The Challenge of Optional Matching in SPARQL

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(joint work with Shqiponja Ahmetaj, Wolfgang Fischl, Markus Kröll, Mantas Simkus, and Sebastian Skritek)

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Roadmap

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SPARQL and optional matching

- SPARQL
  - W3C query language for the semantic web
  - important feature: OPTIONAL
  - significant extension of CQs: non-monotonicity

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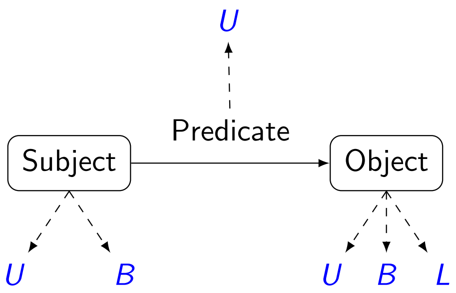
- SPARQL
  - W3C query language for the semantic web
  - important feature: OPTIONAL
  - significant extension of CQs: non-monotonicity
- Various challenges:
  - query answering: becomes harder
  - static query analysis: containment vs. subsumption
  - SPARQL entailment regimes: rethinking of the semantics

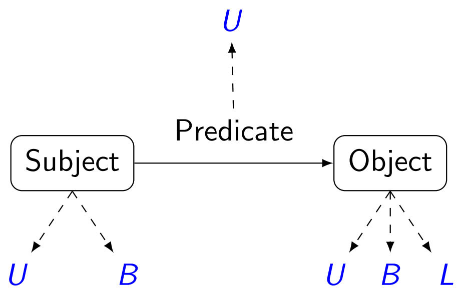
Roadmap

RDF: Resource Description Framework



U = set of Uris
B = set of Blank nodes
L = set of Literals

RDF: Resource Description Framework



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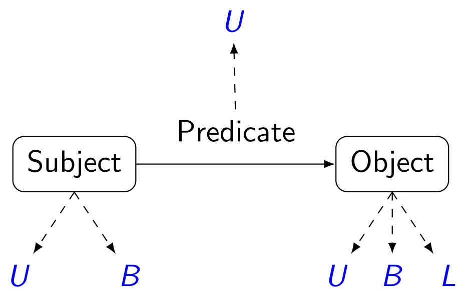

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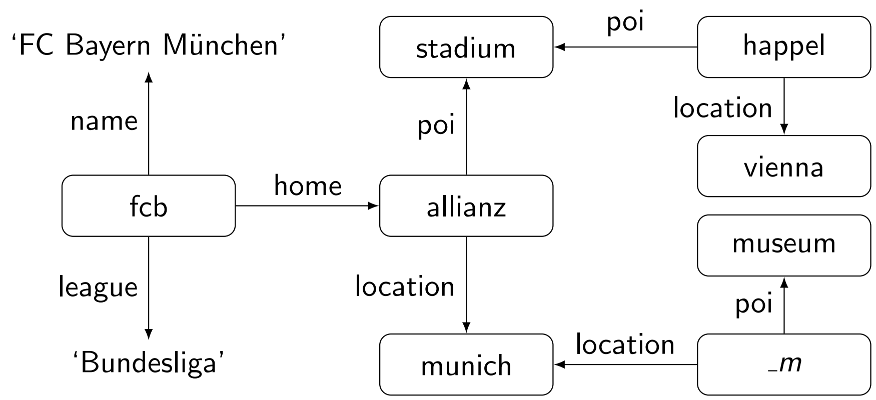
A set of RDF triples is called an RDF graph.

RDF: Example



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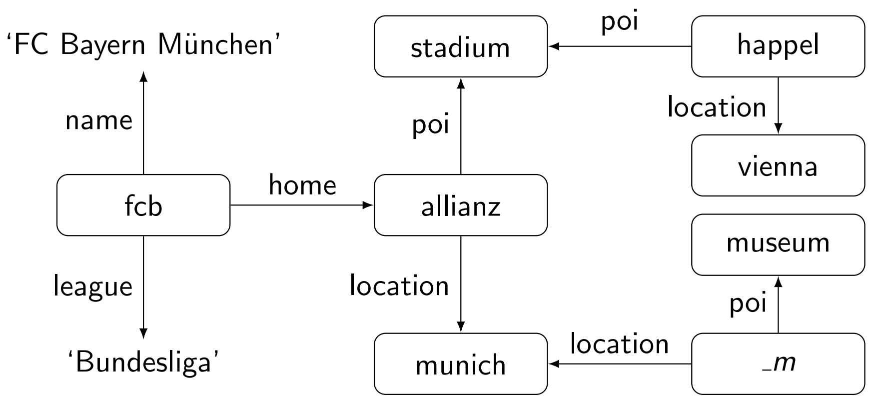

