Database Theory

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1. Introduction: Relational Query Languages

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Outline

A short history of databases

- 1970's:
  - relational model of databases (E. F. Codd)
  - relational query languages (SQL)
- 1980's:
  - relational query optimization
  - constraints, dependency theory
  - datalog (extend the query language with recursion)
- 1990's:
  - new models: temporal databases, OO, OR databases
  - data mining, data warehousing
- 2000's:
  - data integration, data exchange
  - data on the web, managing huge data volumes
  - new data formats: XML, RDF
  - data streams

Database theory

- Cut-crossing many areas in Computer Science and Mathematics
  - *Complexity* → efficiency of query evaluation, optimization
  - *Logics, Finite model theory* → expressiveness
  - *Logic programming, constraint satisfaction (AI)* → Datalog
  - *Graph theory* → (hyper)tree-decompositions
  - *Automata* → XML query model, data stream processing
- Benefit from other fields on the one hand, contribute new results on the other hand

Relational data model

- A database is a collection of relations (or tables)
- Each database has a schema, i.e., the *vocabulary (or signature)*
  - Each relation (table) has a name and a schema;
  - the schema of each relation r is defined by a list of attributes (columns), denoted $schema(r)$
- Each attribute A has a domain (or universe), denoted $dom(A)$
  - We define
$$dom(r) = \bigcup_{A \in schema(r)} dom(A)$$
- Each relation contains a set of tuples (or rows)
  - Formally, a tuple in r is a mapping $t : schema(r) \to dom(r)$ such that $t(A) \in dom(A)$ for all $A \in schema(r)$

Example

- Schema
  - *Author* (*AID* integer, *name* string, *age* integer)
  - *Paper* (*PID* string, *title* string, *year* integer)
  - *Write* (*AID* integer, *PID* string)
- Instance
  - $\{\langle 142, Knuth, 73\rangle, \langle 123, Ullman, 67\rangle, \ldots\}$
  - $\{\langle 181140pods, Querycontainment, 1998\rangle, \ldots\}$
  - $\{\langle 123, 181140pods\rangle, \langle 142, 193214algo\rangle, \ldots\}$

Relational query languages

- Query languages are formal languages with syntax and semantics:
  - Syntax: algebraic or logical formalism or specific query language (like SQL). Uses the vocabulary of the DB schema
  - Semantics: $M[Q]$ a mapping that transforms a database (instance) D into a database (instance) $D' = M[Q](D)$, i.e. the database $M[Q](D)$ is the answer of Q over the DB D.
- Usually, $M[Q]$ produces a single table, i.e., $M[Q]\colon D \to dom(D)^k$
  - in general: $k \geq 0$. We say "Q is a k-ary query".
  - Boolean queries: $k = 0$, i.e.:
    possible values of $M[Q](D)$ are $\{\}$ (= false) or $\{\langle\rangle\}$ (= true).
- Expressive power of a query language: which mappings $M[Q]$ can be defined by queries Q of a given query language?

Relational Algebra (RA)

- $\sigma \to$ *Selection*∗
- $\pi \to$ *Projection*∗
- $\times \to$ *Cross product*∗
- $\bowtie \to$ *Join*
- $\rho \to$ *Rename*∗
- $- \to$ *Difference*∗
- $\cup \to$ *Union*∗
- $\cap \to$ *Intersection*

∗Primitive operations, all others can be obtained from these.

For precise definition of RA see any DB textbook or Wikipedia.

