

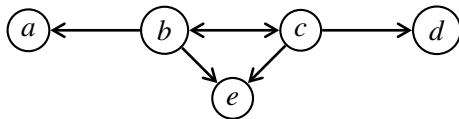
The Hidden Power of Abstract Argumentation Semantics

Thomas Linsbichler, Christof Spanring, Stefan Woltran

The 2015 International Workshop on Theory and Applications of Formal Argument

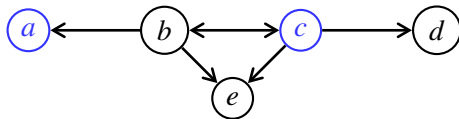
July 25, 2015

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Introduction

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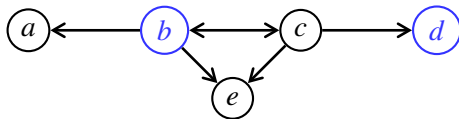


- Evaluation: Argumentation Semantics

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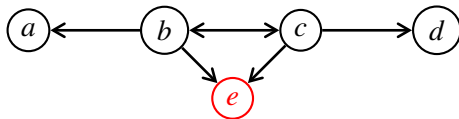


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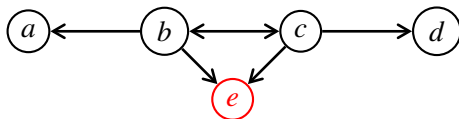
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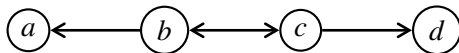
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Problems

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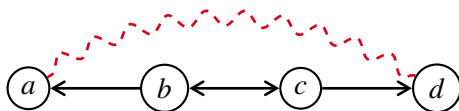
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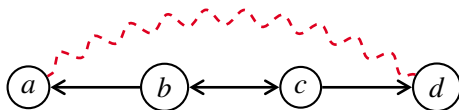
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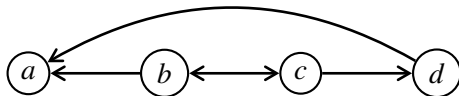
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- Abstract Argumentation Framework [Dung, 1995]:



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Problems

- Can we find an equivalent AF F' without rejected argument e ? Yes.
- Can we make the implicit conflict between a and d explicit? Yes.

- Systematic comparison of argumentation semantics [Baroni and Giacomin, 2007, Dunne et al., 2014].
- Normal form.
 - Succinct representation of argumentation frameworks.
 - Minimal number of arguments \Rightarrow no rejected arguments.
 - Maximal number of attacks \Rightarrow no implicit conflicts.
- Belief revision in abstract argumentation [Coste-Marquis et al., 2014, Diller et al., 2015].
 - Scenarios where the use of additional (rejected) arguments is not permitted.

- Background
- Implicit Conflicts
 - Refute Explicit Conflict Conjecture [Baumann et al., 2014].
 - Conditions for explication of implicit conflicts.
- Rejected Arguments
 - Impact on expressiveness.
 - Comparison of semantics.
- Conclusion & Future work

Background

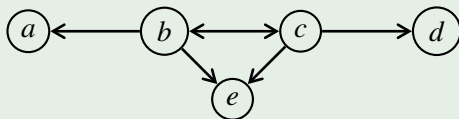
Countably infinite domain of arguments \mathfrak{A} .

Definition

An **argumentation framework** (AF) is a pair (A, R) where

- $A \subseteq \mathfrak{A}$ is a finite set of arguments and
- $R \subseteq A \times A$ is the attack relation representing conflicts.

Example



$$F = (\{a, b, c, d, e\}, \\ \{(b, a), (c, d), (b, c), (c, b), (b, e), (c, e)\})$$

Definition

Given an AF $F = (A, R)$, a set $S \subseteq A$ is

- naive extension if $S \in cf(F)$ and $\nexists T \in cf(F) : T \supset S$,
- stable extension if $S \in cf(F)$ and $S_F^+ = A$,
- stage extension if $S \in cf(F)$ and $\nexists T \in cf(F) : T_F^+ \supset S_F^+$,
- preferred extension if $S \in adm(F)$ and $\nexists T \in adm(F) : T \supset S$,
- semi-stable extension if $S \in adm(F)$ and $\nexists T \in cf(F) : T_F^+ \supset S_F^+$.

