FAKULTÄT FÜR INFORMATIK
Faculty of Informatics

On the Functional Completeness of Argumentation Semantics

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Introduction

- Argumentation has become a major topic in AI research.
- Gives answers to “how assertions are proposed, discussed, and resolved in the context of issues upon which several diverging opinions may be held” [Bench-Capon and Dunne, 2007].
- Connections to other AI formalism: knowledge representation, nonmonotonic reasoning, multiagent systems.
- Dung’s Abstract Argumentation Frameworks (AFs) [Dung, 1995] conceal the concrete contents of arguments; only consider the conflict between them $\Rightarrow$ attack graph.
- Argumentation semantics: rules for identifying sets of acceptable arguments.
- Recent years have seen some work on structural analysis of their capabilities. [Dunne et al., 2015, Dyrkolbotn, 2014]

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- AFs with designated input- and output-arguments.
- Question: Which functions from input assignments to (multiple) output assignments are realizable by such AFs?
- Exact characterization of realizable functions.

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  - Realizability [Dunne et al., 2015]: Capabilities of semantics in terms of expressiveness.
  - Input/Output-AFs [Baroni et al., 2014]: Decomposability and transparency of semantics.

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  - Realizability [Dunne et al., 2015]: Capabilities of semantics in terms of expressiveness.
  - Input/Output-AFs [Baroni et al., 2014]: Decomposability and transparency of semantics.
- Adds to the systematic comparision of semanitcs [Baroni and Giacomin, 2007].
- Modular and dynamic aspects of argumentation.
- Strategic argumentation: Deciding whether achieving a certain goal is possible and, if yes, how to do so.

Outline

- Background
- Realizability
- Input/Output-AFs
- I/O-characterization of extension-based semantics
- I/O-characterization of labelling-based semantics
- Conclusion

Background

Countably infinite set of arguments $\mathfrak{A}$.

Definition

An argumentation framework (AF) is a pair (A, R) where

- $A \subseteq \mathfrak{A}$ is a finite set of arguments and
- $R \subseteq A \times A$ is the attack relation representing conflicts.

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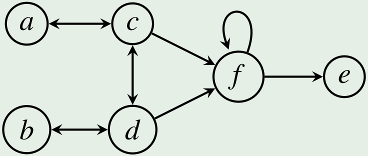

$F = \big(\{a, b, c, d, e, f\},$
$\{(a, c), (c, a), (c, d), (d, c), (d, b), (b, d), (c, f), (d, f), (f, f), (f, e)\}\big)$

Background (ctd.)

Conflict-free Sets

Given an AF $F = (A, R)$, a set $S \subseteq A$ is conflict-free in F, if, for each $a, b \in S$, $(a, b) \notin R$.

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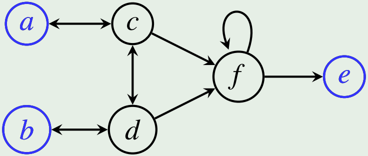

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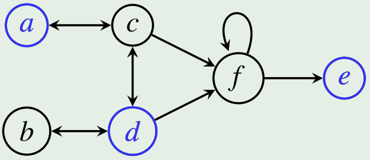

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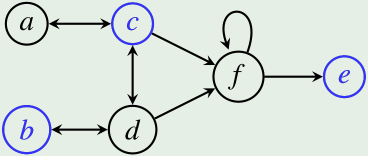

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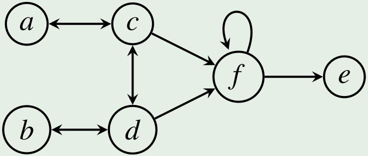

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Given an AF $F = (A, R)$, a set $S \subseteq A$ is a stable extension in F, if

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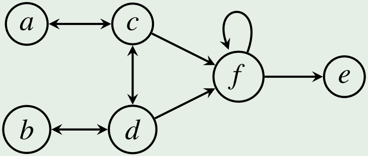

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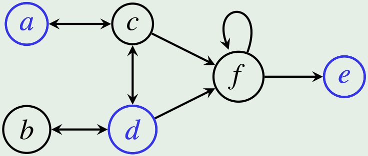

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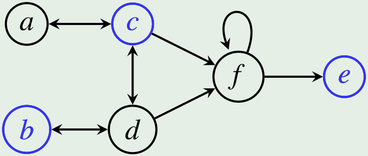

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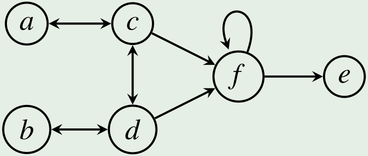

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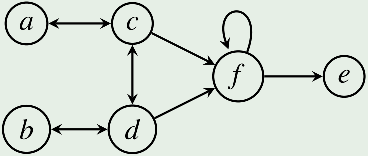

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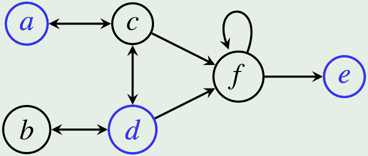