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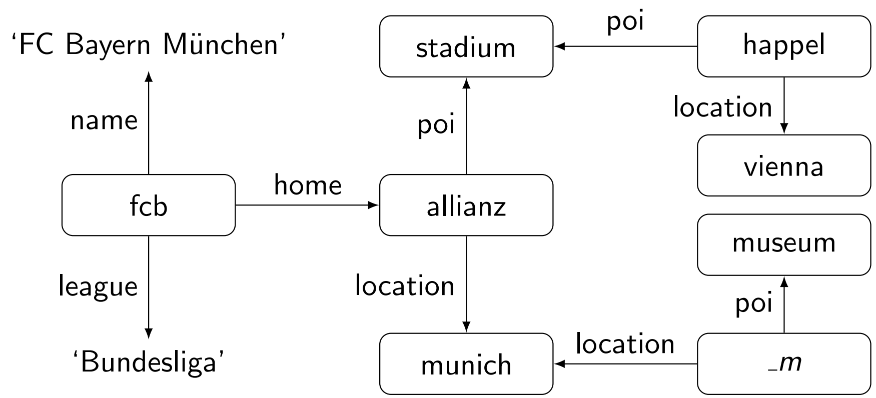

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Remark. Use of algabraic-style notation from [Pérez, Arenas, Gutierrez 2009].

Querying RDF with triple patterns

- Triple patterns: RDF triple + variables

$$(?X, \text{name}, ?N)$$

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$$\{t_1, t_2, \ldots, t_k\}.$$

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Example

$$\{\ (?X, \text{name}, ?N),\ \ (?X, \text{home}, ?S)\ \}$$

The semantics of triple patterns

Definition

The evaluation of a triple pattern t over an RDF graph G yields the set of mappings M such that $\mu \in M$ if

- μ has as domain the variables in t: $dom(\mu) = \text{var}(t)$
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Example

triple pattern: $(?X, \text{location}, ?Y)$

graph

(allianz, location, munich)
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(happel, location, vienna)

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?X	?Y

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μ_1:	allianz	munich
μ_2:	happel	vienna

SPARQL: "SPARQL Protocol and RDF Query Language"

- W3C recommendation for querying RDF:
  - SPARQL 1.0: January 2008
  - SPARQL 1.1: March 2013
- A SPARQL query consists of three parts:
  - Pattern matching: AND, OPTIONAL, UNION, FILTER, ...
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SPARQL Query

Example

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SELECT ?Stadium ?League
WHERE
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Head: Processing of the variable assignments

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$((?X, \text{location}, ?Y) \text{ OPT } (?X, \text{poi}, ?E))$

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INPUT: An RDF graph G, a graph pattern P and a mapping μ.
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Theorem (Schmidt, Meier, Lausen 2010)

EVALUATION remains PSPACE*-complete if P contains operators AND and OPT only.*

Observation

- High complexity due to unrestricted occurrences of OPT-operator.
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RDF graph:

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SPARQL graph pattern:

$(?C, \text{home}, ?S)\ \text{OPT}\ ((?C, \text{league}, ?L)\ \text{OPT}\ (?S, \text{location}, ?P))$

result:

$\{\{?C \to \text{fcb}, ?S \to \text{allianz}\}\}$

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2. Introduction to RDF and SPARQL (14 / 47)

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Well-designed graph patterns

Definition

A query in the AND-OPT-FILTER fragment of SPARQL is well-designed if for every OPT in the pattern:

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if a variable occurs inside Q and anywhere outside the OPT expression, then the variable must also occur inside P.

Well-designed graph patterns

Theorem (Pérez, Arenas, Gutierrez 2009)

EVALUATION is coNP-complete for well-designed graph pattern expressions constructed by using only AND and OPT operators.

Theorem (Letelier, Pérez, P., Skritek 2012)

EVALUATION is Σ_2P–complete for well-designed graph pattern expressions constructed by using only AND and OPT operators and allowing projection.