Definition

Given AF $F = (A, R)$, semantics σ and arguments $a, b \in A$

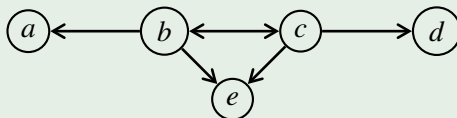
- a and b ($a \neq b$) are in **conflict** $| \begin{smallmatrix} a \\ b \end{smallmatrix} |$ for σ if $a \in \sigma(F) \Rightarrow b \notin \sigma(F)$;
- $| \begin{smallmatrix} a \\ b \end{smallmatrix} |$ is **explicit** if $(a, b) \in R$ or $(b, a) \in R$;
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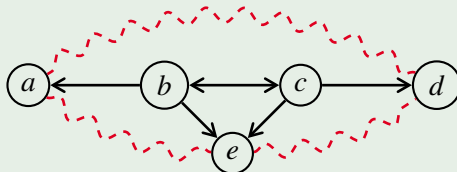
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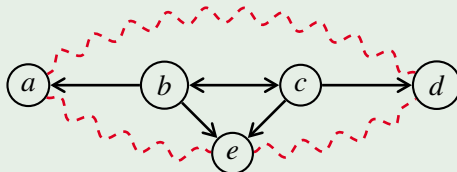
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- explicit conflicts: $|a|_b, |b|_c, |c|_d, |b|_e, |c|_e$.

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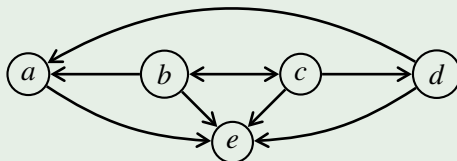
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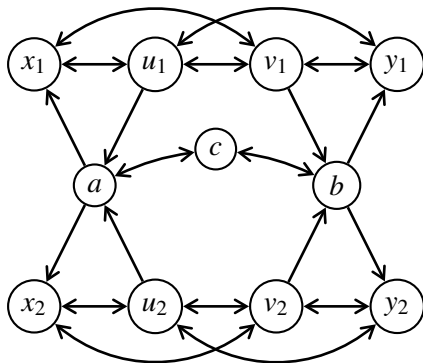
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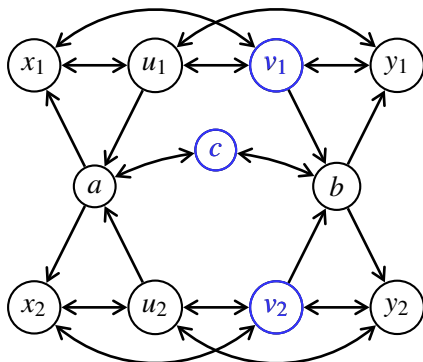
Explicit Conflict Conjecture

For stable semantics every AF is quasi-analytic. [Baumann et al., 2014]