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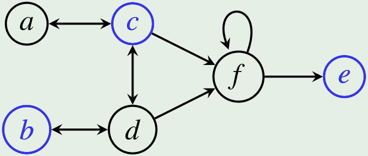

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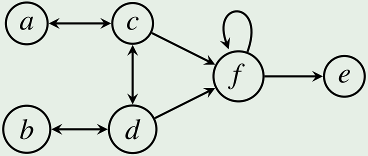

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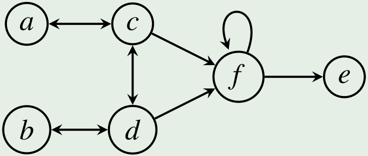

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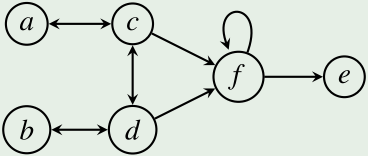

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Given an AF $F = (A, R)$, a set $S \subseteq A$ is a preferred extension in F, if

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$\Rightarrow$ Maximal admissible sets (w.r.t. set-inclusion).

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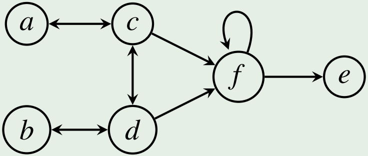

prf$(F) = \{$~~$\{a, b, e\}$~~, $\{a, d, e\}$, $\{b, c, e\}$,
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Background (ctd.)

Further semantics:

- Stage semantics [Verheij, 1996]
- Semi-stable semantic [Caminada et al., 2012]
- Complete semantics
- Grounded semantics
- Ideal semantics

- cf2 semantics [Baroni et al., 2005, Gaggl and Woltran, 2013]
- Resolution-based grounded semantics [Baroni et al., 2011]
- ...

Background (ctd.)

Labelling-based semantics

- More fine-grained evaluation of AFs. [Caminada and Gabbay, 2009]
- A labelling is a function assigning each argument one label among $\mathtt{t}$, $\mathtt{f}$, and $\mathtt{u}$.

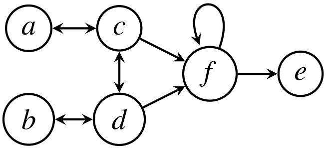
Definition

The labelling-based version of a semantics σ associates to an AF $F = (A, R)$ a set $\mathcal{L}_\sigma(F)$, where any labelling $L \in \mathcal{L}_\sigma(F)$ corresponds to an extension $E \in \sigma(F)$ as follows:

- $L(a) = \mathtt{t}$ iff $a \in E$;
- $L(a) = \mathtt{f}$ iff $\exists b \in E : (b, a) \in R$;
- $L(a) = \mathtt{u}$ iff neither of the above holds.

Background (ctd.)

Labelling-based semantics



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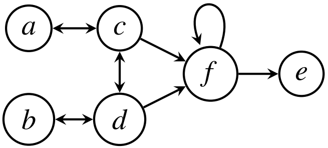

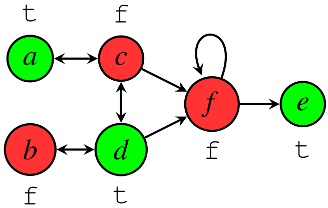

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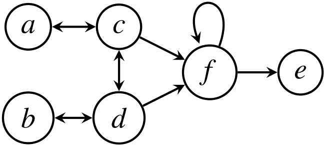

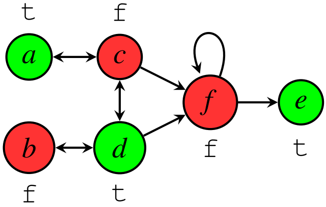

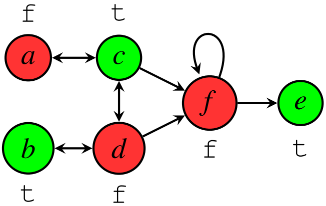

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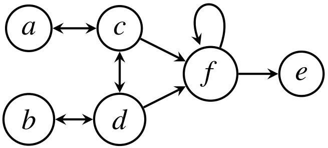

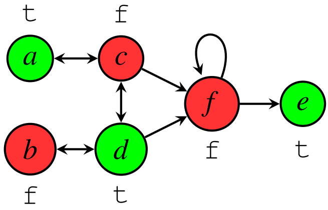

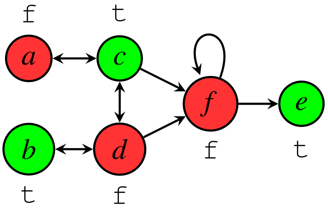

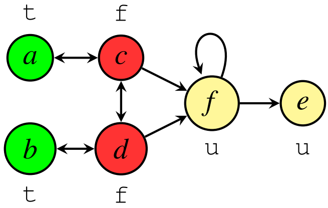

Realizability [Dunne et al., 2015]

Definition

Given a semantics σ, a set $\mathbb{S} \subseteq 2^{\mathfrak{A}}$ is realizable under σ if there exists an AF having $\sigma(F) = \mathbb{S}$.

