Interdependencies of Electricity Market Characteristics and Bidding Strategies of Power Producers

by
Wolfgang Karl Gatterbauer

Submitted to the Engineering Systems Division and the
Department of Electrical Engineering and Computer Science
in Partial Fulfillment of the Requirements for the Degree of

Master of Science in Technology and Policy
Master of Science in Electrical Engineering and Computer Science

at the Massachusetts Institute of Technology
June, 2002

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Abstract

In the context of the ongoing deregulation of the electricity industry, we revisit the commonly held assumption that, under the condition of perfect information, a decentralized unit commitment would lead to the same power quantities traded and the same optimal social welfare as centralized unit commitment. Taking fixed operating costs into account, we show that meeting decentralized performance objectives of the individual market participants can lead to a lower efficiency than minimizing total operating cost in a decentralized way. This result concerns short-term optimization, and does not consider long-term investment issues.

Next, the task of optimally self-scheduling generators to maximize profits in a deregulated electricity market is investigated from the standpoint of an individual market participant. We show how a generator owner can improve his scheduling tactics by using stochastic dynamic programming and assuming a Cauchy error distribution of forecast prices.

Power auction markets require power producers to internalize start-up / shut-down costs of their units when submitting their bids. With the help of an abstract example, we finally show that a generator owner's optimum bid sequence for a centralized auction market is above marginal cost even where is no market power-related strategic bidding. We conclude that marginal production costs cannot be used as baseline for the assessment of market power in electricity markets.

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Acknowledgments

It is a pleasure to thank the people who made this thesis possible.

The general research area of this thesis was proposed to me by my teacher and supervisor, Dr. Marija Ilic, to whom I wish to express my deepest gratitude for the continued guidance and support she has provided throughout the course of this work. I greatly appreciate her patience with my many, many questions.

There is a large group of people, some of them at MIT and some not, without whom my stay in Cambridge would not have been as enjoyable as it was. Their contributions to the thesis range from philosophy to dancing to just being patient with me (sometimes all three at once), and I want to thank especially: Katarina Veljovich, Navid Sabbaghi, Guillaume Quiviger, Michalis Aftias, Guido Meardi, Pankaj Sarim, Yu-Ling Hsu, Karen and E.J. Babin (the greatest Rotarians in Boston!), Carolina Escobar, Gerlinde Gniewosz , Milan Boran, Chris Roberts, Yuka Asada, Andrea Balboni, Gero Nordmann, Nico Voigtlaender, Manolis and Maria Kamvysselis, Chalermmon Lertpattarapong, and Carlos Barrientos.

I gratefully acknowledge financial support for my studies at MIT through a private scholarship from the Rotary Foundation and a fellowship from the MIT graduate student office.

Lastly, and most importantly, I wish to thank my parents, Karl and Christine Gatterbauer. They raised me, supported me, taught me, and loved me. To them I dedicate this thesis.

Cambridge, May 2002

To my parents,

Karl and Christine.

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Nomenclature

Symbol	Meaning
k	Hour
$\widehat{p_k}$	Price forecast for hour k
p_k	Actual price for hour k
$e_k = \Delta p_k$	Deviation between price forecast and actual price in hour k
ε_k	Deviation between price forecast and actual price in hour k that is not explained by correlation
σ	Observed variance in Δp_k (chapter 3.2.1)
σ_ε	Observed variance in regression (chapter 3.2.2)
δ	Observed correlation in Δp_k between successive hours
$\overline{\Delta p_k}$	Expected deviation, given Δp_{k-1} and δ in Method3
$\widetilde{X}$	Median of variable X
x_k	State of the generator in hour k
u_k	Decision to turn the generator on or off during hour k
$f(x_k, u_k)$	State equation
U_k	Set of possible choices for u_k for hour *k* given the state x_k
$\underset{\Delta p_k}{\mathrm{E}}\{X\}$	Expected value of X with regard to random variable Δp_k
π	Hourly profit
d	Size of an interval for discretization
s	Number of different values used for discretization
$\mathrm{p_i}$	Discretized probability distribution (chapter 3.2.1)
$\mathrm{p_{i,j}}$	Discretized probability distribution (chapter 3.2.2)
Φ	Standard Normal cumulative distribution function
F	Cumulative Cauchy distribution
b	Parameter for Cauchy distribution
i,j	Indices for discretization of probability distributions
C(Q)	Production costs as a function of output
Q	Output quantity

1 Introduction

1.1 Research Objective

The electric power industry in the US has undergone a major transition in recent years. Historically, wholesale electricity provision is managed by vertically integrated utilities granted monopoly franchise service territories. Regulators judged both, the economic efficiency and reliability, and approved tariffs charged to the customers.

