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Revision of Abstract Dialectical Frameworks

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Introduction

- Argumentation is a major topic in AI research
- Abstract Argumentation frameworks (AFs) [Dung, 1995]
 - Widely used formalism
 - Various extensions to overcome its limitations

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 - Relation between arguments is modelled via acceptance conditions
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- (Abstract) argumentation is an inherently dynamic process
- **Revision** when new information arises
- [Diller et al., 2015]: AGM style revision of AFs
 - Extension-based revision with respect to semantics σ
 - Minimal change of the extensions of the original AF

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 - **Revision** when new information arises
 - [Diller et al., 2015]: AGM style revision of AFs
 - Extension-based revision with respect to semantics σ
 - Minimal change of the extensions of the original AF
- ⇒ Revision of ADFs under three-valued semantics

Definition

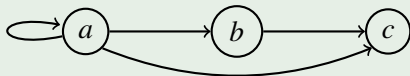
An abstract dialectical framework (ADF) is a triple $F = (S, L, C)$,

- S ... set of statements (correspond to AF arguments)
- $L \subseteq S \times S$... links $(par(s) = L^{-1}(s))$
- $C = \{C_s\}_{s \in S}$... acceptance conditions

- links denote some kind of dependency relation
- acceptance condition: Boolean function $C_s : 2^{par(s)} \rightarrow \{\mathbf{t}, \mathbf{f}\}$
- here: C_s specified by propositional formula φ_s
- here: F represented as sets of tuples $\langle s, \varphi_s \rangle$

Example

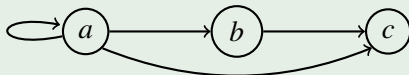
$$\varphi_a = a \quad \varphi_b = a \quad \varphi_c = \neg(a \wedge b)$$



$$F = \{\langle a, a \rangle, \langle b, a \rangle, \langle c, \neg(a \wedge b) \rangle\}$$

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$$F = \{\langle a, a \rangle, \langle b, a \rangle, \langle c, \neg(a \wedge b) \rangle\}$$

- a supports itself
- a supports b
- a and b jointly attack c

Truth values, interpretations

- truth values: true **t**, false **f**, unknown **u**
- interpretation: $v : S \rightarrow \{\mathbf{t}, \mathbf{f}, \mathbf{u}\}$

Information ordering

- $\mathbf{u} <_i \mathbf{t}$ and $\mathbf{u} <_i \mathbf{f}$ (as usual $x \leq_i y$ iff $x <_i y$ or $x = y$)
- **consensus** \sqcap is greatest lower bound w.r.t. \leq_i :
 $\mathbf{t} \sqcap \mathbf{t} = \mathbf{t}$ and $\mathbf{f} \sqcap \mathbf{f} = \mathbf{f}$, otherwise $x \sqcap y = \mathbf{u}$
- information ordering generalized to interpretations:
 $v_1 \leq_i v_2$ iff $v_1(s) \leq_i v_2(s)$ for all $s \in S$
- $[v]_2$: two-valued interpretations w with $v \leq_i w$

Characteristic operator

Given ADF F and interpretation v ,

$$\Gamma_F(v)(s) = \prod_{w \in [v]_2} w(\varphi_s).$$

Abstract Dialectical Frameworks

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Semantics

Given ADF F , an interpretation v is

- **admissible** for F iff $v \leq_i \Gamma_F(v)$,
- **complete** for F iff $v = \Gamma_F(v)$,
- **preferred** for F iff v is \leq_i -maximal admissible for F
- **grounded** for F iff v is \leq_i -minimal complete for F
- a **(supported) model** of F iff v is two-valued and $v = \Gamma_F(v)$,
- a **stable model** of F iff v is a model of F and $v^t = w^t$, where w is the grounded interpretation of $F^v = \{\langle a, \varphi_a[x/\perp : v(x) = \mathbf{f}] \rangle \mid a \in v^{\mathbf{t}}\}$.