Implicit Conflicts



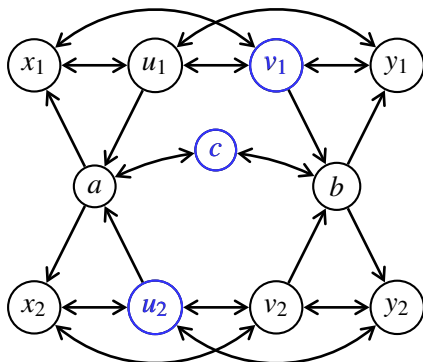
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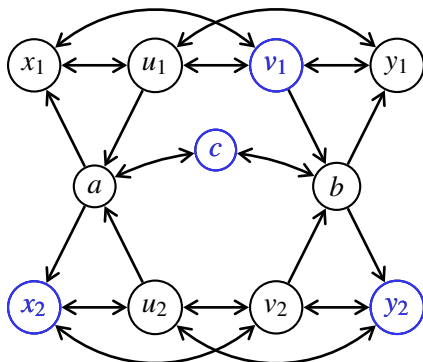
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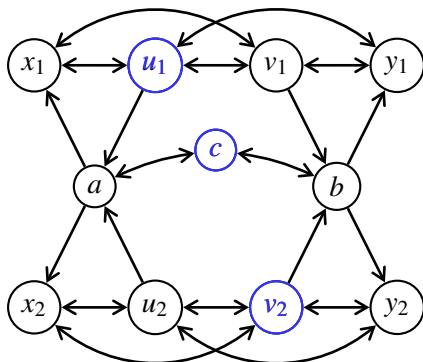
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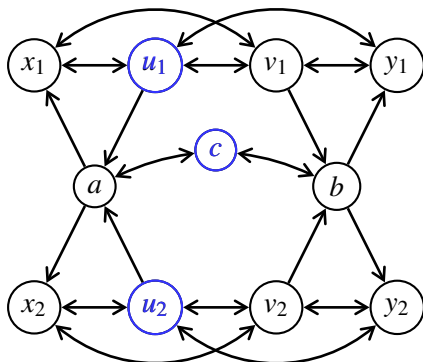
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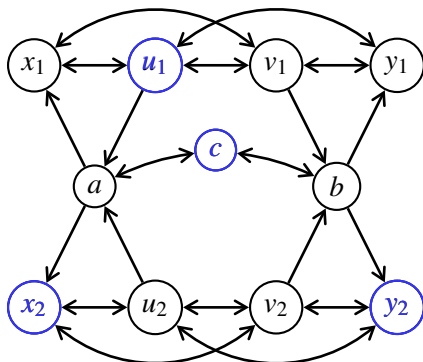
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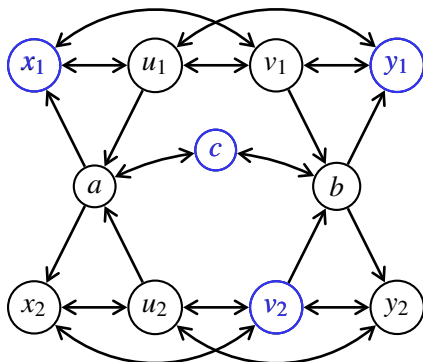
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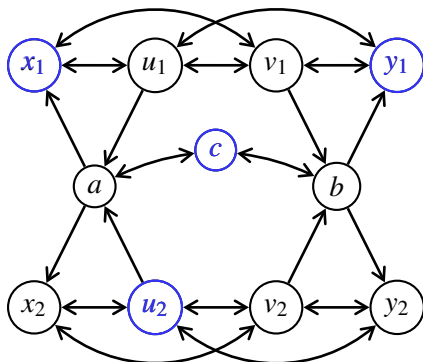
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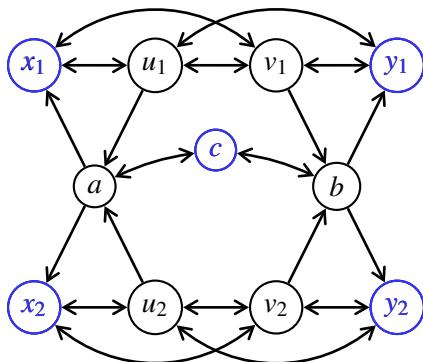
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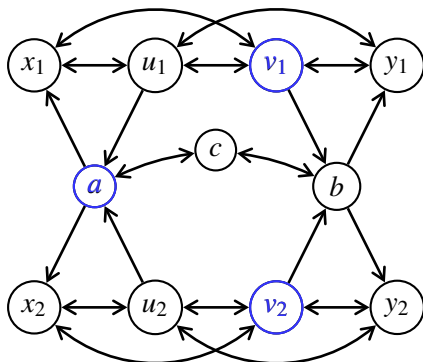
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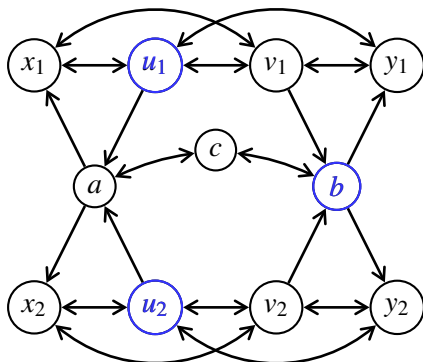
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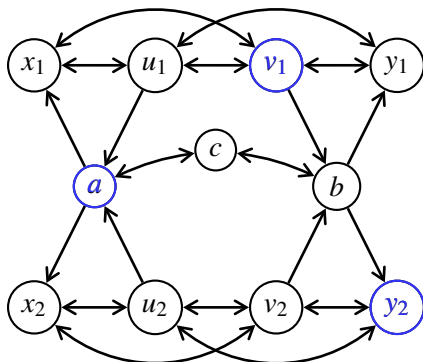