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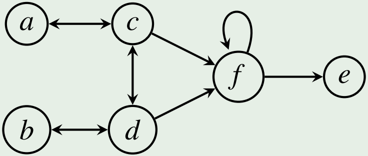
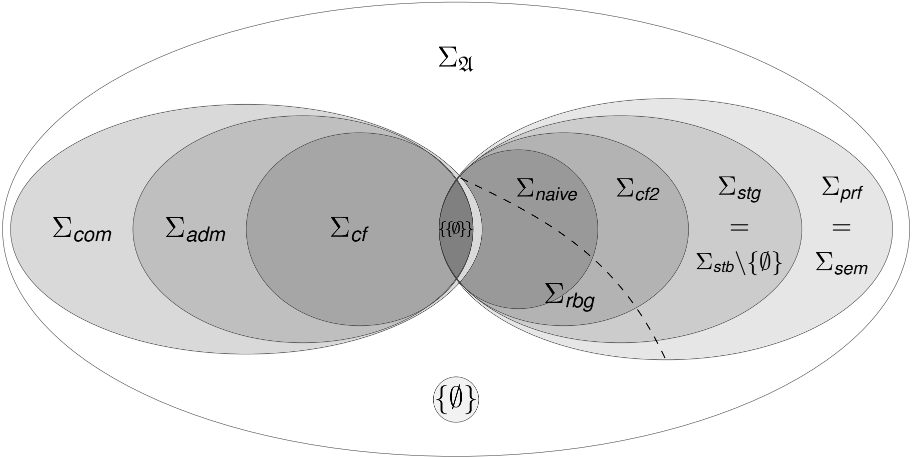


- $\mathbb{S}$ is realizable under *prf*, since *prf*$(F) = \mathbb{S}$.
- $\mathbb{S}$ is *not* realizable under *stb*.

Realizability [Dunne et al., 2015]



$\Sigma_\sigma = \{\sigma(F) \mid F = (A, R)$ is an AF$\}$.

Input/Output AFs [Baroni et al., 2014]

- Investigation of the input/output-behaviour of argumentation semantics
- Decomposability: Given an arbitrary partition of an AF, can the extensions under σ be determined by composition of the partial evaluations?
  - Allows for incremental computation.

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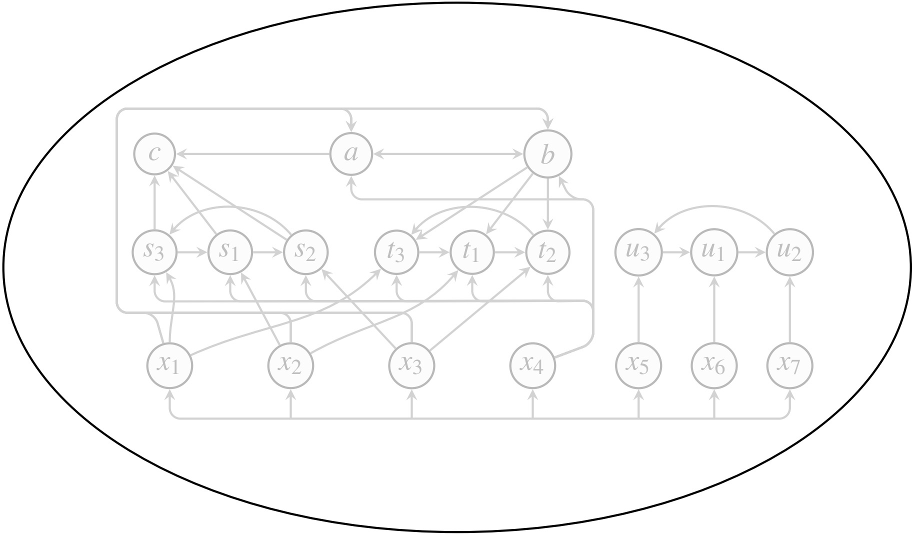

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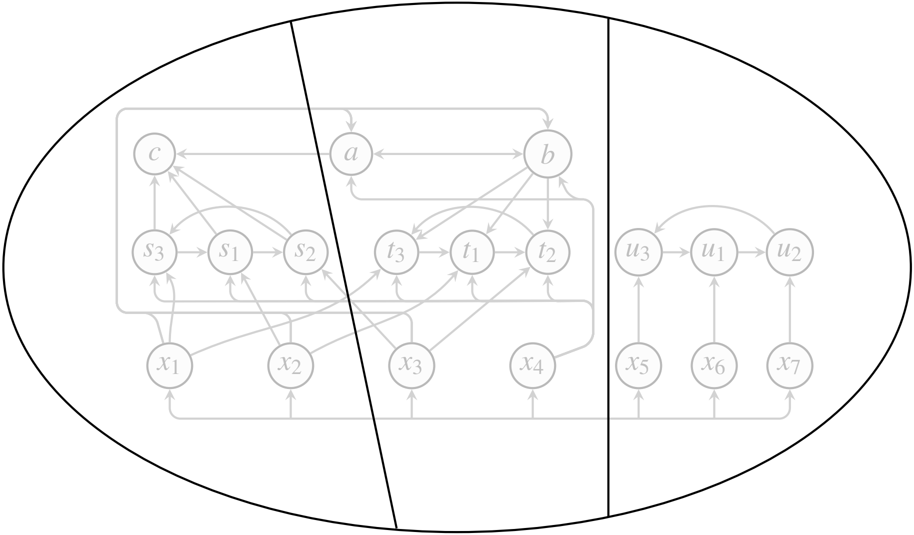