Summary so far

- Syntax of RDF
  - RDF triples (s, p, o)
  - underlying data model: labelled, directed graph
- Basic querying of RDF
  - RDF triple patterns
  - Basic graph patterns (BGPs)
  - Result of evaluation: set of mappings
- SPARQL
  - SPARQL graph patterns
  - principal operators: AND and OPT
  - complexity of SPARQL
  - well-designed SPARQL

Roadmap

OPT normal form [Pérez, Arenas, Gutierrez 2009]

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$$(P_1 \text{ AND } (P_2 \text{ OPT } P_3)) \equiv ((P_1 \text{ AND } P_2) \text{ OPT } P_3)$$

for well-designed graph patterns

Pattern trees: representing well-designed graph patterns

Represent patterns in OPT normal form by trees:

- nodes represent basic graph patterns (BGPs, CQs)
- tree structure represents optionality

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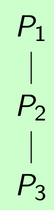
Intuition



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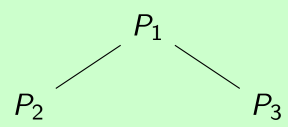

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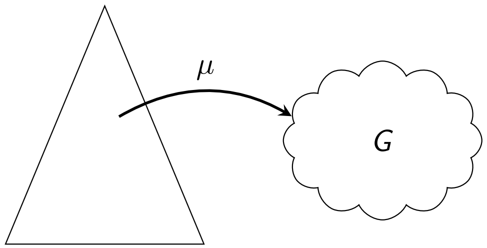
Pattern trees: evaluation of well-designed SPARQL

Mapping $\mu \in [\![\mathcal{T}]\!]_G$ if and only if



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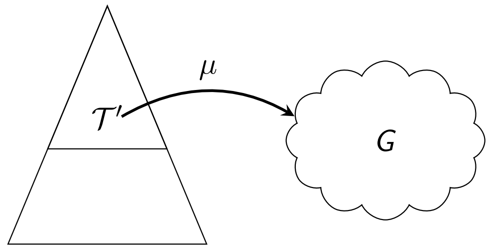

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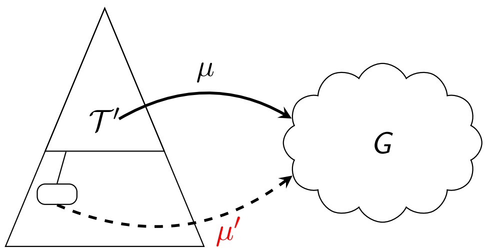

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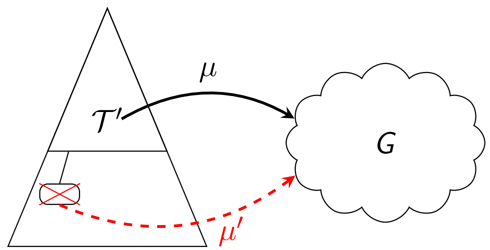

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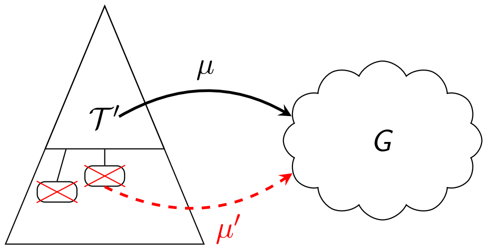

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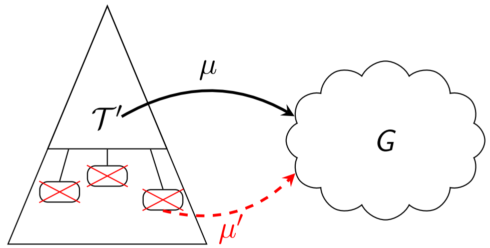

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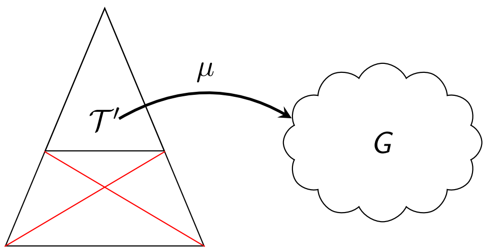
2. there do not exist extensions μ' of μ and $\hat{\mathcal{T}}'$ of $\mathcal{T}'$ s.t.

   $dom(\mu') = vars(\hat{\mathcal{T}}')$ and $\mu'(\hat{\mathcal{T}}') \subseteq G$



Summary so far

- Introduction to RDF and SPARQL
- well-designed SPARQL
  - OPT-normal form
  - pattern trees
  - characterization of query evaluation

Roadmap

SPARQL: Subsumption

Definition

Let P_1 and P_2 be two SPARQL graph patterns. We say that P_1 is subsumed by P_2 ($P_1 \sqsubseteq P_2$) if, for every RDF graph G, every solution of P_1 over G can be extended to a solution of P_2 over G.

