FAKULTÄT
FÜR !NFORMATIK

Faculty of !nformatics

Compact Argumentation Frameworks

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Hannes Strass, Stefan Woltran

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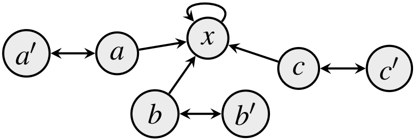
July 19, 2014

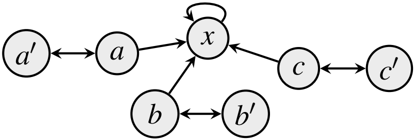
Der Wissenschaftsfonds.

- Abstract Argumentation Framework [Dung, 1995]:



Introduction

- Abstract Argumentation Framework [Dung, 1995]:



- Evaluation: Argumentation Semantics
- *stb*$(F) = \{\{a,b,c\},\{a,b,c'\},\{a,b',c\},\{a',b,c\},\{a,b',c'\},$ $\{a',b,c'\}, \{a',b',c\}\}$.

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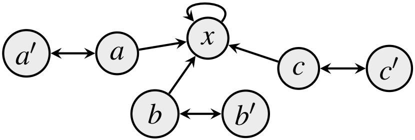

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Problem

Can we find an equivalent AF F' without argument x?

Motivation

- Heavy research on argumentation semantics, i.e. rules for identifying sets of acceptable arguments [Baroni and Giacomin, 2007].
- Structural analysis of their capabilities.
- Realizability [Dunne et al., 2014].
  - Model-based revision.
  - Search space reduction.
- Compact Realizability

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  - Model-based revision.
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- Compact Realizability
- Compact Argumentation Frameworks
  - Attractive for normal-forms.
  - Fairness: desired feature in application area (e.g. decision support [Amgoud et al., 2008]).

Background

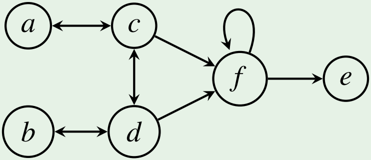
Countably infinite set of arguments $\mathfrak{A}$.

Definition

An argumentation framework (AF) is a pair (A, R) where

- $A \subseteq \mathfrak{A}$ is a finite set of arguments and
- $R \subseteq A \times A$ is the attack relation representing conflicts.

Example



$$F = \big(\{a, b, c, d, e, f\},$$
$$\{(a, c), (c, a), (c, d), (d, c), (d, b), (b, d), (c, f), (d, f), (f, f), (f, e)\}\big)$$

Background (ctd.)

Conflict-free Sets

Given an AF $F = (A, R)$, a set $S \subseteq A$ is conflict-free in F, if, for each $a, b \in S$, $(a, b) \notin R$.

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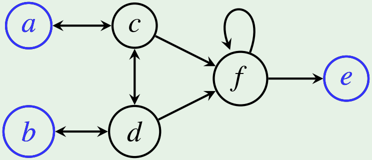

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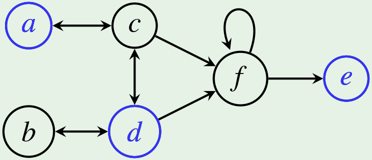

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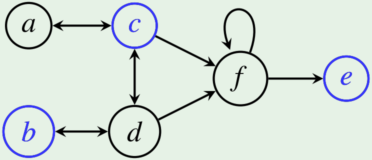

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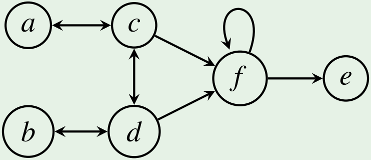

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Naive Extensions

Given an AF $F = (A, R)$, a set $S \subseteq A$ is a naive extension in F, if

- S is conflict-free in F and
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$\Rightarrow$ Maximal conflict-free sets (w.r.t. set-inclusion).