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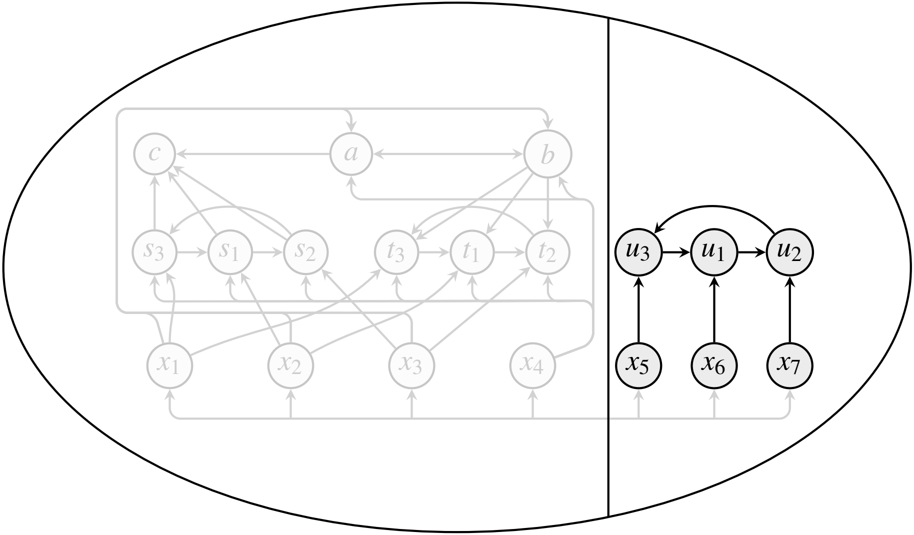

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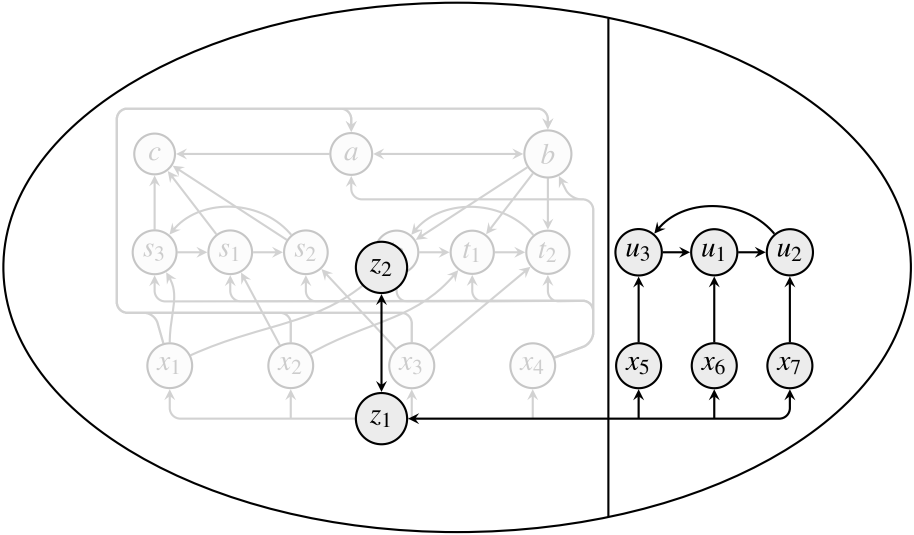

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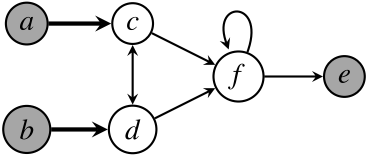
	stb	*prf*	*com*	*grd*	*sem*	*id*
Decomposability	Yes	No	Yes	No	No	No
SCC-Decomposability	Yes	Yes	Yes	Yes	No	No
Transparency	Yes	No*	Yes	Yes	No	No
SCC-Transparency	Yes	Yes	Yes	Yes	No	No

* Yes under additional mild conditions

Extension-based I/O-characterization

Definition

Given input arguments I and output arguments O with $I \cap O = \emptyset$, an I/O-gadget is an AF $F = (A, R)$ such that $I, O \subseteq A$ and $I_F^- = \emptyset$.