Example

- Recall the schema:
  - *Author* (*AID* integer, *name* string, *age* integer)
  - *Paper* (*PID* string, *title* string, *year* integer)
  - *Write* (*AID* integer, *PID* string)
- Example query: *PID*s of the papers NOT written by *Knuth*

$$\pi_{PID}(Paper) - \pi_{PID}(Write \bowtie \sigma_{name=\text{"}Knuth\text{"}}(Author))$$

- Example query: *AID*s of authors who wrote exactly one paper

$$S_2 = Write \bowtie_{AID=AID' \wedge PID \neq PID'} \rho_{AID' \leftarrow AID, PID' \leftarrow PID}(Write)$$
$$S = \pi_{AID} Write - \pi_{AID} S_2$$

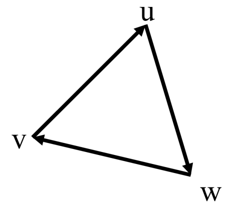
Recall First-order Logic (FO)

Formulas built using:

- Quantifiers: $\forall$, $\exists$,
- Boolean connectives: $\wedge$, $\vee$, $\neg$
- Parentheses: (,)
- Atoms: $R(t_1, \ldots, t_n)$, $t_1 = t_2$

Example database (i.e. a first-order structure):

- Schema: E(*FROM* string, *TO* string)
- Instance: $\{\langle v, u\rangle, \langle u, w\rangle, \langle w, v\rangle\}$



Example sentences of FO:

- $\forall x \exists y E(x, y)$
- $\exists x \forall y E(x, y)$
- $\forall x \exists y \exists z (E(z, x) \wedge E(x, y))$
- $\forall x \exists y \exists z (\neg(y = z) \wedge E(x, y) \wedge E(x, z))$

Relational (Domain) Calculus

If φ is an FO formula with free variables $\{x_1, \ldots, x_k\}$, then

$$\{\langle x_1, \ldots, x_k\rangle \mid \varphi\}$$

is a k-ary query of the domain calculus. On database $\mathcal{A}$ with domain A, it returns the set of all tuples $\langle a_1, \ldots, a_k\rangle \in (A)^k$ such that the sentence $\varphi[a_1, \ldots, a_k]$ obtained from φ by replacing each x_i by a_i evaluates to true in the structure $\mathcal{A}$.

Notational simplifications.

- We often simply write φ rather than $\{\langle x_1, \ldots, x_k\rangle \mid \varphi\}$ (i.e., the free variables of a formula are considered as the output).
- In particular, we usually write φ rather than $\{\langle\rangle \mid \varphi\}$ for Boolean queries ($k = 0$).

Example

- Recall the schema:
  - *Author* (*AID* integer, *name* string, *age* integer)
  - *Paper* (*PID* string, *title* string, *year* integer)
  - *Write* (*AID* integer, *PID* string)
- Example query: "*PID*s of the papers NOT written by *Knuth*"

$$\{PID \mid \exists T \exists Y (Paper(PID, T, Y) \wedge$$
$$\wedge \neg(\exists A \exists AID(Write(AID, PID) \wedge Author(AID, \text{"}Knuth\text{"}, A))))\}$$

- Example query: "*AID*s of authors who wrote exactly one paper"

$$\{AID \mid \exists PID(Write(AID, PID) \wedge \neg \exists PID2(Write(AID, PID2) \wedge PID \neq PID2))\}$$

SQL (Structured Query Language)

- A standardized language:
  - most database management systems (DBMSs) implement SQL
- SQL is not only a query language:
  - supports constructs to manage the database (create/delete tables/rows)
- Query constructs of SQL (SELECT/FROM/WHERE/JOIN) are based on relational algebra

- Example query: "*AIDs* of the co-authors of Knuth"

```
SELECT W1.AID
FROM Write W1, Write W2, Author
WHERE W1.PID=W2.PID AND W1.AID <> W2.AID
AND W2.AID=Author.AID AND Author.Name="Knuth"
```