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Example

- $G = \{(fcb, n, 'bayern'), (fcb, h, allianz)\}$
- $P_1 = (?C, n, ?N)$
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- $[\![P_1]\!]_G = \{\mu\}$ with $\mu = \{?C \to fcb, ?N \to \text{`}bayern'\}$,
- $[\![P_2]\!]_G = \{\mu'\}$ with $\mu' = \{?C \to fcb, ?N \to \text{`}bayern', ?H \to allianz\}$

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We have $P_1 \sqsubseteq P_2$ but $P_1 \not\subseteq P_2$.

Complexity

Language fragments considered

- well-designed SPARQL (AND, OPT): wd-SPARQL[$\emptyset$]
- extension by UNION: wd-SPARQL[$\{\cup\}$]
- extension by projection: wd-SPARQL[$\{\pi\}$] (i.e., via an appropriate SELECT clause)
- extension by both: wd-SPARQL[$\{\cup, \pi\}$]

Decision problems: for $S_1, S_2 \subseteq \{\cup, \pi\}$

CONTAINMENT[S_1,S_2]

INPUT: $Q_1 \in$ wd-SPARQL[S_1], $Q_2 \in$ wd-SPARQL[S_2]
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SUBSUMPTION$[S_1, S_2]$

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QUESTION: Does $Q_1 \sqsubseteq Q_2$ hold?

Complexity [P., Skritek 2014]

CONTAINMENT$[S_1, S_2]$

$\downarrow S_1$ / $S_2 \rightarrow$	$\emptyset$	$\{\cup\}$	$\{\pi\}$	$\{\cup, \pi\}$
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$\{\cup\}$	Π_2P-compl.	Π_2P-compl.	–	–
$\{\pi\}$	Π_2P-compl.	Π_2P-hard	undec.	–
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*[Letelier, Pérez, P., Skritek 2012]

Complexity of subsumption

Theorem (Letelier, Pérez, P., Skritek 2012)

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SUBSUMPTION$[S_1, S_2]$ *is in* Π_2P *even for* $S_1 = S_2 = \{\cup, \pi\}$.

Overview of Results

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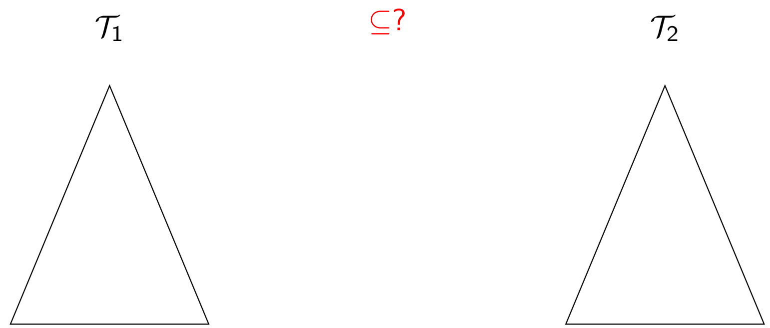
Decidable Containment



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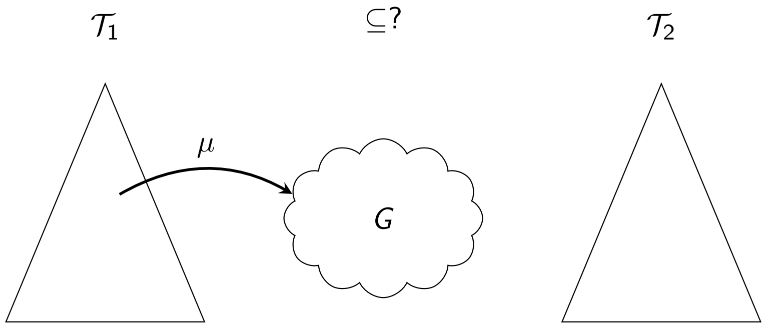