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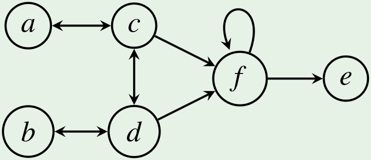

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Stable Extensions

Given an AF $F = (A, R)$, a set $S \subseteq A$ is a stable extension in F, if

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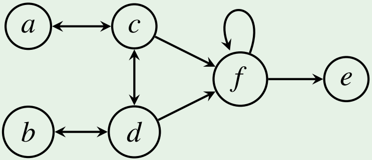

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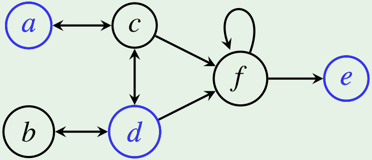

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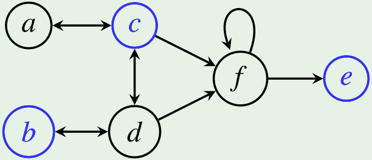

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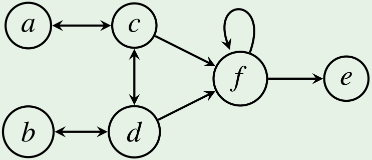

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Further semantics:

- Stage semantics
- Admissible sets
- Complete semantics
- Preferred semantics
- Semi-stable semantics

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- Stage semantics
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- Grounded semantics
- Ideal semantics
- cf2 semantics
- Resolution-based grounded semantics

Realizability

Definition

Given a semantics σ, an extension-set $\mathbb{S} \subseteq 2^{\mathfrak{A}}$ is called σ-realizable if there exists an AF F such that $\sigma(F) = \mathbb{S}$.

Signature: $\Sigma_\sigma = \{\sigma(F) \mid F \in AF_{\mathfrak{A}}\}$.

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Definition

Given an extension-set $\mathbb{S}$,

- $\mathit{Args}_{\mathbb{S}} = \bigcup_{S \in \mathbb{S}} S$, and
- $\mathit{Pairs}_{\mathbb{S}} = \{(a,b) \mid \exists S \in \mathbb{S} : \{a,b\} \subseteq S\}$.

Definition

$\mathbb{S}^+ = \max_{\subseteq}\{S \subseteq \mathit{Args}_{\mathbb{S}} \mid \forall a,b \in S : (a,b) \in \mathit{Pairs}_{\mathbb{S}}\}$.
$\mathbb{S}^- = (\mathbb{S}^+ \setminus \mathbb{S})$.

Example

$\mathbb{T} = \{\{a, b, c'\}, \{a, b', c\}, \{a', b, c\}\}$.

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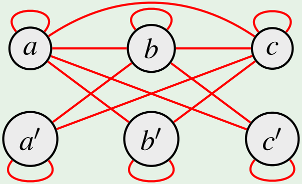

Realizability & Signatures

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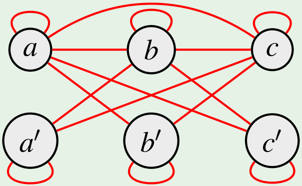

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Theorem [Dunne et al., 2014]

- $\Sigma_{naive} = \{\mathbb{S} \subseteq 2^{\mathfrak{A}} \mid \mathbb{S} = \mathbb{S}^{+}, \mathbb{S} \neq \emptyset\}$

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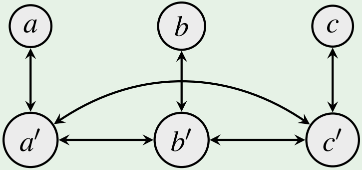

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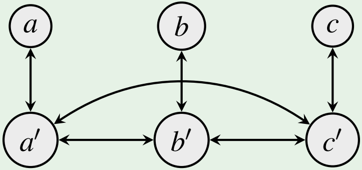

$\mathbb{T} = \{\{a, b, c'\}, \{a, b', c\}, \{a', b, c\}, \sout{\{a, b, c\}}\} \neq \mathbb{T}^+$?

