, 2010])

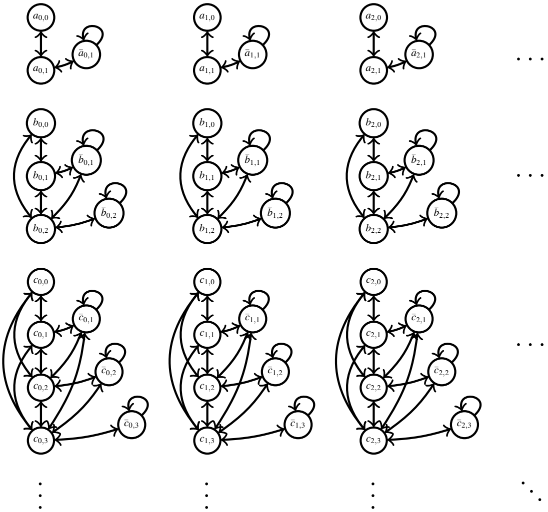
A set $S \subseteq A$ is called conflict conflict-free, $S \in cf(F)$, if $S \times S \cap R = \emptyset$. $S \in cf(F)$ is called

- admissible, $S \in adm(F)$, if $a \rightarrowtail S$ implies $S \rightarrowtail a$;
- a stable extension, $S \in stb(F)$, if $S^+ = A$;
- a stage extension, $S \in stg(F)$, if it is maximal in range.

An set $S \in adm(A)$ is called

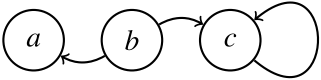
- a semi-stable extension, $S \in sem(F)$, if it is maximal in range.

Some Infinite AF

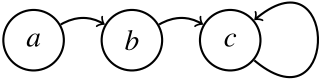


some infinite argumentation framework

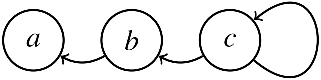
Stable, Stage and Semi-Stable Semantics



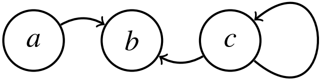
$$\mathit{stb} : \{\{b\}\}$$
$$\mathit{sem} : \{\{b\}\}$$
$$\mathit{stg} : \{\{b\}\}$$



$$\mathit{stb} : \emptyset$$
$$\mathit{sem} : \{\{a\}\}$$
$$\mathit{stg} : \{\{a\}, \{b\}\}$$

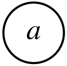


$$\mathit{stb} : \emptyset$$
$$\mathit{sem} : \{\emptyset\}$$
$$\mathit{stg} : \{\{b\}\}$$



$$\mathit{stb} : \emptyset$$
$$\mathit{sem} : \{\{a\}\}$$
$$\mathit{stg} : \{\{a\}\}$$

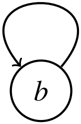
Stable, Stage and Semi-Stable Semantics ctd.



$stb : \{\{a\}\}$

$sem : \{\{a\}\}$

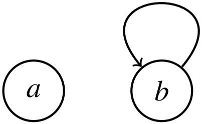
$stg : \{\{a\}\}$



$stb : \emptyset$

$sem : \{\emptyset\}$

$stg : \{\emptyset\}$

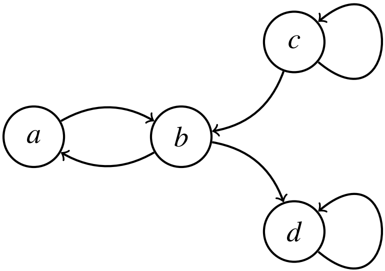


$stb : \emptyset$

$sem : \{\{a\}\}$

$stg : \{\{a\}\}$

Stage and Semi-Stable Semantics



$$sem : \{\{a\}\}$$
$$stg : \{\{b\}\}$$

Known Classes of Infinite AFs

Definition (Standard Classes)

Consider odd-cycle/even-cycle/cycle free AFs, finite AFs, bipartite AFs, coherent ($\mathit{stb} = \mathit{prf}$) AFs, well-founded ($\mathit{grd} = \mathit{stb}$) AFs. bipartite (and well-founded) AFs are coherent. Granted AC, coherent AFs provide sem and stg extensions.

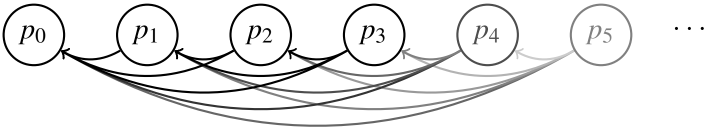
Definition (Finitary AFs [Dung, 1995])

An AF F is called finitary if for each argument b we have $|\{a \rightarrowtail b\}| < \infty$.