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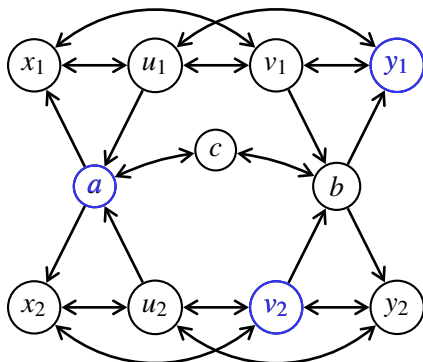
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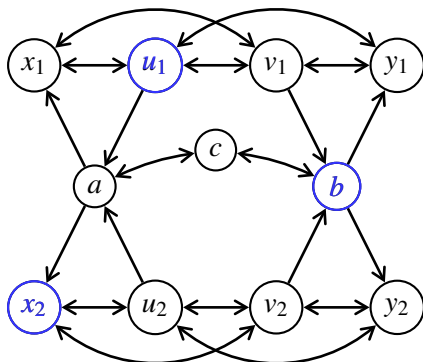
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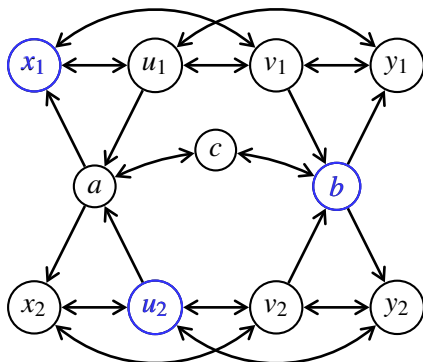
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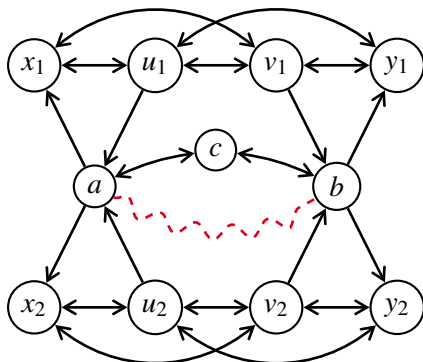
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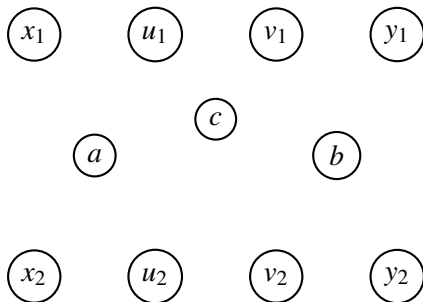
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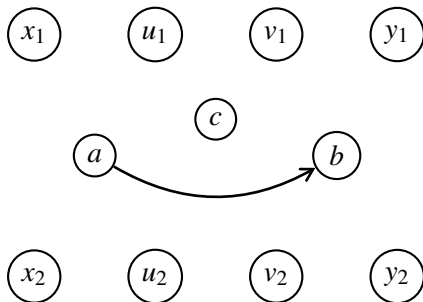
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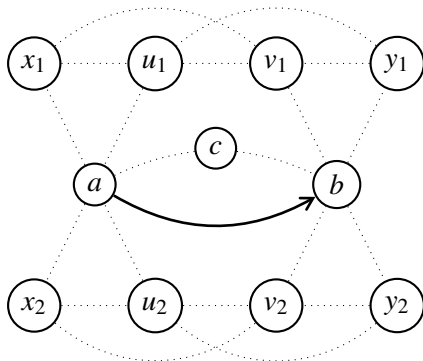
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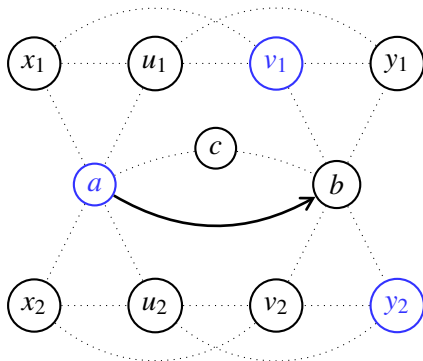
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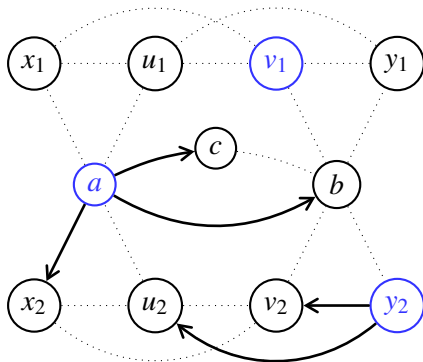
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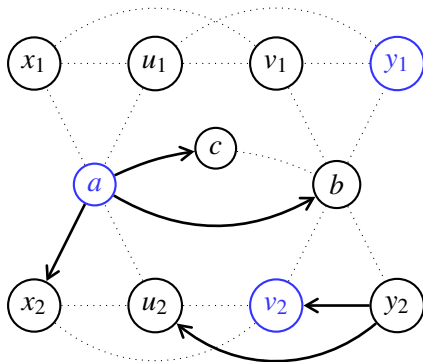
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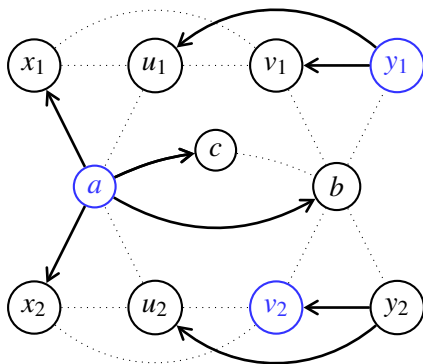
Implicit Conflicts