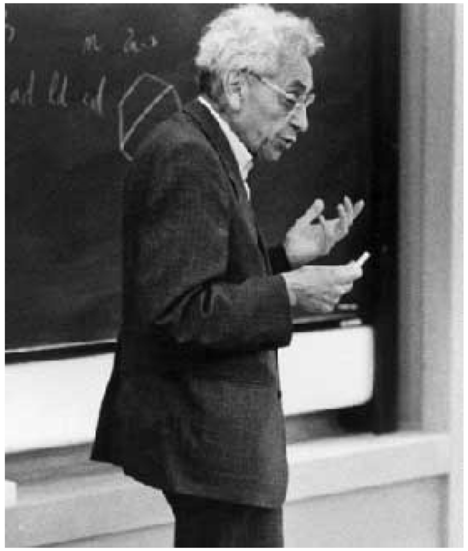

Relational Algebra vs. Relational Calculus vs. SQL

Theorem (following Codd 1972)

Relational algebra, relational calculus, and SQL queries essentially *have equal expressive power.*

- Restrictions apply: no "group by" and aggregation in SQL queries, "safety" requirements for relational calculus.
- Languages with this expressive power are "relational complete".
- all 3 languages have their advantages:
  1. use the flexible syntax of relational calculus to specify the query
  2. use the simplicity of relational algebra for query simplification/optimization
  3. use SQL to implement the query over a DB
- More expressive query languages:
  - many interesting queries cannot be formulated in these languages
  - Example: no recursive queries (SQL now offers a recursive construct)

Towards other query languages



Paul Erdös (1913-1996), one of the most prolific writers of mathematical papers, wrote around 1500 mathematical articles in his lifetime, mostly co-authored. He had 509 direct collaborators

Erdös number

- The *Erdös number*, is a way of describing the "collaborative distance", in regard to mathematical papers, between an author and Erdös.
- An author's *Erdös number* is defined inductively as follows:
  - *Paul Erdös* has an *Erdös number* of zero.
  - The *Erdös number* of author *M* is one plus the minimum among the *Erdös numbers* of all the authors with whom *M* co-authored a mathematical paper.
- Rothschild B.L. co-authored a paper with Erdös → Rothschild B.L.'s Erdös number is 1.
  - Kolaitis P.G. co-authored a paper with Rothschild B.L. → Kolaitis P.G.'s Erdös number is 2.
  - Pichler R. co-authored a paper with Kolaitis P.G. → Pichler R.'s Erdös number is 3.
- Rowling J.K.'s Erdös number is ∞

Queries about the Erdös number

- Recall the schema:
  - *Author* (*AID* integer, *name* string, *age* integer)
  - *Paper* (*PID* string, *title* string, *year* integer)
  - *Write* (*AID* integer, *PID* string)
- Assume that Erdös's *AID* is 17
- Query "*AIDs* of the authors whose Erdös number ≤ 1"

$$P_1 = \pi_{PID}(\sigma_{AID=17} Write)$$

$$A_1 = \pi_{AID}(P_1 \bowtie Write)$$

- Query "*AIDs* of the authors whose Erdös number ≤ 2"

$$P_2 = \pi_{PID}(A_1 \bowtie Write)$$

$$A_2 = \pi_{AID}(P_2 \bowtie Write)$$

Queries about the Erdös number (continued)

- What about Q1="*AIDs* of the authors whose Erdös number $< \infty$"?
- What about Q2="*AIDs* of the authors whose Erdös number $= \infty$"?
- Can we express Q1 and Q2 in relational calculus (or equivalently in RA)?
  - We cannot!
  - Formal methods to prove this negative result will be presented in the course.
- Are there query languages that allow us to express Q1 and Q2?
  - Yes, we can do this in DATALOG (the topic of the next lecture).

Some fundamental aspects of query languages

Questions dealt with in this lecture

- Expressive power of a query language
- Comparison of query languages
- Complexity of query evaluation
- Undecidability of important properties of queries (e.g., redundancy, safety)
- Important special cases (conjunctive queries)
- Inexpressibility results

Learning objectives

- Short recapitulation of
  - the notion of a relational database,
  - the notion of a query language and its semantics,
  - relational algebra,
  - first-order logic,
  - relationcal calculus,
  - SQL.
- Some fundamental aspects of query languages