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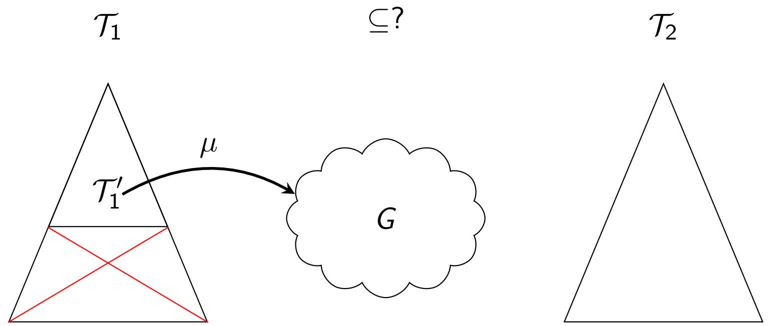

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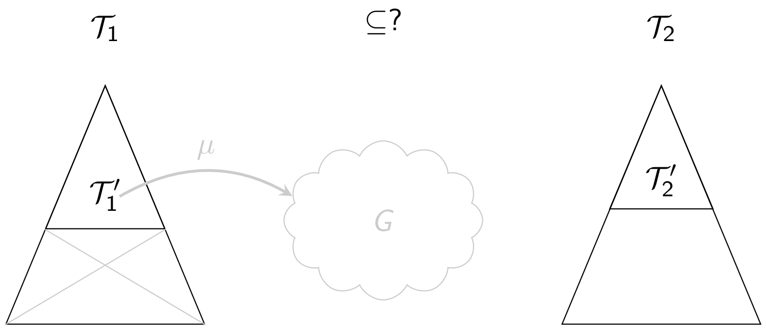

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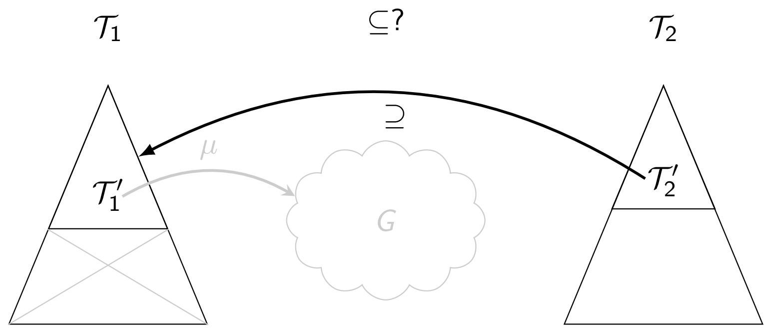

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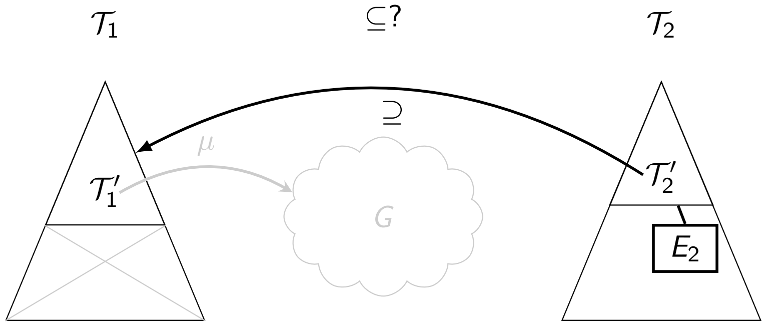

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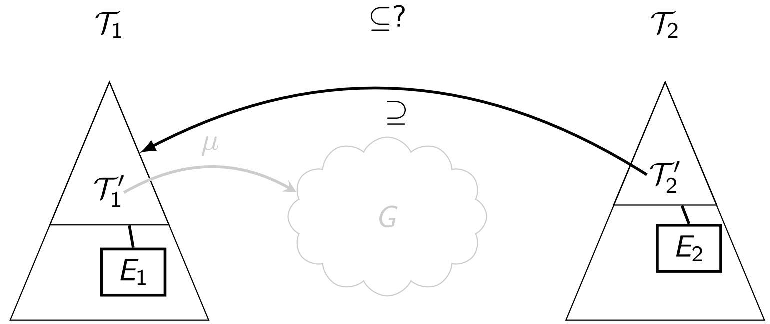

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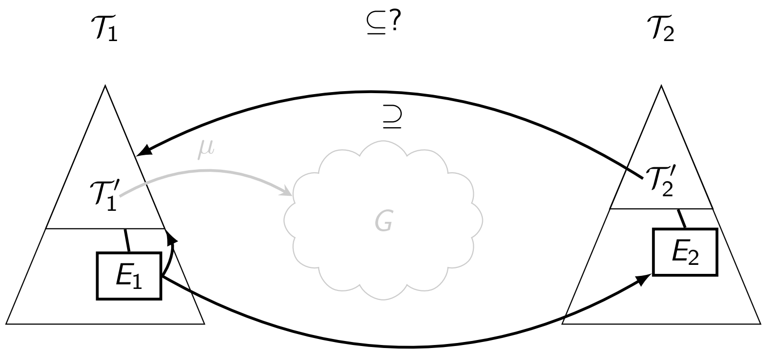