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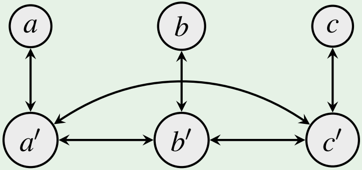

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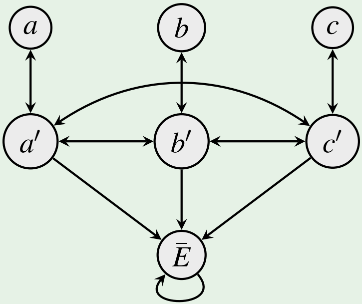

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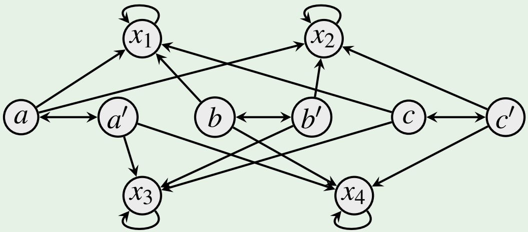

Additional (self-attacking) arguments lead to “artificial” constructions.

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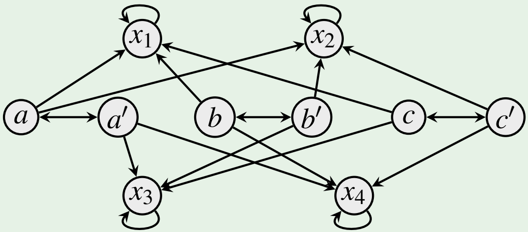

Additional (self-attacking) arguments lead to “artificial” constructions.

Definition

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C-Signature: $\Sigma^c_\sigma = \{\sigma(F) \mid F \in AF_{\mathfrak{A}}, \mathit{Args}_{\sigma(F)} = A_F\}$.

Stable Semantics: C-Signature

- $\Sigma^c_{naive} = \Sigma_{naive} \subset \Sigma^c_{stb}$:
  - For any AF F: $naive(F) = naive(F^{sym}) = stb(F^{sym})$.

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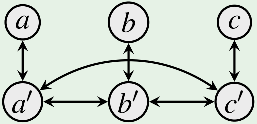

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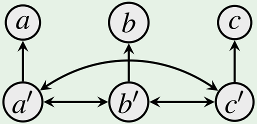

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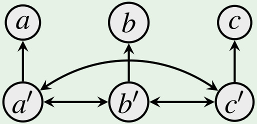

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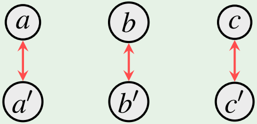

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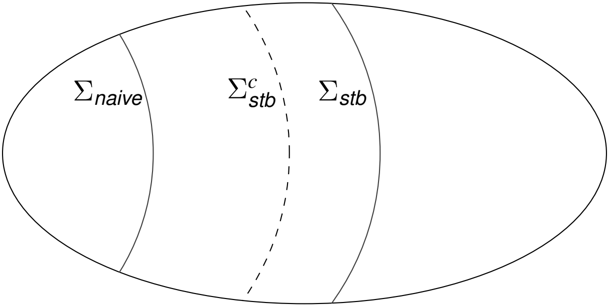

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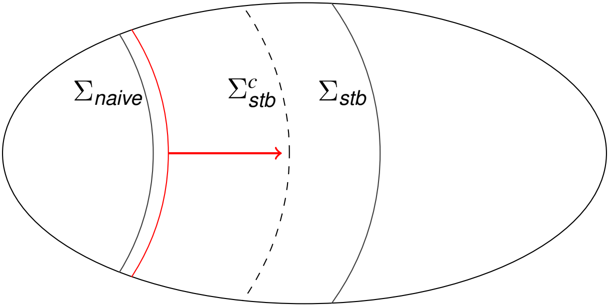

Stable Semantics: C-Signature

Proposition

For every extension-set $\mathbb{S}$ with $\mathbb{S} \subseteq \mathbb{S}^+$ it holds that if $|\mathbb{S}| \leq 3$ then $\mathbb{S} \in \Sigma^c_{stb}$.