In the deregulated industry, competition between market participants should improve economic efficiency and lower prices for customers. Tasks that were performed in a centralized, coordinated fashion, are now performed by market participants and questions arise as to how much of the control and planning should remain in the hands of a centralized organization, and how much should be taken care of by "the market," in which each market participant will try to commit its resources in such a way as to maximize his profits.

This thesis contributes to current debates in the context of deregulation by supporting the following points:

- A centralized unit commitment can be economically more efficient in the short run than a decentralized unit commitment under the assumption of complete knowledge about demand and the generators' marginal cost and neglecting the differences with respect to capacity investments.

- Higher than marginal cost bids in electricity auctions do not necessarily indicate the exercise of market power, but can be explained by the fact that rational market participants have to internalize the cost of being on when not selling and the uncertainties of market outcomes into their market bids.

In addition, we will show how a self-scheduling generation owner can use stochastic dynamic programming for determining the hours for which it is most profitable to switch the unit on. Price uncertainties are simulated with Gaussian and non-Gaussian probability distributions.

1.2 Data Set Used

The example data for calibrating the bidding algorithms using stochastic dynamic programming come from daily announcements by the independent System Operator of New England [ISO-NE-www].

1.3 Organization of the Thesis

Chapter 2 compares theoretical efficiencies of a centralized economic unit commitment, such as in the PoolCo type markets, with those of a decentralized commitment, such as in a Power Exchange, for which individual generators schedule themselves. The chapter shows examples in which a decentralized unit commitment is economically more efficient than a centralized unit commitment, even in the presence of complete knowledge about demand and all generators' marginal costs, and thereby adds to current literature on power systems restructuring [Ilic98], [Christie01], [Day02], [Stoft02].

Chapter 3 shows how stochastic dynamic programming can be used to determine the optimal hours to turn a generator on if the generator is able to self-schedule its unit. Only recently have variations of stochastic dynamic programming been proposed for optimal unit commitment in a deregulated market [Takriti97], [Allen99], [Tseng99], [Valenzuela01]. This chapter uses actual price forecast data and observed correlations between hourly prices from the ISO New England Market to solve the optimal commitment problem and shows the difference in efficiencies when a generator self-schedules its units at different points in time.

Chapter 4 addresses the issue of assumed market power when bids in energy markets exceed marginal cost. By using a simple example, it will be shown that in energy auctions observed bidding prices above marginal cost are not a consequence of market power, such as commonly assumed [Borenstein00], but of the rational internalization of uncertainties and intertemporal constraints of electricity units into the bid sequences.

Chapter 5 sums up the major conclusions of the thesis, and the Appendices contain an overview of dynamic programming, source codes and numerical examples for the simulations.

2 Centralized versus Decentralized Unit Commitment and Economic Dispatch

2.1 Background

This chapter investigates the day-ahead unit commitment problem for the deterministic case in which demand is inelastic and known. We formulate a model for the generators that considers linear marginal cost curves, fixed costs, switching costs and intertemporal time constraints.

Two market structures are compared: the decentralized case in which individual generators schedule themselves (DC), and the centralized version of a PoolCo type market in which an Independent System Operator (ISO) determines which generators are on during the 24 hours given the knowledge about each generator's constraints.

The model is implemented in Matlab assuming a simple forecast methodology for the resulting prices of the individual generators in the decentralized version. An interesting example for deviations between DC and ISO is singled out and mathematically generalized for the unconstrained problem. The commonly held belief that a competitive market process and an unconstrained economic dispatch lead to the same optimal social welfare [Oren95] is argued against and disproved by examples; general boundaries for deviations are established in the case of quadratic cost curves and two generators.