Extension-based I/O-characterization

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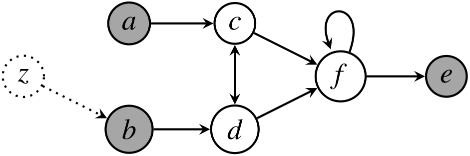
Given an I/O-gadget $F = (A, R)$ the injection of $J \subseteq I$ to F is the AF $\triangleright(F, J) = (A \cup \{z\}, R \cup \{(z, i) \mid i \in (I \setminus J)\})$.

Injection of $\{a\}$:



Extension-based I/O-characterization

Definition

An I/O-specification consists of two sets $I, O \subseteq \mathfrak{A}$ and a total function $\mathfrak{p} : 2^I \mapsto 2^{2^O}$.

Input ($I = \{a, b\}$)	Output ($O = \{e\}$)
$\emptyset$	$\{\{e\}\}$
$\{a\}$	$\{\{e\}\}$
$\{b\}$	$\{\{e\}\}$
$\{a, b\}$	$\{\emptyset\}$

Extension-based I/O-characterization

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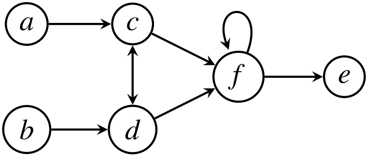
$\emptyset$	$\{\{e\}\}$
$\{a\}$	$\{\{e\}\}$
$\{b\}$	$\{\{e\}\}$
$\{a, b\}$	$\{\emptyset\}$

Definition

The I/O-gadget F satisfies I/O-specification $\mathfrak{p}$ under semantics σ iff $\forall J \subseteq I : \sigma(\triangleright(F, J))|_O = \mathfrak{p}(J)$.

Extension-based I/O-characterization

Input ($I = \{a, b\}$)	Output ($O = \{e\}$)
$\emptyset$	$\{\{e\}\}$
$\{a\}$	$\{\{e\}\}$
$\{b\}$	$\{\{e\}\}$
$\{a, b\}$	$\{\emptyset\}$



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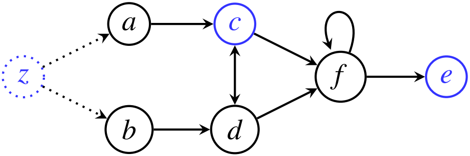

Extension-based I/O-characterization

Input ($I = \{a, b\}$)	Output ($O = \{e\}$)
$\emptyset$	$\{\{e\}\}$
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$\{b\}$	$\{\{e\}\}$
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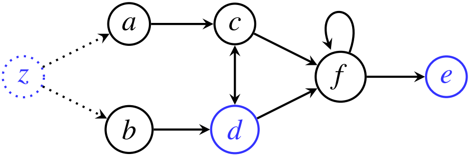

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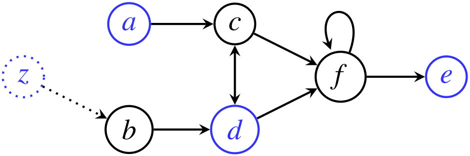

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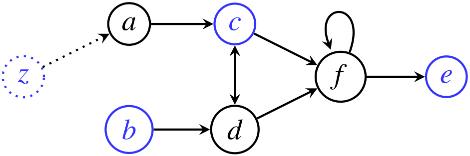

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Input ($I = \{a, b\}$)	Output ($O = \{e\}$)
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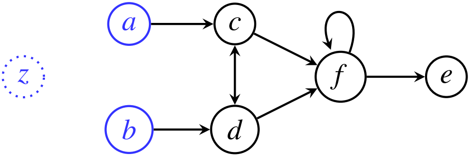
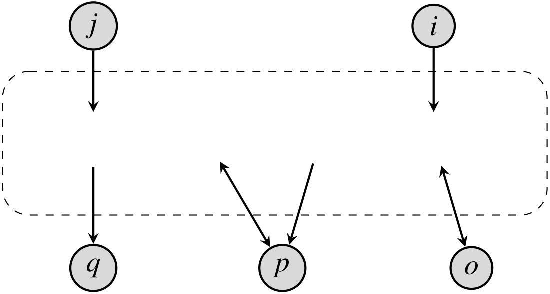


Extension-based I/O-characterization

Another I/O-specification:

Input ($I = \{i,j\}$)	Output ($O = \{o,p,q\}$)
$\emptyset$	$\{\emptyset\}$
$\{i\}$	$\{\{o,q\}\}$
$\{j\}$	$\{\{o,p,q\},\{p,q\}\}$
$\{i,j\}$	$\{\{o,p,q\},\{o,p\}\}$



Extension-based I/O-characterization

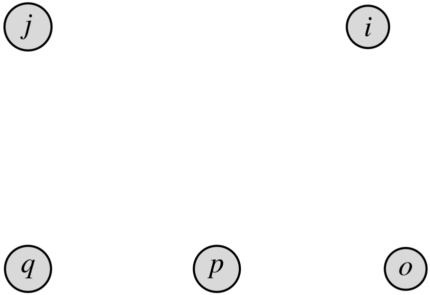
Theorem

An I/O-specification $\mathfrak{p}$ is satisfiable under σ iff

stb:	$\top$
prf, *sem*, *stg*:	$\forall J \subseteq I : \lvert\mathfrak{p}(J)\rvert \geq 1$
com:	$\forall J \subseteq I : \lvert\mathfrak{p}(J)\rvert \geq 1 \wedge \bigcap \mathfrak{p}(J) \in \mathfrak{p}(J)$
grd, *id*:	$\forall J \subseteq I : \lvert\mathfrak{p}(J)\rvert = 1$