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Proposition

For every extension-set $\mathbb{S}$ such that $\mathbb{S} \subseteq \mathbb{S}^+$ and for each $S \in \mathbb{S}$ there is an $a \in S$ with $\forall T \in (\mathbb{S} \setminus \{S\}) : a \notin T$ then $\mathbb{S} \in \Sigma^c_{stb}$.

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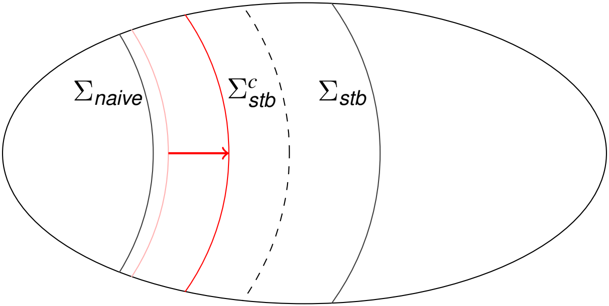

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Recall:

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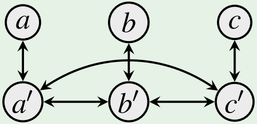

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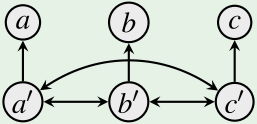

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Stable Semantics: C-Signature

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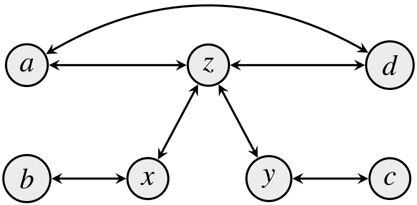

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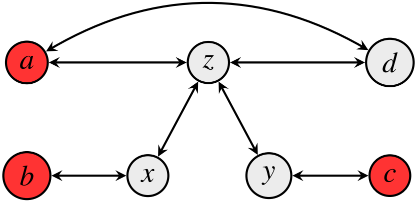

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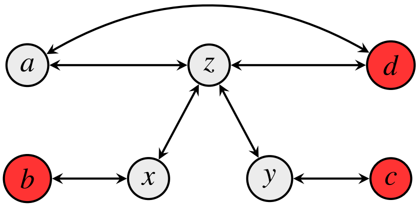

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Stable Semantics: C-Signature

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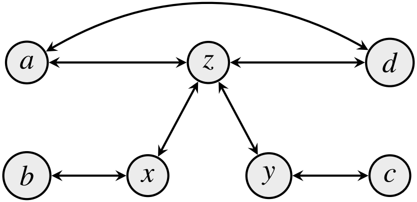

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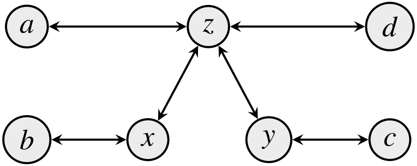

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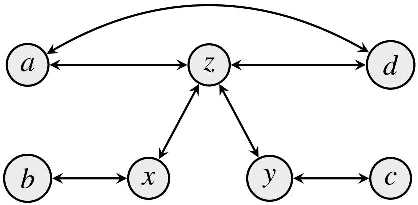

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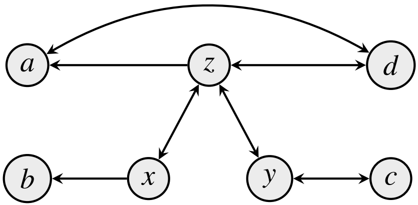

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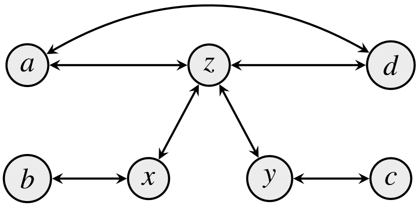