2.2 Unit Commitment and Economic Dispatch

Unit commitment is the process of deciding in advance whether to turn on or off each generator on the power grid at a given hour. It becomes an intricate mathematical decision process because the hourly decisions are interdependent. A power plant, for example, has to stay on line or remain switched off a given number of hours after startup / shutdown due to thermal stress or nuclear concerns. We include these constraints in our model. Economic Dispatch is the process of allocating the required load demand between the available generation units such that the cost of operation is minimized.

We will compare two markets. The first type is an outgrowth of the centralized power pools in the Northeast US, such as ISOs of New England (NE), New York, and Pennsylvania-New Jersey-Maryland (PJM). Here the bidders explicitly provide generation specific constraints and costs, and the ISO uses centralized unit commitment to select bids so that total cost is minimized subject to operating constraints and costs and determines the quantities each unit should produce.

The second market is California style in which market rules require specification of convex supply functions, independent of the type of generation technology. The bids assume that the plants would be available at the hour for which the supply function is specified. It is up to the bidders to account for these constraints for bidding day ahead;

this decision making process at a bidder's level could be viewed as a decentralized unit commitment.

2.3 The Basic Economic Argument Assuming Perfect Information

In the following equations C_i is the cost function and Q_i the quantity produced by generator i. Q_D is the total demand, n the total number of available generators, and u_i a binary variable that states whether a generation unit i is turned on or off at a given moment.

2.3.1 Conventional Economic Dispatch

Mathematically, a centralized economic dispatch is the problem of minimizing the total generation cost, using the quantities produced by each of the possible generators as decision variables [Ilic98],

$$\min_{Q} \sum_{i=1}^{n} C_i(Q_i)\,, \tag{2.1}$$

such that total generation equals total load:

$$\sum_{i=1}^{n} Q_i = Q_D. \tag{2.2}$$

This basic version of an unconstrained economic dispatch finds a solution to this optimization problem for a system of arbitrary size. A necessary condition for solving this basic economic dispatch problem is:

$$\frac{\delta C_1}{\delta Q_1} = \ldots = \frac{\delta C_n}{\delta Q_n} = \lambda. \tag{2.3}$$

This condition defines the least generation cost for meeting given demand. The term λ is known as the short-run marginal cost (SRMC) and, at the optimum, all unit marginal costs are equal to it.

2.3.2 Conventional Centralized Unit Commitment

The basic unit commitment problem (without start-up costs or minimum up/down time constraints) is as follows [Ilic98]

$$\min_{u_i, Q} \sum_{i=1}^{n} u_i C_i(Q_i), \tag{2.4}$$

subject to

$$\sum_{i=1}^{n} Q_i = Q_D, \tag{2.5}$$

where u_i equals 0 or 1 depending on whether the unit if off or on. Following the Lagrangian relaxation method, one first forms the Lagrangian function,

$$L(u, Q, \lambda) = \sum_{i=1}^{n} u_i \left(C_i(Q_i) - \lambda Q_i \right) + \lambda Q_D . \tag{2.6}$$

By minimizing (2.6) over Q first, one obtains the conventional economic dispatch equal incremental condition, that is,

$$\frac{\delta C_1}{\delta Q_1} = \ldots = \frac{\delta C_n}{\delta Q_n} = \lambda. \tag{2.7}$$

which permits one to solve for Q in terms of λ, the system incremental cost. The Lagrangian can be written as

$$L(u, \lambda) = \sum_{i=1}^{n} u_i \left(C_i\left(Q_i(\lambda) \right) - \lambda Q_i(\lambda) \right) + \lambda Q_D . \tag{2.8}$$

Finally, the Lagrangian method minimizes $L(u, \lambda)$ with respect to u giving the switching curve law

$$u_i = \begin{cases} 0 & \text{if } C_i - \lambda Q_i > 0 \\ 1 & \text{if } C_i - \lambda Q_i < 0, \end{cases} \tag{2.9}$$

that is, the unit is off if the average cost $C_i / Q_i > \lambda$ and on otherwise. Once on, a conventional economic dispatch is used to adjust to demand changes if these are monitored more frequently.