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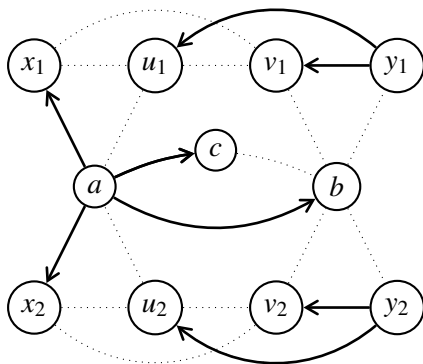
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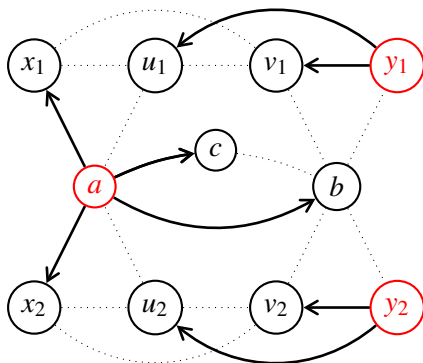
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Theorem

There are non-analytic AFs for stable semantics.

Theorem

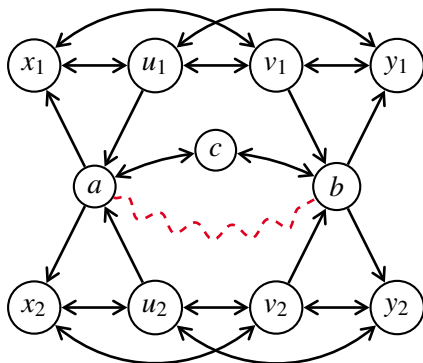
There are non-analytic AFs for stable, preferred, semi-stable semantics.

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What about stage semantics?

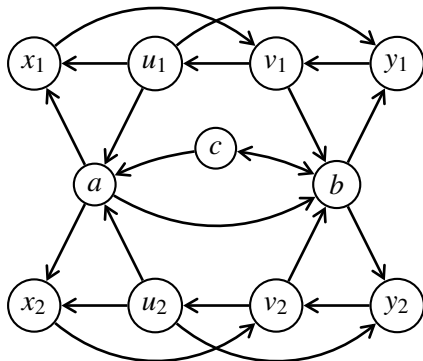
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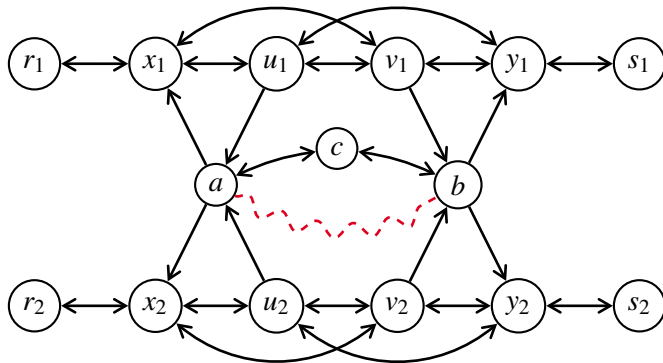
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Implicit Conflicts



Theorem

There are non-analytic AFs for stable, preferred, semi-stable semantics.

Theorem

There are non-analytic AFs for stable, preferred, semi-stable and stage semantics.