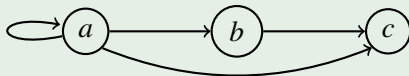
Abstract Dialectical Frameworks

Semantics

- admissible for F iff $v \leq_i \Gamma_F(v)$

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$$\varphi_a = a \quad \varphi_b = a \quad \varphi_c = \neg(a \wedge b)$$



$$ad(F) = \{$$

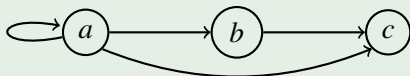
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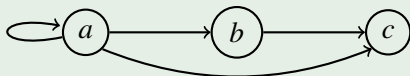
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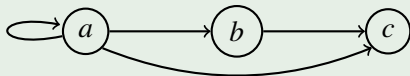
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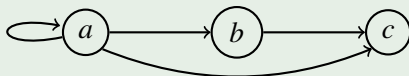
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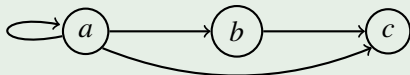
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$$ad(F) = \{uuu, \mathbf{tuu}, \mathbf{fuu}, \mathbf{tuf}, \mathbf{ttu}, \mathbf{ffu}, \mathbf{ttf}, \mathbf{fff}\}$$

Belief Revision

- Initial beliefs: ψ
- New piece of information (observation, . . .): μ

$$\begin{array}{ccc} \text{Inputs} & & \text{Outputs} \\ \left. \begin{array}{l} \psi \\ \mu \end{array} \right\} & \Longrightarrow & \psi \circ \mu \end{array}$$

- s.t. $\psi \circ \mu$ implies μ (success principle)
- s.t. $\psi \circ \mu$ is as close as possible to ψ (minimal change principle)
- “good” operators are characterized by postulates + representation theorem

[Alchourrón et al., 1985, Katsuno and Mendelzon, 1991]:

(R1) $\psi \circ \mu \models \mu$.

(R2) If $\psi \wedge \mu$ is satisfiable, then $\psi \circ \mu = \psi \wedge \mu$.

(R3) If μ is satisfiable, then $\psi \wedge \mu$ is also satisfiable.

(R4) If $\psi_1 \equiv \psi_2$ and $\mu_1 \equiv \mu_2$, then $\psi_1 \circ \mu_1 \equiv \psi_2 \circ \mu_2$.

(R5) $(\psi \circ \mu) \wedge \phi \models \psi \circ (\mu \wedge \phi)$.

(R6) If $(\psi \circ \mu) \wedge \phi$ is satisfiable, then $\psi \circ (\mu \wedge \phi) \models (\psi \circ \mu) \wedge \phi$.

Definition

Given a formula ψ , a preorder \preceq_ψ is a **faithful ranking** for ψ if it is total and for all interpretations $v_1, v_2 \in \mathcal{V}$ it holds that

- (i) if $v_1, v_2 \in \text{Mod}(\psi)$ then $v_1 \approx_\psi v_2$, and
- (ii) if $v_1 \in \text{Mod}(\psi)$ and $v_2 \notin \text{Mod}(\psi)$ then $v_1 \prec_\psi v_2$.

Function mapping each formula to a faithful ranking: **faithful assignment**.

Belief Revision

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Function mapping each formula to a faithful ranking: **faithful assignment**.

Theorem [Katsuno and Mendelzon, 1991]

A revision operator \circ satisfies *R1 – R6*

iff

there is an assignment mapping each formula ψ to a faithful ranking \preceq_ψ such that $\text{Mod}(\psi \circ \mu) = \min(\text{Mod}(\mu), \preceq_\psi)$

- $\min(V, \preceq) = \{v_1 \in V \mid \nexists v_2 \in V : v_2 \prec v_1\}$.