2.3.3 Decentralized Economic Dispatch

When competitive bilateral transactions take place, each party's objective is to maximize its profit,

$$\max_{Q_i} \pi_i(Q_i), \tag{2.10}$$

where $\pi_i = PQ_i - C_i(Q_i)$ stands for the profit made by the market participant *i* through some sort of trading process, given known price *P*. In the above equations C_i is the cost function and Q_i the quantity produced by generator *i*. Thus, under perfect conditions, when the market converges to a single electricity price, one can maximize π_i to yield:

$$\frac{\delta C_1}{\delta Q_1} = \ldots = \frac{\delta C_n}{\delta Q_n} = P. \tag{2.11}$$

This is simply obtained by each market participant optimizing its own profit for the assumed exogenous market price *P*. The process of bilateral decisions will stabilize *P* at the systemwide economic equilibrium under a perfect information exchange among all market participants.

2.3.4 Decentralized Unit Commitment

We assume a generator owner to be a price taker in a competitive market place. He must make a unit commitment decision typically by certain time day ahead, before actually knowing the spot price of the next hour. After the spot price is known, the generator decides how much power to sell in order to maximize profit. The only control for the problem is u_k whether to turn on or off at stage *k*. The expected generation level $\widehat{Q_k}$ may be regarded as a function of the control u_k and the expected price $\hat{p}$.

In the case of assuming deterministic price, and ignoring start-up costs, must-run time constraints, etc. it can be shown that an individual decision maker would arrive at the same average cost versus market price decision rule as the rule often used by a system operator scheduling plants in a coordinate way. The proof for this goes as follows [Ilic98]:

Given a generator *i*, its profit while on is

$$\widehat{\pi_{on}} = \hat{p} \cdot \widehat{Q_i} - C_i(\widehat{Q_i}) \tag{2.12}$$

The generator will turn on only if $\widehat{\pi_{on}} > 0$, which is equivalent to

$$\widehat{p} > \frac{C_i(\widehat{Q}_i)}{\widehat{Q}_i} \tag{2.13}$$

which is the average cost rule used for coordinated unit commitment.

Based on this derivation, one could conclude that under perfect market assumptions and when neglecting minimum run time constraints, startup costs, etc. a system operator would schedule the same units as the individual power producers would in a decentralized way. Thus, both a centralized and a decentralized commitment process should lead to the same power quantities traded, and to the same total social welfare optimum. Most importantly, in this case the optimal electricity price is reached under the same conditions as the social welfare is maximized. The performance objectives of the individual market participants (to maximize profits) and the objective of a centralized entity (to minimize total operating cost) are considered to be equivalent.

2.4 The Model

In what follows, we will describe and simulate the optimal unit commitment for generators with intertemporal time constraints both in a centralized and decentralized way. We will show that a decentralized and a centralized commitment process can actually lead to different outcomes. To do so, we will introduce a simplified state model of a generator.

2.4.1 Model of a Generator

We assume as the only intertemporal constraint on the generator the fact that once turned on/off, it has to remain turned on/off for at least two hours. Each generator can therefore be in one of 4 states in every hour *k*. State x_k^j contains information about generator *j* in the previous hour:

$$x_k^j = \begin{cases} 1 & \text{if } \textit{shut on for exactly } 1 \textit{ hour at the end of hour } k-1 \\ 2 & \text{if } \textit{shut on for at least } 2 \textit{ hours at the end of hour } k-1 \\ 3 & \text{if } \textit{shut off for exactly } 1 \textit{ hour at the end of hour } k-1 \\ 4 & \text{if } \textit{shut off for at least } 2 \textit{ hours at the end of hour } k-1. \end{cases} \tag{2.14}$$

The constraints on the ability to turn on or off for a specific hour *k* can be formulated as a constrained control space U_k^j at each stage:

$$U_k^j(x_k^j) = \begin{cases} \{0\} & \text{if } x_k^j = 3 \\ \{1\} & \text{if } x_k^j = 3 \\ \{0,1\} & \text{else.} \end{cases} \tag{2.15}$$