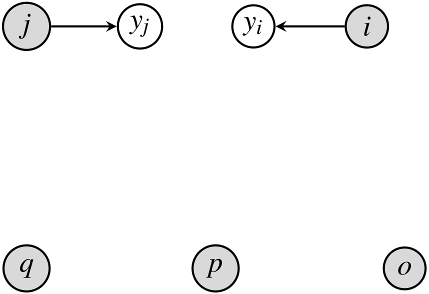
Extension-based I/O-characterization

Input ($I = \{i,j\}$)	Output ($O = \{o,p,q\}$)
$\emptyset$	$\{\emptyset\}$
$\{i\}$	$\{\{o,q\}\}$
$\{j\}$	$\{\{o,p,q\},\{p,q\}\}$
$\{i,j\}$	$\{\{o,p,q\},\{o,p\}\}$



Extension-based I/O-characterization

Input ($I = \{i, j\}$)	Output ($O = \{o, p, q\}$)
$\emptyset$	$\{\emptyset\}$
$\{i\}$	$\{\{o, q\}\}$
$\{j\}$	$\{\{o, p, q\}, \{p, q\}\}$
$\{i, j\}$	$\{\{o, p, q\}, \{o, p\}\}$



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Input ($I = \{i,j\}$)	Output ($O = \{o,p,q\}$)
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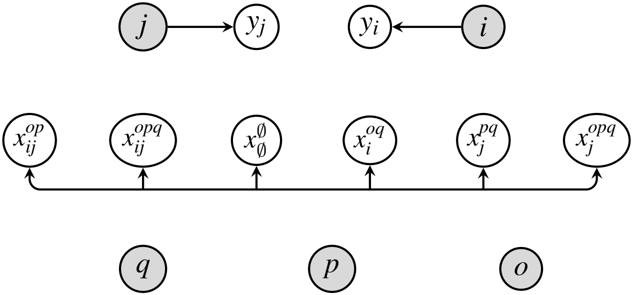
$\{i\}$	$\{\{o,q\}\}$
$\{j\}$	$\{\{o,p,q\},\{p,q\}\}$
$\{i,j\}$	$\{\{o,p,q\},\{o,p\}\}$



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Input ($I = \{i, j\}$)	Output ($O = \{o, p, q\}$)
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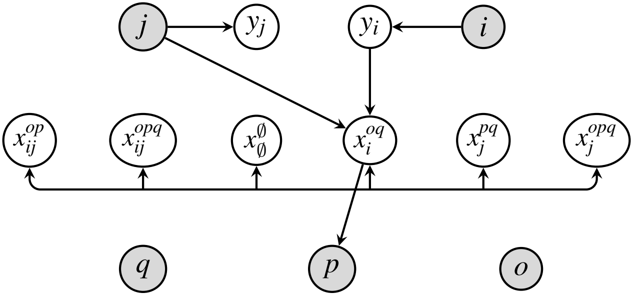

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Input ($I = \{i, j\}$)	Output ($O = \{o, p, q\}$)
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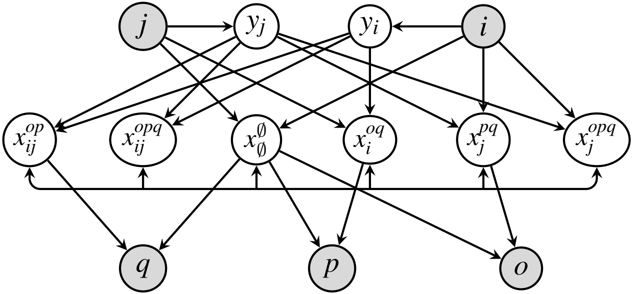

Extension-based I/O-characterization

Input ($I = \{i,j\}$)	Output ($O = \{o,p,q\}$)
$\emptyset$	$\{\emptyset\}$
$\{i\}$	$\{\{o,q\}\}$
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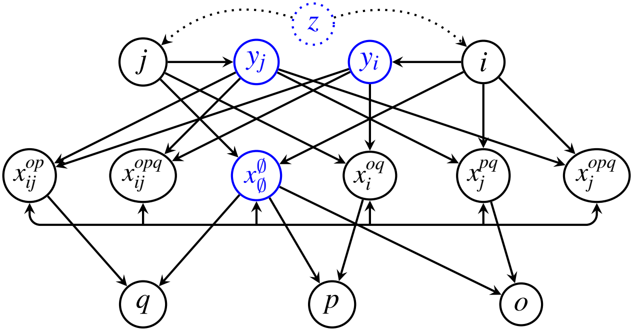