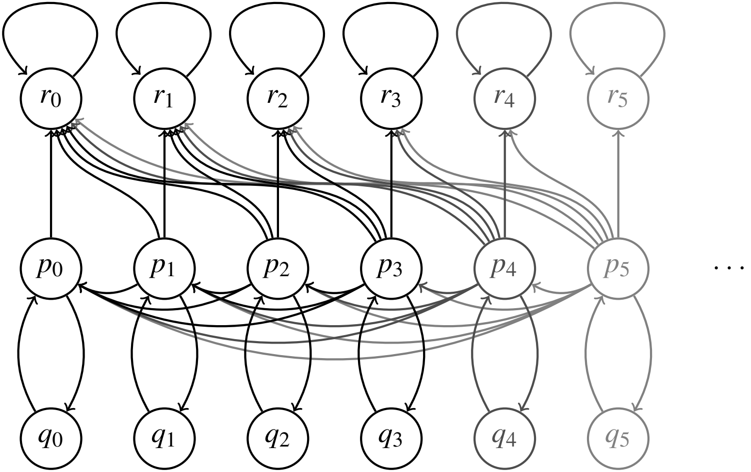
Definition (Finitarily Superseded)

An AF F is called finitarily superseded, if there is a finitary AF $F' \subseteq F$ and mapping $f : A_F \mapsto A_{F'}$ such that $a \rightarrowtail b$ implies $f(a) \rightarrowtail b$ and $a \rightarrowtail f(b)$ implies $a \rightarrowtail b$.

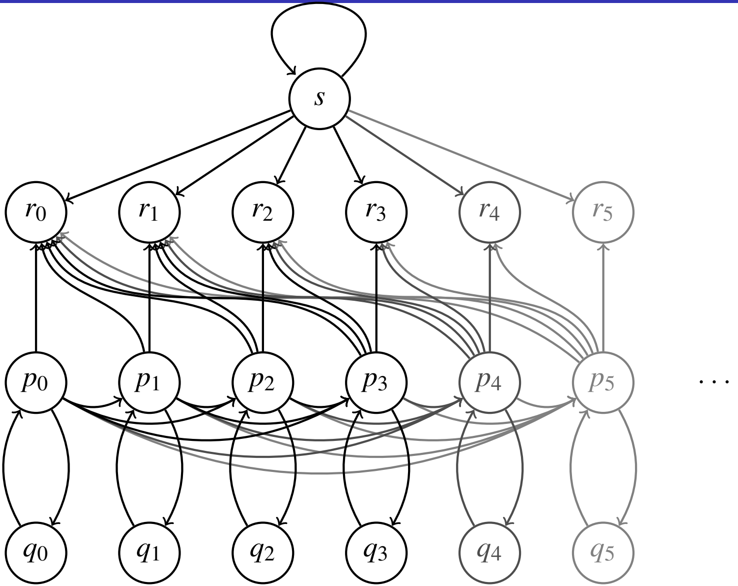
Crash of Stage Semantics [Verheij, 2003]



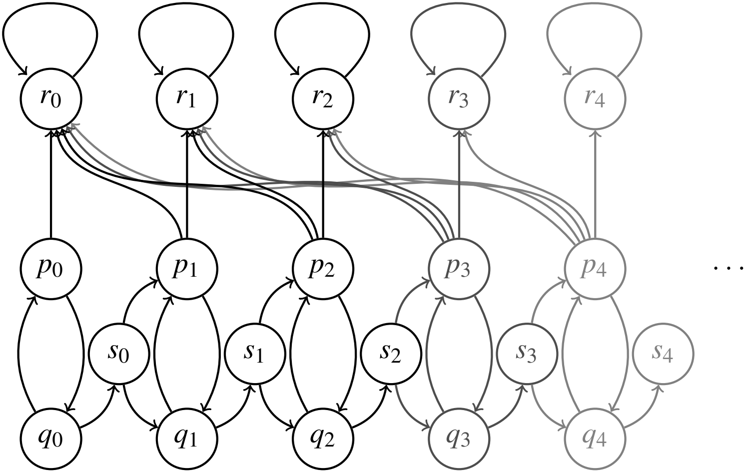
Crash of Semi-Stable and Stage Semantics [Verheij, 2003]