The state of the generator evolves according to:

$$x_{k+1}^j = \mathrm{f}(x_k^j, u_k^j) = \begin{cases} 1 & \text{if } (x_{k+1}^j = 4 \wedge u_{k+1}^j = 1) \\ 2 & \text{if } (x_{k+1}^j = 1) \vee (x_{k+1}^j = 2 \wedge u_{k+1}^j = 1) \\ 3 & \text{if } (x_{k+1}^j = 2 \wedge u_{k+1}^j = 0) \\ 4 & \text{if } (x_{k+1}^j = 3) \vee (x_{k+1}^j = 4 \wedge u_{k+1}^j = 0). \end{cases} \tag{2.16}$$

We model marginal cost as proportionally increasing with the generator's output Q^j: $\mathrm{MC}^j(\mathrm{Q}^j) = a^j Q^j$. This is not necessarily a realistic assumption, but simplifies the implementation of the model. We will later extend the result to a quadratic cost function. Given the price of electricity P_k for hour *k*, generator *j* decides to adjust his unit's output Q_k^j so that $P_k = MC^j(Q_k^j)$ if it is on during the considered hour. Taking the maximum output $Q_{\max}^j$ into account, its output becomes:

$$Q_k^j(u_k^j, P_k) = \begin{cases} \min[Q_{\max}^j, \dfrac{P_k}{a^j}] & \text{if } u_k^j = 1 \\ 0 & \text{if } u_k^j = 0. \end{cases} \tag{2.17}$$

Given chosen output Q_k^j, the variable costs incurred during one hour become:

$$C_{V,k}^j(\mathrm{Q}_k^j) = \frac{a^j}{2} Q_k^{j2}. \tag{2.18}$$

If the generator is turned on during a given hour, it incurs fixed costs C_F^j:

$$C_{F,k}^j(u_k) = \begin{cases} C_F^j & \text{if } u_k^j = 1 \\ 0 & \text{if } u_k^j = 0. \end{cases} \tag{2.19}$$

Together, these two cost components represent a quadratic cost function $C(Q) = aQ^2 + bQ + c$ with the linear coefficient *b* equal to zero. In addition to the above described hourly costs, the generator incurs "switching costs" every time it changes its state from on to off or v.v.:

$$C_{S,k}(\mathrm{x}_k, u_k) = \begin{cases} C^j_{Son} & \text{if } (x^j_k = 4 \wedge u^j_k = 1) \\ C^j_{Soff} & \text{if } (x^j_k = 2 \wedge u^j_k = 0) \\ 0 & \text{else.} \end{cases} \tag{2.20}$$

Together with revenues $R^j_k(P_k, Q^j_k) = P_k Q^j_k$, the profit for each hour can be written as:

$$G^j_k(\mathrm{x}^j_k, u^j_k, P_k) = R^j_k(\mathrm{P}_k, Q^j_k) - C^j_{F,k}(\mathrm{u}^j_k) - C_{S,k}(\mathrm{x}_k, u_k) - C_{V,k}(\mathrm{Q}^j_k). \tag{2.21}$$

2.4.2 The Decentralized Commitment Process

In order to decide ahead on which of the 24 hours of the next day the generator should schedule its unit, he has to make an assumption of how prices will develop given the known electricity demand. Generator *j* has to use a methodology to map the vector *Q* into the vector of forecast prices $\widehat{P}$.

Our model assumes global knowledge of the marginal cost curves of all generators participating in the market. This assumption is backed by the fact that market participants are publicly known and marginal cost functions can easily be estimated for all technologies used.

Hence, generator *j* can calculate a forecast market price for a given hour, assuming all generators are committing their units. In our model, we assume that all generators base their decisions on the above defined $\widehat{P}(Q)$.

The objective of generator *j* is to find an optimal decision vector $(u^j_1, u^j_2, ..., u^j_{24})$ for a given initial state x^j_1 that maximizes the profit over a given 24-hour time frame:

$$J^j(\mathrm{x}^j_1) = \max_{u^j_k} \sum_{k=1}^{24} G^j_k(\mathrm{x}^j_k, u^j_k, P_k). \tag{2.22}$$

It follows that, under these assumptions, the actual price P_k during one hour can never be smaller than the forecast price $\widehat{P}(Q)$ as the market supply curve can only be shifted upwards by generators deciding not to participate during hour *k*.