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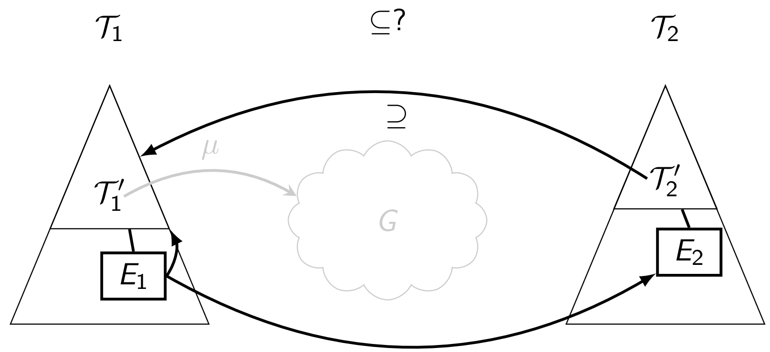

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NP-completeness

Theorem (P., Skritek 2014)

Let $S_1 \subseteq \{\cup, \pi\}$ and $S_2 = \emptyset$.
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Proof idea.

- apply the algorithm sketched above
- show that it suffices to test polynomially many subtrees of $\mathcal{T}_1$

Overview of Results:

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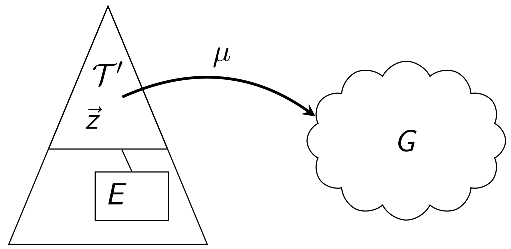
Undecidable Containment

Evaluation of $\mathcal{T} \in$ wd-SPARQL$[\{\pi\}]$:



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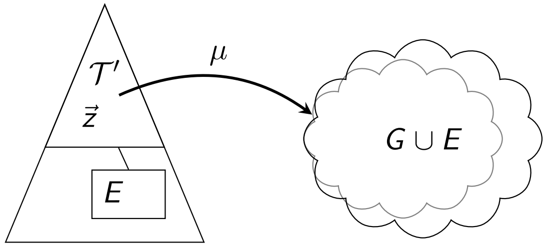

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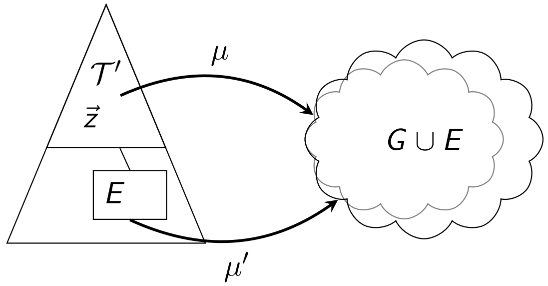

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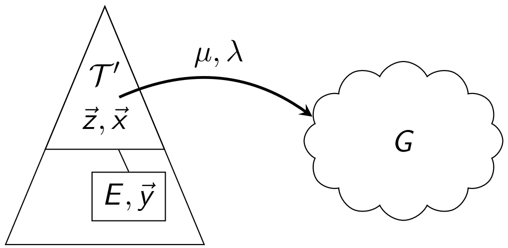

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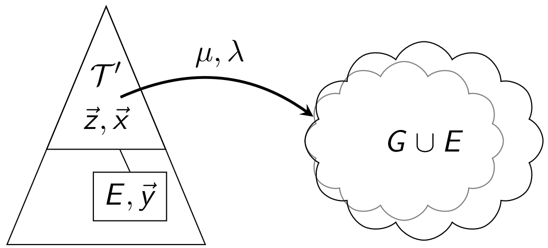