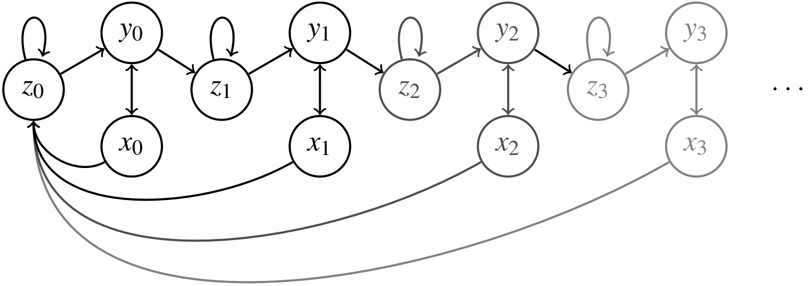
Crash of Semi-Stable Semantics



Crash of Semi-Stable and Stage Semantics



Crash of Semi-Stable Semantics



Insights

Theorem ([Baumann and Spanring, 2015, Weydert, 2011])

Any finitary (no argument with infinitely many attackers) argumentation framework provides semi-stable and stage extensions.

Theorem (Not yet published)

For any framework-property that is subframework-valid and guarantees existence of stage extensions, we can have any finite amount of arguments violating this property without loosing the guarantee for the existence of stage extensions.

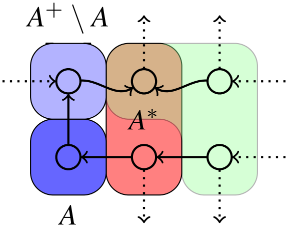
Corollary (Conjecture)

If for some argumentation framework there is no stage extension, then there is an infinite amount of arguments with infinitely many attackers.

Finitary AFs

Theorem ([Baumann and Spanring, 2015, Weydert, 2011])

Any finitary (no argument with infinitely many attackers) argumentation framework provides semi-stable and stage extensions.



Definition (Concepts)

For Σ a set of sets of arguments, $\Sigma^+ = \bigcup_{S \in \Sigma} S^+$ the range of Σ, define *keeper* (occurs range-unbounded) and *outsider* (otherwise). $\Sigma = (S_i)_i$ with $S_i^+ \subseteq S_j^+$ whenever $i \leq j$ is called a *range chain* and Σ^+ the corresponding *chain range*.

Existence of Stage Extensions

Theorem

Given some AF $F = (A, R)$ *and argument* $x \in A$*. If* $stg(F|_{A\setminus\{x\}}) \neq \emptyset$ *and* $stg(F|_{A\setminus x^+})$ *then* $stg(F) \neq \emptyset$*.*

Proof Sketch.

For $S \in nav(F)$ exactly one of the following holds:

1. $x \in S$,
2. $S \rightarrowtail x$,
3. $x \notin S^+$ holds;

Wlog. unbounded rangechains make use of only of these. □

Existence of Stage Extensions

Corollary (slightly weaker version)

Finite extensions of stage-perfect AFs are still stage-perfect.

Corollary (implications of finite AFs)

If stage semantics crashes then there is an infinite amount of arguments.

Corollary (implications of finitary AFs)

If stage semantics crashes there is an infinite amount of arguments with infinitely many incoming attacks.

Corollary (impliciations of bipartite AFs)

If stage semantics crashes there is an infinite amount of relatively independent (undirected) odd cycles.

Summary, Existence Conditions

Theorem (Preferred and Naive Semantics)

Axiom of Choice.

Theorem (Coherent AFs)

Stable and Preferred Semantics agree (and thus Semi-Stable and Stage as well). This includes symmetric AFs, bipartite AFs, well-founded AFs, odd-cycle free AFs where each path has a source...

Theorem (Semi-Stable Semantics)

Coherence, Finitariness, others?

Theorem (Stage Semantics)

Coherence, Finitariness, Finite expansions of stage-perfect AFs, others?

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Jeffery Raphael

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by year Trier 1

2010 – today

Refine list

2015

[c3] Jeffery Raphael, Simon Maskell, Elizabeth Sklar:
From Goods to Traffic: First Steps Toward an Auction-Based Traffic Signal Controller. PAAMS 2015: 187-198

[c2] Jeffery Raphael, Simon Maskell, Elizabeth Sklar:
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[c1] Jeffery Raphael, Eric Schneider, Simon Parsons, Elizabeth I. Sklar:
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Coauthor Index

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