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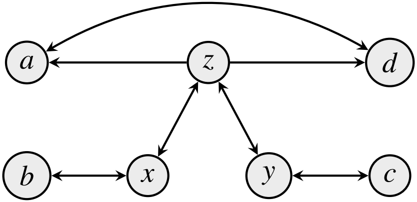

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Stable Semantics: C-Signature

Definition

Given an extension-set $\mathbb{S}$, an exclusion-mapping is the set

$$\mathfrak{R}_{\mathbb{S}} = \bigcup_{S \in \mathbb{S}^-} \{(s, \mathfrak{f}_{\mathbb{S}}(S)) \mid s \in S \text{ s.t. } (s, \mathfrak{f}_{\mathbb{S}}(S)) \notin \mathit{Pairs}_{\mathbb{S}}\}$$

where $\mathfrak{f}_{\mathbb{S}} : \mathbb{S}^- \to \mathit{Args}_{\mathbb{S}}$ is a function with $\mathfrak{f}_{\mathbb{S}}(S) \in (\mathit{Args}_{\mathbb{S}} \setminus S)$.

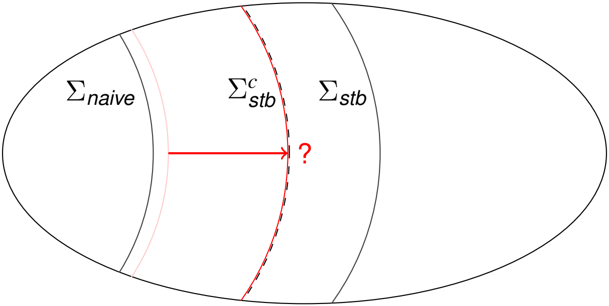
An extension-set $\mathbb{S}$ is called independent if there exists an exclusion-mapping $\mathfrak{R}_{\mathbb{S}}$ such that

- $\mathfrak{R}_{\mathbb{S}}$ is antisymmetric, and
- $\forall S \in \mathbb{S} \forall a \in (\mathit{Args}_{\mathbb{S}} \setminus S) : \exists s \in S : (s, a) \notin (\mathfrak{R}_{\mathbb{S}} \cup \mathit{Pairs}_{\mathbb{S}})$.

Theorem

For every independent extension-set $\mathbb{S}$ with $\mathbb{S} \subseteq \mathbb{S}^+$ it holds that $\mathbb{S} \in \Sigma^c_{stb}$.

Stable Semantics: C-Signature

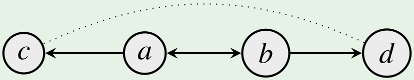


Stable Semantics: C-Signature

Definition

We call an AF $F = (A, R)$ conflict-explicit under semantics σ iff for each $a, b \in A$ such that $(a, b) \notin \mathit{Pairs}_{\sigma(F)}$, we find $(a, b) \in R$ or $(b, a) \in R$ (or both).

Example



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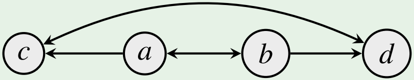

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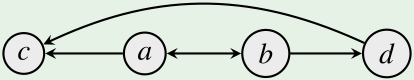

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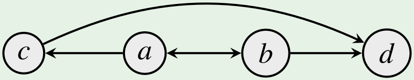

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Explicit-Conflict-Conjecture

For each AF $F = (A, R)$ there exists an AF $F' = (A, R')$ which is conflict-explicit under the stable semantics such that $\mathit{stb}(F) = \mathit{stb}(F')$.