Main Contributions

- $* : \mathcal{F}_A \times \mathcal{F}_A \mapsto \mathcal{F}_A$:
- **Representation theorems:** Correspondence between revision operators captured by rankings and revision operators given by (extended) set of AGM postulates.
- Revision under **preferred** semantics
- Revision under **admissible** semantics
- **Hybrid** approach

Main Contributions

- $* : \mathcal{F}_A \times \mathcal{F}_A \mapsto \mathcal{F}_A$:
- **Representation theorems**: Correspondence between revision operators captured by rankings and revision operators given by (extended) set of AGM postulates.
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Tool-Kit:

- **Realizability** results for ADF semantics [Strass, 2015, Pührer, 2015]
 - Exact characterization of realizable interpretation sets
- **Horn** belief revision [Delgrande and Peppas, 2015]
 - How to modify postulates and rankings in order to stay in the fragment

$* : \mathcal{F}_A \times \mathcal{F}_A \mapsto \mathcal{F}_A$:

(A1 $_{\sigma}$) $\sigma(F * G) \subseteq \sigma(G)$.

(A2 $_{\sigma}$) If $\sigma(F) \cap \sigma(G) \neq \emptyset$, then $\sigma(F * G) = \sigma(F) \cap \sigma(G)$.

(A3 $_{\sigma}$) If $\sigma(G) \neq \emptyset$, then $\sigma(F * G) \neq \emptyset$.

(A4 $_{\sigma}$) If $\sigma(G) = \sigma(H)$, then $\sigma(F * G) = \sigma(F * H)$.

(A5 $_{\sigma}$) $\sigma(F * G) \cap \sigma(H) \subseteq \sigma(F * f_{\sigma}(\sigma(G) \cap \sigma(H)))$.

(A6 $_{\sigma}$) If $\sigma(F * G) \cap \sigma(H) \neq \emptyset$, then
 $\sigma(F * f_{\sigma}(\sigma(G) \cap \sigma(H))) \subseteq \sigma(F * G) \cap \sigma(H)$.

Revision of ADFs – Obstacles

- Limited expressiveness: not every set of interpretations has a realizing ADF

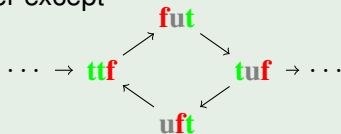
Proposition [Pührer, 2015, Strass, 2015]

A set of interpretations V is realizable under

- *ad* iff $V \neq \emptyset$ and $V = cl(V)$;
 - *pr* iff $V \neq \emptyset$ and V is incompatible;
 - *mo* iff $V \subseteq \mathcal{V}^2$;
 - *st* iff $V \subseteq \mathcal{V}^2$ and $v_1^t \not\subseteq v_2^t, v_2^t \not\subseteq v_1^t$ for all $v_1, v_2 \in V$.
-
- Postulates may be satisfied by operators induced by pseudo-preorder
 - Three-valued interpretations: alternative distance measures needed

Example

- \preceq_F : Faithful preorder except



- $f_{pr}(\min(pr(G), \preceq_F))$ well defined for any ADF G .
 - Operator induced by \preceq_F satisfies $R1 - R6$.
 - Any binary relation \preceq'_F inducing the same operator must contain this cycle.
-
- Revision operators characterizing non-transitive rankings

- Additional postulate [Delgrande and Peppas, 2015]:

(*Acyc_σ*) If for $1 \leq i < n$, $\sigma(F * G_{i+1}) \cap \sigma(G_i) \neq \emptyset$ and $\sigma(F * G_1) \cap \sigma(G_n) \neq \emptyset$ then $\sigma(F * G_n) \cap \sigma(G_1) \neq \emptyset$.

Revision under Preferred Semantics

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- Faithfulness adapted:

Definition

Given a semantics σ and an ADF F , a preorder \preceq_F is a **i-max-faithful** ranking for F if it is **i-max-total** and for all **incompatible** interpretations $v_1, v_2 \in \mathcal{V}$ it holds that

- (i) if $v_1, v_2 \in \sigma(F)$ then $v_1 \approx_F v_2$, and
- (ii) if $v_1 \in \sigma(F)$ and $v_2 \notin \sigma(F)$ then $v_1 \prec_F v_2$.