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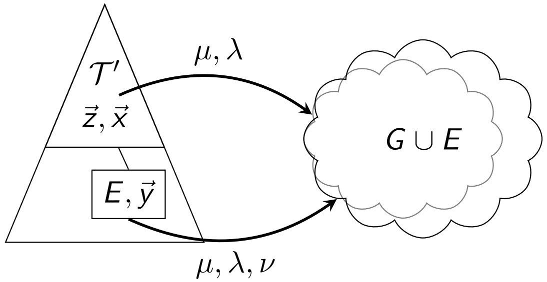

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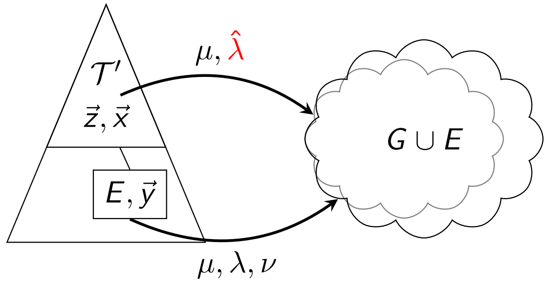

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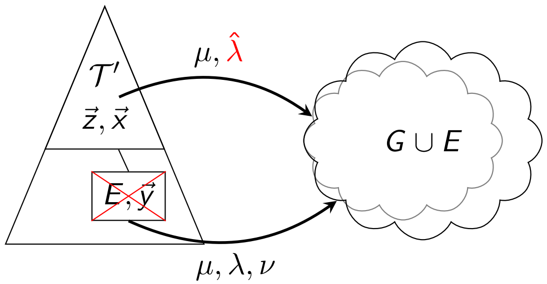

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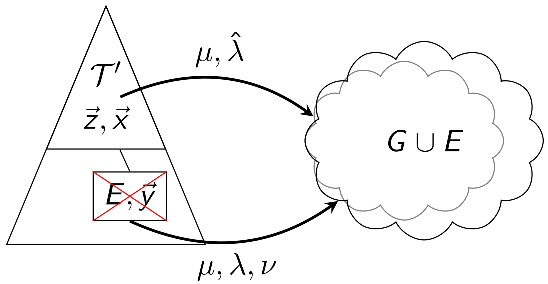

similar to TGDs: $\forall \vec{x} \varphi(\mu(\vec{z}), \vec{x}) \rightarrow \exists \vec{y} \psi(\mu(\vec{z}), \vec{x}, \vec{y})$

Undecidability

BCQ-UNDER-TGDS

INPUT: Instance I, BCQ $\exists\vec{q}\, Q(\vec{q})$,
tgd $\tau\colon \forall\vec{x}(\varphi(\vec{x}) \rightarrow \exists\vec{y}\psi(\vec{x},\vec{y}))$

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Let $S_1 \subseteq \{\cup, \pi\}$ *and* $\pi \in S_2$.
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Proof idea.

by reduction from BCQ-UNDER-TGDS

Roadmap

1. Motivation
2. Introduction to RDF and SPARQL
3. SPARQL Pattern Trees
4. Static Analysis of SPARQL Queries
5. Certain answers for OWL2 Entailment
6. Conclusion and Future Work

Motivation

OBDA with SPARQL

- W3C defines various entailment regimes for SPARQL
  - examples of entailment regimes: RDF simple entailment, RDFS, OWL2 profiles (like OWL2-QL), etc.
  - W3C standard defines evaluation of BGPs
  - resulting semantics may not be intuitive
- Challenge with SPARQL
  - non-monotonicity of the OPT operator
  - adapt certain answer semantics to wd-SPARQL
  - adapt query rewriting to wd-SPARQL

Example 1: existentially quantified variables

Example

Consider RDF graph $G = \{(marc, \text{a}, Prof)\}$
with OWL2 ontology $\mathcal{O} = \{Prof \sqsubseteq \exists teaches\}$.

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Remark. blank nodes are “immediately” projected out
$\Rightarrow$ binding to labelled nulls in the canonical model is allowed.

Example 2: join across the OPT operator

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Again, consider RDF graph $G = \{(\text{marc,a,Prof})\}$
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What about $\mu = \{?x \rightarrow marc, ?z \rightarrow marc\}$?

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What about $\mu = \{?x \rightarrow marc, ?z \rightarrow marc\}$?

Remark. SPARQL does not allow the join of blank nodes across BGP boundaries.

Certain answers via subsumption?

Idea: following [Arenas, Pérez 2011]

Certain answers of query Q over RDF graph G w.r.t. ontology $\mathcal{O}$ as greatest lower bound w.r.t. $\sqsubseteq$ over all models of $G \cup \mathcal{O}$.