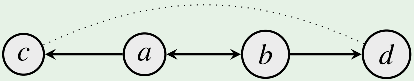
Theorem

Under the assumption that the EC-conjecture holds,

$$\Sigma^c_{\mathit{stb}} = \{\mathbb{S} \mid \mathbb{S} \subseteq \mathbb{S}^+ \land \mathbb{S} \text{ is independent}\}.$$

Explicit-Conflict-Conjecture

Example



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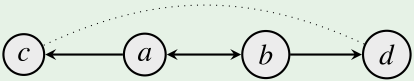

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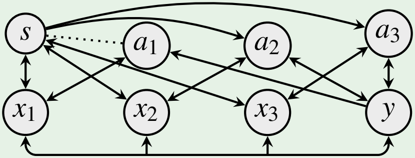

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Explicit-Conflict-Conjecture

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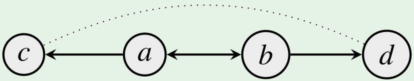

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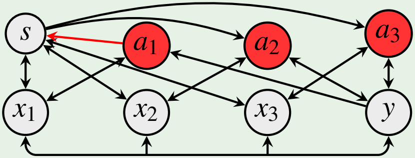

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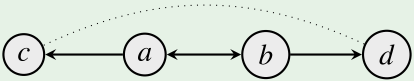

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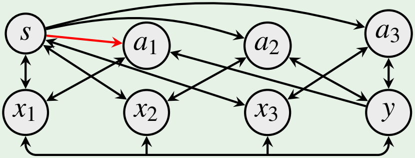

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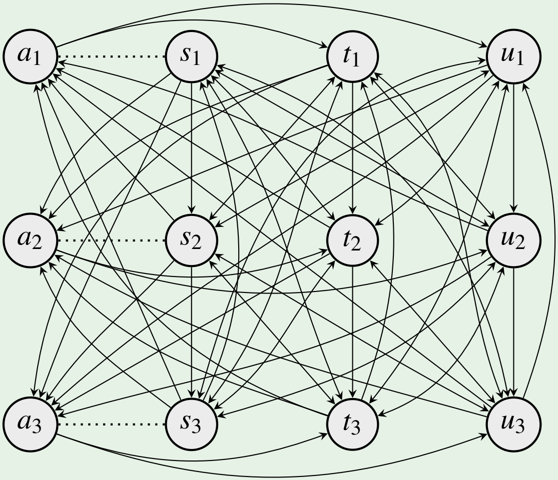
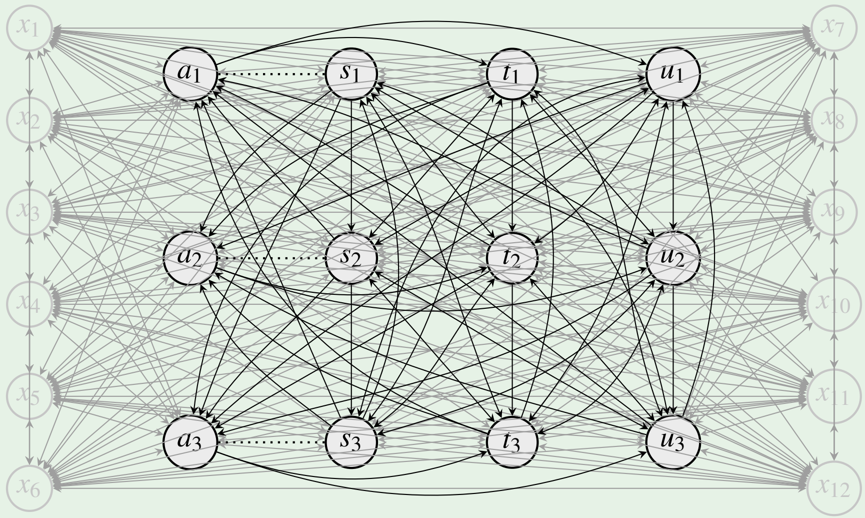
Example