A function mapping each ADF to a **i-max-faithful** ranking is called **i-max-faithful assignment**.

Theorem

An operator \star_{pr} satisfies postulates $A1_{pr} - A6_{pr}$ and $Acyc_{pr}$

iff

there exists an assignment mapping each ADF F to an *i-max-faithful* ranking \preceq_F such that $pr(F \star_{\sigma} G) = \min(pr(G), \preceq_F)$.

Symmetric distance function Δ :

- $t\Delta f = 1$,
- $t\Delta u = f\Delta u = \frac{1}{2}$, and
- $x\Delta x = 0$ for $x \in \{t, f, u\}$.

$$v_1\Delta v_2 = \sum_{a \in A} v_1(a)\Delta v_2(a).$$

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$$v_1 \Delta v_2 = \sum_{a \in A} v_1(a) \Delta v_2(a).$$

$$v_1 \preceq_F^\sigma v_2 \Leftrightarrow \min_{v \in \sigma(F)} (v \Delta v_1) \leq \min_{v \in \sigma(F)} (v \Delta v_2).$$

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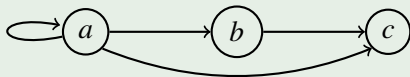
$$v_1 \preceq_F^\sigma v_2 \Leftrightarrow \min_{v \in \sigma(F)} (v \Delta v_1) \leq \min_{v \in \sigma(F)} (v \Delta v_2).$$

$\Rightarrow \preceq_F^{pr}$ is i-max-faithful.

$\Rightarrow *_{pr}^D : F *_{pr}^D G = f_{pr}(\min(pr(G), \preceq_F^{pr}))$ satisfies the postulates.

Example

$$\varphi_a = a \quad \varphi_b = a \quad \varphi_c = \neg(a \wedge b)$$

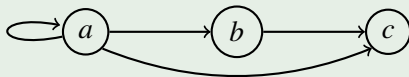


- $pr(F) = \{\mathbf{ttf}, \mathbf{fff}\}$
- $\mathbf{ttf} \approx_F^{pr} \mathbf{fff} \prec_F^{pr} \text{others.}$

Revision under Preferred Semantics

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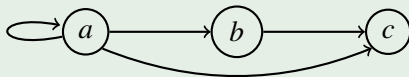


- $pr(F) = \{\mathbf{ttf}, \mathbf{fff}\}$
- $\mathbf{ttf} \approx_F^{pr} \mathbf{fff} \prec_F^{pr} \text{others.}$
- Revision by $G = \{\langle a, \top \rangle, \langle b, \neg a \rangle, \langle c, \neg b \rangle\}$
 - $ad(G) = \{\mathbf{tft}, \mathbf{tfu}, \mathbf{tuu}, \mathbf{uuu}\}$

Revision under Preferred Semantics

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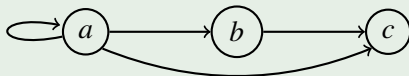
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- Revision by $G = \{\langle a, \top \rangle, \langle b, \neg a \rangle, \langle c, \neg b \rangle\}$
 - $ad(G) = \{\mathbf{tft}, \mathbf{tfu}, \mathbf{tuu}, \mathbf{uuu}\}$
- $F \star_{pr}^D G = f_{pr}(\{\mathbf{tft}\})$.
- \mathbf{tuu} is admissible in both F and G and has less distance.

Theorem

An operator \star_{ad} satisfies postulates $A1_{ad} - A6_{ad}$ iff it is defined as $F \star_{ad} G = f_{ad}(ad(F) \cap ad(G))$.