Definition

Given AF $F = (A, R)$, semantics σ and arguments $a, b \in A$

- F is **quasi-analytic** for σ if there is some analytic AF $F' = (A', R')$ with $A = A'$ and $\sigma(F) = \sigma(F')$;

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Implicit Conflicts

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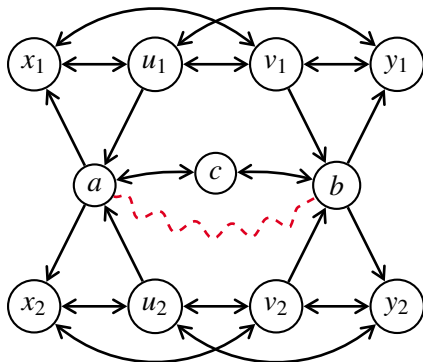
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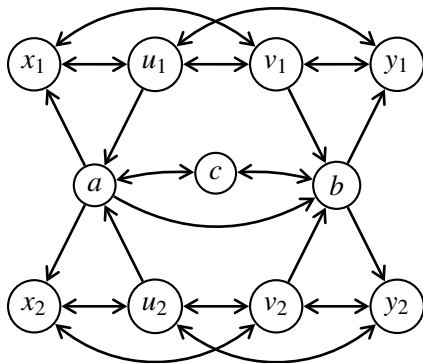
Proposition

For stable semantics and some AF $F = (A, R)$, if there is an implicit conflict $|^a_b|$ for *stb* then there is an AF $F' = (A', R')$ with $stb(F) = stb(F')$, $|A'| = |A| + 1$, $R' \supseteq R$ and $(a, b) \in R'$.

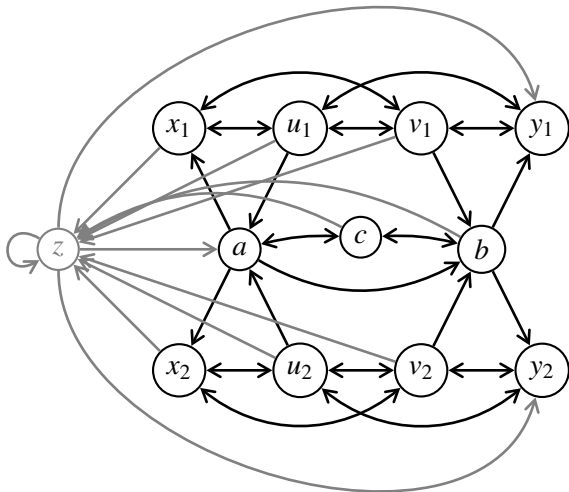
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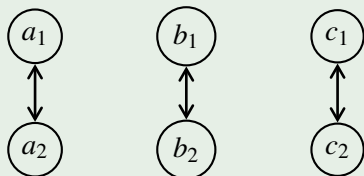
Theorem

Every AF F is quasi-analytic' for stable and stage semantics.

Note: This does not hold for preferred and semi-stable semantics.

Rejected Arguments

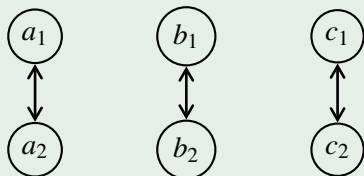
Example



- $stb(F) = \{\{a_i, b_j, c_k\} \mid i, j, k \in \{1, 2\}\}$.

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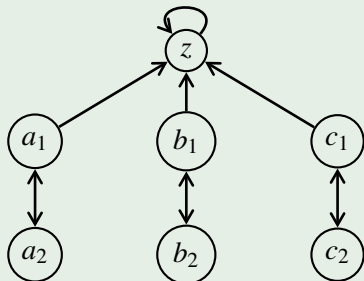
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- $stb(F) = \{\{a_i, b_j, c_k\} \mid i, j, k \in \{1, 2\}\}$.
- Revision eliminating $\{a_2, b_2, c_2\}$.

Rejected Arguments

Example



- $stb(F) = \{\{a_i, b_j, c_k\} \mid i, j, k \in \{1, 2\}\}$.
- Revision eliminating $\{a_2, b_2, c_2\}$.
- Possible by introduction of rejected arguments.
- Not possible without.

Definition

Given a semantics σ , an extension-set $\mathbb{S} \subseteq 2^{\mathcal{A}}$ is called

- σ -realizable if there exists an AF F such that $\sigma(F) = \mathbb{S}$.
- compactly σ -realizable if there exists an AF $F = (\bigcup \mathbb{S}, R)$ such that $\sigma(F) = \mathbb{S}$.