Extension-based I/O-characterization

Input ($I = \{i, j\}$)	Output ($O = \{o, p, q\}$)
$\emptyset$	$\{\emptyset\}$
$\{i\}$	$\{\{o, q\}\}$
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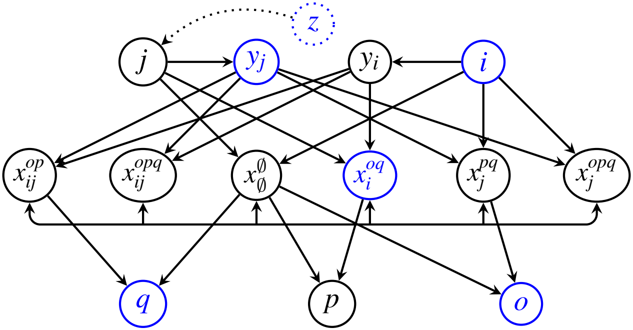
$\{i, j\}$	$\{\{o, p, q\}, \{o, p\}\}$



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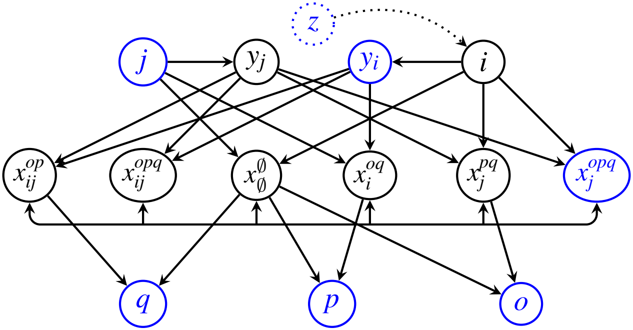
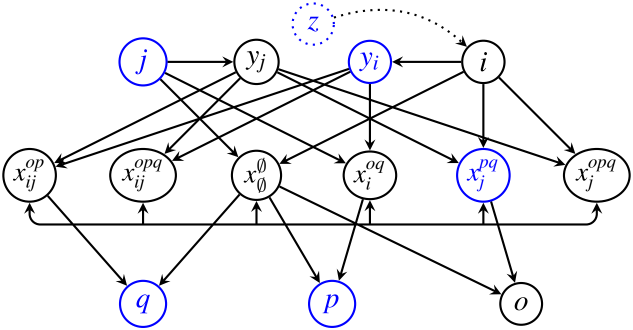


Extension-based I/O-characterization

Input ($I = \{i, j\}$)	Output ($O = \{o, p, q\}$)
$\emptyset$	$\{\emptyset\}$
$\{i\}$	$\{\{o, q\}\}$
$\{j\}$	$\{\{o, p, q\}, \{p, q\}\}$
$\{i, j\}$	$\{\{o, p, q\}, \{o, p\}\}$



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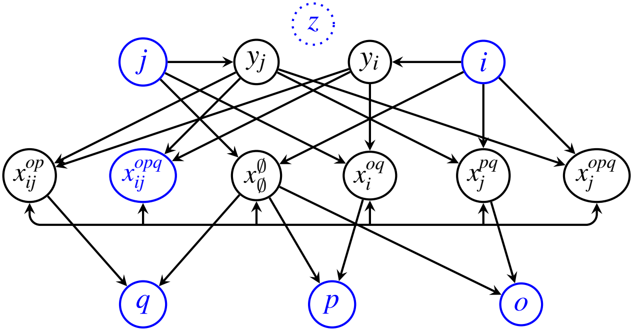

Extension-based I/O-characterization

Input ($I = \{i,j\}$)	Output ($O = \{o,p,q\}$)
$\emptyset$	$\{\emptyset\}$
$\{i\}$	$\{\{o,q\}\}$
$\{j\}$	$\{\{o,p,q\},\{p,q\}\}$
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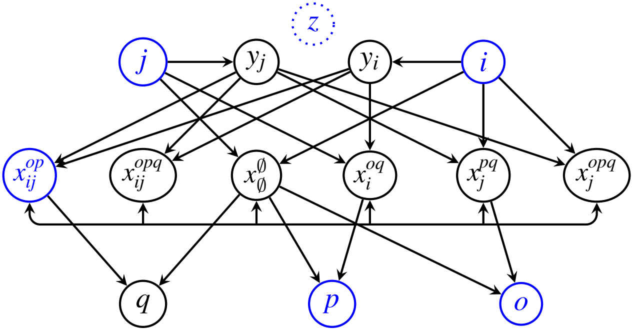

Labelling-based I/O-characterization

Definition

An 3-valued I/O-specification consists of two sets $I, O \subseteq \mathfrak{A}$ and a total function $\mathfrak{p} : \mathcal{L}(I) \mapsto 2^{\mathcal{L}(O)}$.