Example

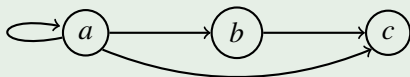
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- Revision by G'
 - $ad(G') = \{\mathbf{utf}, \mathbf{uuu}\}$

Example

$$\varphi_a = a \quad \varphi_b = a \quad \varphi_c = \neg(a \wedge b)$$



- $pr(F) = \{\mathbf{ttf}, \mathbf{fff}\}$; $ad(F) = \{\mathbf{ttf}, \mathbf{fff}, \mathbf{ttu}, \mathbf{tuf}, \mathbf{ffu}, \mathbf{tuu}, \mathbf{fuu}, \mathbf{uuu}\}$
- Revision by G'
 - $ad(G') = \{\mathbf{utf}, \mathbf{uuu}\}$
 - $F \star_{ad}^D G' = f_{ad}(\{\mathbf{uuu}\})$.
 - From pr -perspective questionable:
 - $\mathbf{utf} \Delta \mathbf{ttf} = \frac{1}{2}$
 - $\mathbf{uuu} \Delta \mathbf{ttf} = \frac{3}{2}$

$F \star G$:

- Selecting from $ad(G)$.
- Distance based on $pr(F)$.

Hybrid Approach

$F \star G$:

- Selecting from $ad(G)$.
- Distance based on $pr(F)$.

(P1) $pr(F \star G) \subseteq ad(G)$.

(P2) If $pr(F) \cap ad(G) \neq \emptyset$, then $pr(F \star G) = pr(F) \cap ad(G)$.

(P3) If $ad(G) \neq \emptyset$, then $pr(F \star G) \neq \emptyset$.

(P4) If $ad(G) = ad(H)$, then $pr(F \star G) = pr(F \star H)$.

(P5) $pr(F \star G) \cap ad(H) \subseteq pr(F \star f_{ad}(ad(G) \cap ad(H)))$.

(P6) If $pr(F \star G) \cap ad(H) \neq \emptyset$, then
 $pr(F \star f_{ad}(ad(G) \cap ad(H))) \subseteq pr(F \star G) \cap ad(H)$.

(Acyc) If for $1 \leq i < n$, $pr(F \star G_{i+1}) \cap ad(G_i) \neq \emptyset$ and
 $pr(F \star G_1) \cap ad(G_n) \neq \emptyset$ then $pr(F \star G_n) \cap ad(G_1) \neq \emptyset$.

Example

- $F = \{\langle a, \perp \rangle, \langle b, \perp \rangle\}$: $pr(F) = \{\mathbf{ff}\}$
- $G = \{\langle a, \top \rangle, \langle b, \top \rangle\}$: $ad(G) = \{\mathbf{uu}, \mathbf{ut}, \mathbf{tu}, \mathbf{tt}\}$
- $\mathbf{ff} \prec \text{others} \prec \mathbf{tu} \approx \mathbf{ut} \prec \mathbf{tt} \prec \mathbf{uu}$: faithful ranking for F .
- $F \star G = f_{pr}(\{\mathbf{ut}, \mathbf{tu}\})$, but $\{\mathbf{ut}, \mathbf{tu}\}$ is not realizable under pr .

$\Rightarrow \star$ violates $P5$.

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$\Rightarrow \star$ violates $P5$.

Definition

A preorder \preceq is σ - τ -compliant if, for every ADF $F \in \mathcal{F}_A$, $\min(\tau(F), \preceq)$ is realizable under σ .

Theorem

*An operator \star satisfies postulates P1 – P6 and *Acyc**

iff

*there exists an assignment mapping each ADF F to a *faithful and pr-ad-compliant* ranking \preceq_F such that $pr(F \star G) = \min(ad(G), \preceq_F)$.*

- Valid operator induced from \preceq : $pr(F)$ followed by \prec -chain.
- Three-valued Dalal operator not applicable: does not yield *pr-ad-compliant* rankings in general.
- Refinement need to obtain concrete operator.









Summary:

- Combining recent results in argumentation and belief revision
- Revision of ADFs under preferred and admissible semantics
 - Three-valued version of Dalal's operator
- Hybrid approach
- No AGM revision operators for complete and grounded semantics
- Discussion of revision under stable and supported models

Future work:

- Refinement of ranking for hybrid approach
- Hybrid approach for revision in other formalisms
- Syntactic aspects
- Revision of AFs under three-valued semantics
- Complexity of Dalal's operator under preferred semantics

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