Signature: $\Sigma_{\sigma} = \{\sigma(F) \mid F = (A, R) \text{ is an AF}\}$.

Compact signature: $\Sigma_{\sigma}^c = \{\sigma(F) \mid F = (A, R) \text{ is an AF, } A = \bigcup \sigma(F)\}$.

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Proposition [Baumann et al., 2014]

For $\sigma \in \{stb, pref, sem, stg\}$, $\Sigma_\sigma^c \subset \Sigma_\sigma$.

Rejected Arguments

Theorem [Dunne et al., 2014]

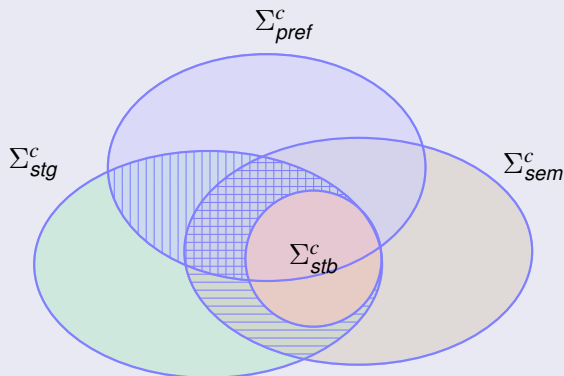
$$\Sigma_{stg} = (\Sigma_{stb} \setminus \{\emptyset\}) \subset \Sigma_{pref} = \Sigma_{sem}.$$

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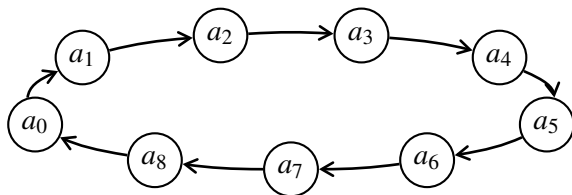
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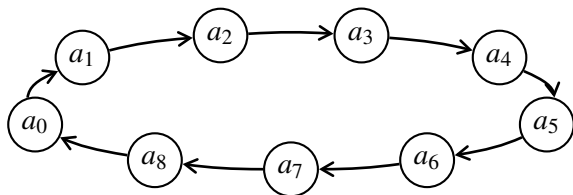
$$\Sigma_{stg}^c \setminus (\Sigma_{stb}^c \cup \Sigma_{pref}^c \cup \Sigma_{sem}^c) \neq \emptyset:$$



$$stg(F) = \{\{a_i, a_{i\oplus 2}, a_{i\oplus 4}, a_{i\oplus 6}\} \mid 0 \leq i < 9\} \text{ with } a \oplus b = (a + b) \bmod 9.$$

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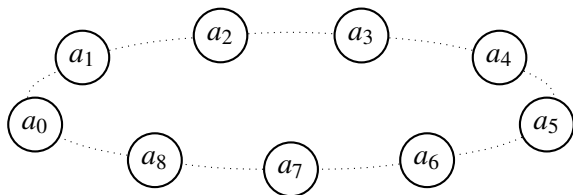


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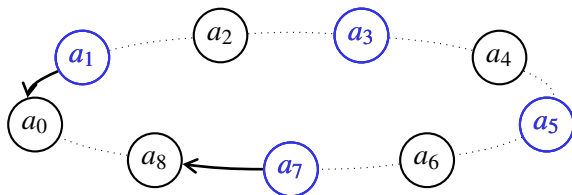


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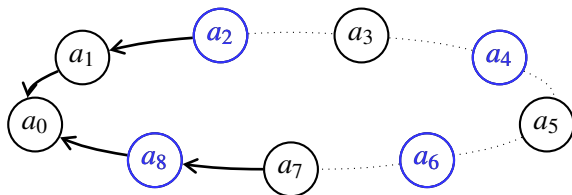


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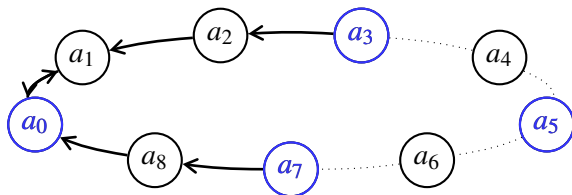


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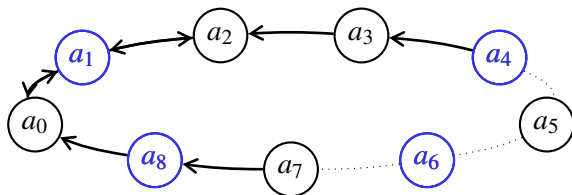