Example


x_1
x_2
x_3
x_4
x_5
x_6
a_1
s_1
t_1
u_1
a_2
s_2
t_2
u_2
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s_3
t_3
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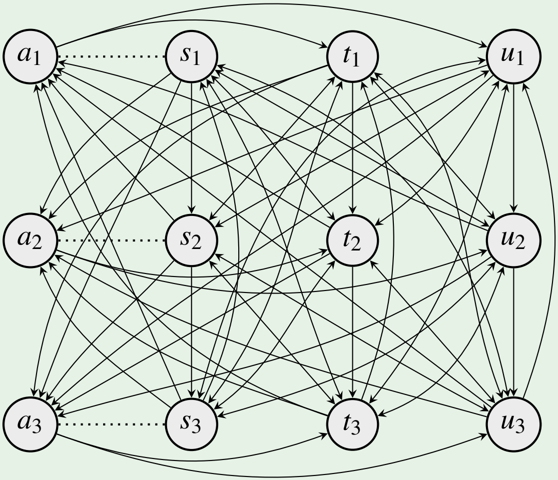
x_7
x_8
x_9
x_{10}
x_{11}
x_{12}

Example



Impossibility Results

- Decision procedure for compact realizability supposed to be hard.
- Shortcuts can be achieved by impossible numbers.
- Possible numbers for non-compact frameworks:
  [Baumann and Strass, 2014].
- Based on results for maximal independent sets [Griggs et al., 1988].

- Subsequent results hold for $\sigma \in \{$*stb*, *sem*, *pref*, *stage*, *naive*$\}$.

Impossibility Results

$\sigma_{\text{max}}(n) = \max\{|\sigma(F)| \mid F \in \mathsf{AF}_n\}$

Theorem

$$\sigma_{\text{max}}(n) = \begin{cases} 1, & \text{if } n = 0 \text{ or } n = 1, \\ 3^s, & \text{if } n \geq 2 \text{ and } n = 3s, \\ 4 \cdot 3^{s-1}, & \text{if } n \geq 2 \text{ and } n = 3s+1, \\ 2 \cdot 3^s, & \text{if } n \geq 2 \text{ and } n = 3s+2. \end{cases}$$

$\sigma_{\text{max}}^{\text{con}}(n) = \max\{|\sigma(F)| \mid F \in \mathsf{AF}_n, F \text{ connected}\}$

Theorem

$$\sigma_{\text{max}}^{\text{con}}(n) = \begin{cases} n, & \text{if } n \leq 5, \\ 2 \cdot 3^{s-1} + 2^{s-1}, & \text{if } n \geq 6 \text{ and } n = 3s, \\ 3^s + 2^{s-1}, & \text{if } n \geq 6 \text{ and } n = 3s+1, \\ 4 \cdot 3^{s-1} + 3 \cdot 2^{s-2}, & \text{if } n \geq 6 \text{ and } n = 3s+2. \end{cases}$$

Impossibility Results

Proposition

Given an extension-set $\mathbb{S}$, the component-structure $\mathcal{K}(F)$ of any AF F compactly realizing $\mathbb{S}$ under σ is given by the equivalence classes of the transitive closure of $\overline{Pairs_{\mathbb{S}}}$, i.e. $\left(\overline{Pairs_{\mathbb{S}}}\right)^*$.

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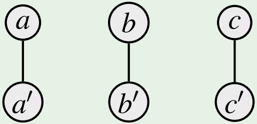

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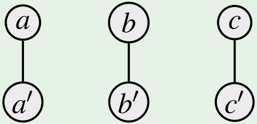


Proposition

Given an extension-set $\mathbb{S}$ where $|\mathbb{S}|$ is odd, it holds that if $\exists K \in \mathcal{K}(\mathbb{S}) : |K| = 2$ then $\mathbb{S}$ is not compactly realizable under semantics σ.

Impossibility Results

Definition

We denote the set of possible numbers of σ-extensions of a compact and connected AF with n arguments as $\mathcal{P}^c(n)$.

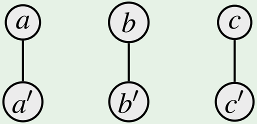
- $\forall p \in \mathcal{P}^c(n) : p \leq \sigma^{\text{con}}_{\text{max}}(n)$.
- Exact contents of $\mathcal{P}^c(n)$ unknown.