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- subsumed answers in the presence of projection and/or union

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Let $M_1 = \{\mu_1\}$, $M_2 = \{\mu_1, \mu_2\}$, and $M_3 = \{\mu_1, \mu_3\}$ with
$\mu_1 = \{?x \rightarrow a, ?y \rightarrow b, ?z \rightarrow c\}$,
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$\mu_1 = \{?x \to a, ?y \to b, ?z \to c\}$,
$\mu_2 = \{?x \to a, ?y \to b\}$, and
$\mu_3 = \{?x \to a, ?z \to c\}$.
Then $M_1 \sqsubseteq M_2$, $M_2 \sqsubseteq M_1$, $M_2 \sqsubseteq M_3$, $M_3 \sqsubseteq M_1$, etc.

Certain answers via subsumption?

Example (continued)

Let $M_1 = \{\mu_1\}$ and $M_2 = \{\mu_1, \mu_2\}$ as before.
Suppose that in some possible worlds the set of solutions to a query Q over G w.r.t. $\mathcal{O}$ is M_1 and in some it is M_2.
Which of M_1 and M_2 should be the set of certain answers?

Certain answers via subsumption?

Example (continued)

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Answer: it depends!

Two reasons why in some possible worlds only μ_1 is a solution:
Case 1: μ_2 "gets lost";
Case 2: μ_2 can be extended to μ_1.

Proposed solution

Definition of certain answers

Given SPARQL pattern tree $\mathcal{T}$ (or, equivalently, wd-SPARQL graph pattern), RDF graph G, and ontology $\mathcal{O}$. We define:

$\mu \in \text{CertainAnswers}(\mathcal{T}, G, \mathcal{O})$ if

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3. μ is maximal with properties 1 and 2.

Computation of the certain answers

Idea: following [Calvanese et al., 2007]

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Essentially, this means that μ is a certain answer if the following conditions hold:

1. $\{\mu\} \sqsubseteq [\![\mathcal{T}]\!]_{Can(G \cup \mathcal{O})}$
2. $rg(\mu) \subseteq Voc(G)$
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4. μ is maximal with properties $1 - 3$.

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Step 2: Extension of the "Perfect Reformulation" algorithm to allow joins over blank node _b between OPT-operands even if _b is mapped to a labelled null.
Idea: store "path information" via Skolem terms.

Computation of the certain answers

Example

$G = \{(marc, \text{a}, Prof)\}$
$\mathcal{O} = \{Prof \sqsubseteq \exists teaches\}$
Q: $SELECT\ ?x, ?z\ WHERE\ (?x, teaches, \_b)$ OPT $(?z, teaches, \_b)$

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Rewriting:
$(?x, teaches, \_b) \Rightarrow (?x, \text{a}, Prof)$ with "path" $\_b \rightarrow f_{teaches}(?x)$,
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Evaluation of Q:
$\mu = \{?x \rightarrow marc, ?z \rightarrow marc\}$, because μ_1 and μ_2 are compatible.

Problems Studied

OWL 2 QL-Evaluation

Input: pattern tree $(\mathcal{T}, \mathcal{X})$, graph G, ontology $\mathcal{O}$, mapping μ.
Question: $\mu \in \text{CertainAnswers}((\mathcal{T}, \mathcal{X}), G, \mathcal{O})$?

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OWL 2 QL-Subsumption/OWL 2 QL-Containment/OWL 2 QL-Equivalence

Input: pattern trees $(\mathcal{T}_1, \mathcal{X}_1)$, $(\mathcal{T}_2, \mathcal{X}_2)$, ontology $\mathcal{O}$.
Question: $(\mathcal{T}_1, \mathcal{X}_1) \sqsubseteq_{\mathcal{O}} / \subseteq_{\mathcal{O}} / \equiv_{\mathcal{O}} (\mathcal{T}_2, \mathcal{X}_2)$?

Complexity

[Ahmetaj, Fischl, P., Simkus, Skritek, 2015]

Theorem

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Theorem

The problems OWL 2 QL-SUBSUMPTION, OWL 2 QL-CONTAINMENT, *and* OWL 2 QL-EQUIVALENCE *are* $\Pi_2 P$*-complete. Hardness holds even for empty ontologies.*

Roadmap

Conclusion and Future Work

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- non-monotonicity poses various challenges
- query answering, static analysis, entailment regimes

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Future Work

- closing gaps in complexity analyses
- extensions (patterns, ontologies)
- restrictions to achieve tractability