Input ($I = \{i_1, i_2\}$)	Output ($O = \{o_1, o_2\}$)
$\{i_1 \leftarrow \mathtt{u}, i_2 \leftarrow \mathtt{u}\}$	$\{\{o_1 \leftarrow \mathtt{u}, o_2 \leftarrow \mathtt{u}\}\}$
$\{i_1 \leftarrow \mathtt{t}, i_2 \leftarrow \mathtt{u}\}$	$\{\{o_1 \leftarrow \mathtt{t}, o_2 \leftarrow \mathtt{u}\}\}$
$\{i_1 \leftarrow \mathtt{u}, i_2 \leftarrow \mathtt{t}\}$	$\{\{o_1 \leftarrow \mathtt{u}, o_2 \leftarrow \mathtt{t}\}\}$
$\{i_1 \leftarrow \mathtt{f}, i_2 \leftarrow \mathtt{u}\}$	$\{\{o_1 \leftarrow \mathtt{u}, o_2 \leftarrow \mathtt{u}\}\}$
$\{i_1 \leftarrow \mathtt{u}, i_2 \leftarrow \mathtt{f}\}$	$\{\{o_1 \leftarrow \mathtt{u}, o_2 \leftarrow \mathtt{f}\}\}$
$\{i_1 \leftarrow \mathtt{t}, i_2 \leftarrow \mathtt{t}\}$	$\{\{o_1 \leftarrow \mathtt{t}, o_2 \leftarrow \mathtt{f}\}\}$
$\{i_1 \leftarrow \mathtt{t}, i_2 \leftarrow \mathtt{f}\}$	$\{\{o_1 \leftarrow \mathtt{t}, o_2 \leftarrow \mathtt{f}\}\}$
$\{i_1 \leftarrow \mathtt{f}, i_2 \leftarrow \mathtt{t}\}$	$\{\{o_1 \leftarrow \mathtt{u}, o_2 \leftarrow \mathtt{t}\}\}$
$\{i_1 \leftarrow \mathtt{f}, i_2 \leftarrow \mathtt{f}\}$	$\{\{o_1 \leftarrow \mathtt{t}, o_2 \leftarrow \mathtt{f}\}\}$

Definition

The I/O-gadget F satisfies the 3-valued I/O-specification $\mathfrak{p}$ under semantics σ iff $\forall L \subseteq \mathcal{L}(I) : \mathcal{L}_\sigma(\blacktriangleright(F, L))|_O = \mathfrak{p}(L)$.

Labelling-based I/O-characterization

Definition

A 3-valued I/O-specification $\mathfrak{p}$ is monotonic iff for all L_1 and L_2 such that $L_1 \sqsubseteq L_2$ it holds that $\forall K_1 \in \mathfrak{p}(L_1) \exists K_2 \in \mathfrak{p}(L_2) : K_1 \sqsubseteq K_2$.

Example

Input ($I = \{i_1, i_2\}$)	Output ($O = \{o_1, o_2\}$)
$\{i_1 \leftarrow \mathtt{u}, i_2 \leftarrow \mathtt{u}\}$	$\{\{o_1 \leftarrow \mathtt{u}, o_2 \leftarrow \mathtt{u}\}\}$
$\{i_1 \leftarrow \mathtt{t}, i_2 \leftarrow \mathtt{u}\}$	$\{\{o_1 \leftarrow \mathtt{u}, o_2 \leftarrow \mathtt{u}\}, \{o_1 \leftarrow \mathtt{u}, o_2 \leftarrow \mathtt{f}\}\}$
$\vdots$	$\vdots$
$\{i_1 \leftarrow \mathtt{t}, i_2 \leftarrow \mathtt{t}\}$	$\{\{o_1 \leftarrow \mathtt{u}, o_2 \leftarrow \mathtt{u}\}\}$
$\{i_1 \leftarrow \mathtt{t}, i_2 \leftarrow \mathtt{f}\}$	$\{\{o_1 \leftarrow \mathtt{t}, o_2 \leftarrow \mathtt{f}\}, \{o_1 \leftarrow \mathtt{t}, o_2 \leftarrow \mathtt{t}\}\}$
$\vdots$	$\vdots$

Labelling-based I/O-characterization

Theorem

A 3-valued I/O-specification $\mathfrak{p}$ is satisfiable under σ iff

stb: $\forall L \in \mathcal{L}(I) \forall K \in \mathfrak{p}(L) \forall o \in O : K(o) \neq \mathtt{u}$

prf: $\mathfrak{p}$ is monotonic

grd: $\mathfrak{p}$ is monotonic and $\forall L \subseteq \mathcal{L}(I) : |\mathfrak{p}(L)| = 1$

Conclusion

Summary

- First step toward a combination of recent lines of research.
  - Input/Output argumentation frameworks.
  - Realizability of argumentation semantics.
- I/O-characterizations: Exact conditions for satisfiability.
  - Extension-based: most prominent semantics.
  - Labelling-based: preferred, stable and grounded semantics.
- Constructions for satisfiable I/O-specifications.

Conclusion

Summary

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Future Work

- 3-valued I/O-characterization of complete semantics.
- Partial I/O-specifications.
- Construction of I/O-gadgets from compact representations of I/O-specifications, such as Boolean (resp. 3-valued) formulas or circuits.
- Identification of minimal I/O-gadgets.

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