Proposition

Let $\mathbb{S}$ be an extension-set that is compactly realizable under semantics σ where $\mathcal{K}_{\geq 2}(\mathbb{S}) = \{K_1, \ldots, K_n\}$. Then for each $1 \leq i \leq n$ there is a $p_i \in \mathcal{P}^c(|K_i|)$ such that $|\mathbb{S}| = \prod_{i=1}^{n} p_i$.

Example

$\mathbb{U} = \{\{a, b, c'\}, \{a, b', c\}, \{a', b, c\}, \{a', b', c'\}\}$.



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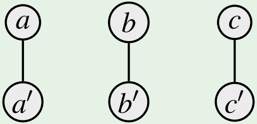

Corollary

Let extension-set $\mathbb{S}$ with $|Args_{\mathbb{S}}| = n$ be compactly realizable under σ. If $|\mathbb{S}|$ is a prime number, then $|\mathbb{S}| \leq \sigma^{\text{con}}_{\text{max}}(n)$.

Impossibility Results

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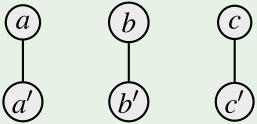

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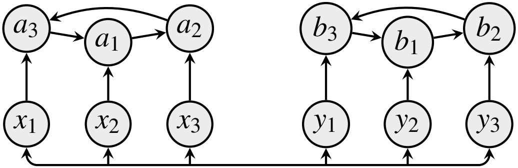
Corollary

Let extension-set $\mathbb{S}$ be compactly realizable under σ and $f_1^{z_1} \cdot \ldots \cdot f_m^{z_m}$ be the integer factorization of $|\mathbb{S}|$, where $f_1, \ldots, f_m$ are prime numbers. Then $z_1 + \ldots + z_m \geq |\mathcal{K}_{\geq 2}(\mathbb{S})|$.

Theorem

1. $CAF_{sem} \subset CAF_{pref}$
2. $CAF_{stb} \subset CAF_{\sigma} \subset CAF_{naive}$ for $\sigma \in \{pref, sem, stage\}$
3. $CAF_{\theta} \not\subseteq CAF_{stage}$ and $CAF_{stage} \not\subseteq CAF_{\theta}$ for $\theta \in \{pref, sem\}$



Theorem

For $\sigma \in \{pref, sem, stage\}$, AF $F = (A, R) \in CAF_{\sigma}$ and $E \subseteq A$, it is coNP-complete to decide whether $E \in \sigma(F)$.

Summary

- Signatures of argumentation semantics
- Compact signatures
  - Exact characterizations hard to find
  - Missing step for stable semantics: EC-Conjecture
- Shortcuts via impossible numbers of extensions
- Full picture of relations between compact AFs under semantics providing incomparability

Future Work

- Exact characterizations of compact signatures.
- Closing the gap between general and compact realizability with fragments of ADFs.
- Explicit-Conflict-Conjecture.

References I

Amgoud, L., Dimopoulos, Y., and Moraitis, P. (2008).
Making decisions through preference-based argumentation.
In KR, pages 113–123.

Baroni, P. and Giacomin, M. (2007).
On principle-based evaluation of extension-based argumentation semantics.
Artif. Intell., 171(10-15):675–700.

Baumann, R. and Strass, H. (2014).
On the Maximal and Average Numbers of Stable Extensions.
In TAFA 2013, volume 8306 of LNAI, pages 111–126.

Dung, P. M. (1995).
On the acceptability of arguments and its fundamental role in nonmonotonic reasoning, logic programming and n-person games.
AIJ, 77(2):321–357.

Dunne, P. E., Dvořák, W., Linsbichler, T., and Woltran, S. (2014).
Characteristics of multiple viewpoints in abstract argumentation.
In KR, pages 72–81.

References II

Griggs, J. R., Grinstead, C. M., and Guichard, D. R. (1988).
The number of maximal independent sets in a connected graph.
Discrete Mathematics, 68(2–3):211–220.

$\Sigma_{stage} \not\subseteq \Sigma_{sem}$