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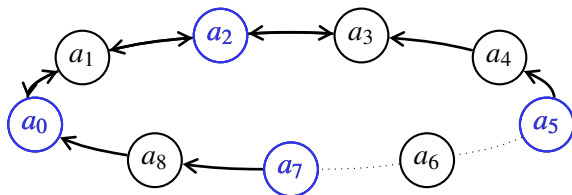


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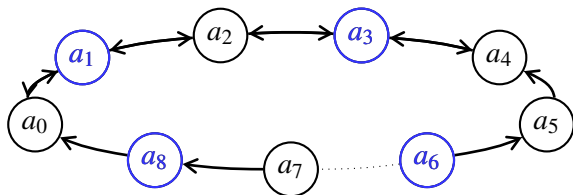


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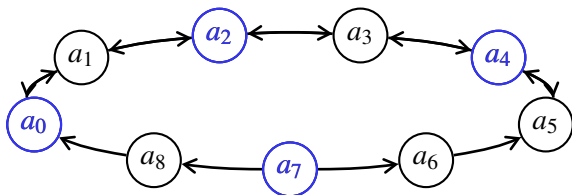


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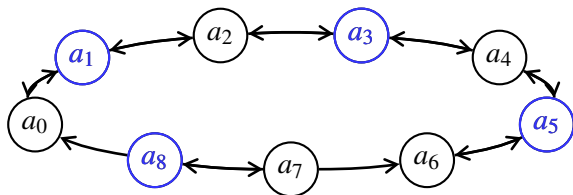


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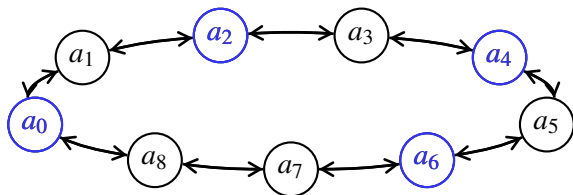


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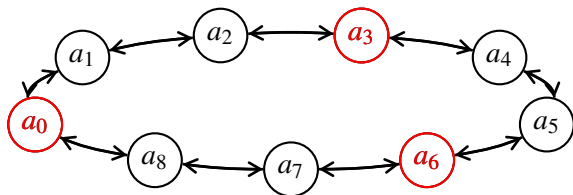


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$\sigma \in \{stb, pref, sem\}$: $\exists F' : \sigma(F') = stg(F)$? **No.**

$\sigma(F') = stg(F) \cup \{\{a_i, a_{i\oplus 3}, a_{i\oplus 6}\} \mid 0 \leq i < 3\}$.

Summary

- Implicit Conflicts
 - In general not explicable.
 - Possible under certain conditions.
 - For *stb* and *stg* possible with additional arguments.
- Rejected Arguments
 - Relation between semantics differs depending on the permission of rejected arguments.

Future Work

- Exact characterizations of compact signatures.
- Concrete use in implementations.
- Analysis in the context of instantiations.

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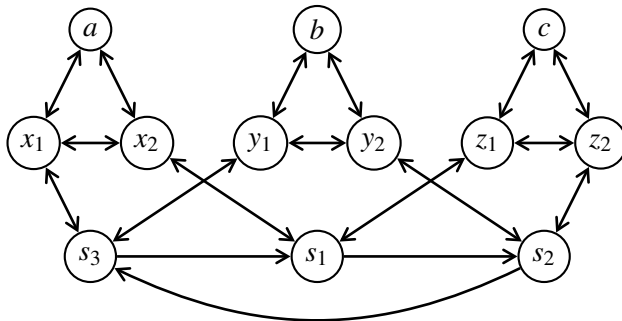
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- $\sigma \in \{\textit{stb}, \textit{pref}, \textit{sem}, \textit{stg}\}$ and F is determined for σ (i.e. $\forall S \in \sigma(F) \exists a \in S \forall S' \in \sigma(F) : S' \neq S \Rightarrow a \notin S'$).

Rejected Arguments

$$\Sigma_{stb}^c \setminus \Sigma_{pref}^c \neq \emptyset:$$



Rejected Arguments

$$(\Sigma_{pref}^c \cap \Sigma_{sem}^c) \setminus (\Sigma_{stb}^c \cup \Sigma_{stg}^c) \neq \emptyset:$$