The optimal strategy can be formulated as one that solves a dynamic programming problem. Given the forecast prices $\widehat{P}$, the optimal commitment policy $\pi^j = \left\{u_1^j, u_2^j, ..., u_{24}^j\right\}$ leading to the maximum profit $J^j(x_1^j) = J_1^j(x_1^j)$ is calculated by the DP-Algorithm

$$
\begin{aligned}
&J_k^j(\mathrm{x}_k) = \max_{u_k^j}\left[G_k^j(\mathrm{x}_k^j, u_k^j, P_k) + J_{k+1}^j\left(\mathrm{f}(\mathrm{x}_k^j, u_k^j)\right)\right], \\
&J_{25}^j = 0.
\end{aligned}
\tag{2.23}
$$

In our notation, x_k describes the physical state of the generator at the end of the previous period and constrains the possible decisions for the same hour *k*.

We have implemented the commitment process in Matlab. The basic structure of the program is shown in Table 2-1 and the source code is printed in Appendix B

Calculate $\widehat{P}_k$ given deterministic demand Q_k for each hour k.
Apply DP-algorithm for each of the generators to calculate optimal cost-to-go matrices for each stage and state
Calculate matrices for decisions and states in function of stage and state
Calculate actual market prices P_k determined by the market supply function of scheduled generators during hour *k*
Calculate profits and outputs for each generator and total costs incurred by all generators to supply given demand

Table 2-1. Structure of program *mainDC.m*

2.4.3 The Centralized Commitment Process

The objective of the centralized commitment process becomes to schedule generation units in such a way as to minimize the total cost necessary for supplying the given inelastic demand *Q* and thereby to maximize welfare. Higher prices than necessary make customers worse off, but generators better off by the same amount. Hence, under given assumptions maximum welfare is achieved by minimizing only costs incurred by generators:

$$
\min_{x_k^j, u_k^j} \sum_{k=1}^{24} \sum_j C_{F,k}^j(u_k^j) + C_{S,k}^j(\mathrm{x}_k^j, u_k^j) + C_{V,k}^j(Q_k^j). \tag{2.24}
$$

Constraints are the intertemporal ones described in Section 2.4.1. The constraint of total output equaling given demand is intrinsically taken care of by the formulation and does not have to be explicitly formulated. This implies that there is enough supply capacity for the solution to exist.

The centralized unit commitment process requires a DP-Algorithm with the variable size of the state and decision spaces depending exponentially on the number of generators. In the decentralized commitment, each generator can be in 4 states and choose from at most 2 decisions at 24 stages. Calculating the total decision matrix for all *l* generators requires an amount of calculation proportional to

$$t_{calc} \sim (4 \cdot 2) \cdot l \cdot 24. \tag{2.25}$$

The centralized DP-Algorithm compares all possible states and decision combinations of all generators. E.g., in a calculation with 3 generators, the algorithm has to compare 4^3 states combined with 2^2 decisions. With *l* generators, the amount of calculation is proportional to

$$t_{calc} \sim (4 \cdot 2)^l \cdot 24. \tag{2.26}$$

Use DP-algorithm to calculate optimal cost-to-go matrix for each of 4^l states and 24 stages, considering 2^l choices at each stage
Calculate optimal state and decision matrices
Calculate actual market prices *P* determined by the generators that are scheduled during each hour
Calculate profits and outputs for each generator and total costs incurred by all generators to supply given demand

Table 2-2. Structure of program *mainISO.m*

2.5 Limits of the Model

The model will deliver unrealistic results for the decentralized case in the following scenario: high fixed costs, flat total supply function, similar generators. If the anticipated total market supply function is so flat that it leads to such small prices that no generator assumes being able to cover the fixed costs for being on, no generator schedules and the system breaks down. This result is a direct outcome of the assumption of linear marginal costs with no intercept. We will change this assumption in Subsection 2.7.3 where we consider marginal cost functions with positive intercept.

This scenario can be prevented by using different parameters for the generators. One can interpret this limit of the model as an indication of why a market with heterogeneous market participants works best by leading to less variability. The actions of participants are less correlated and hence tend to equal out.

The model is also limited with regard to the forecast methodology in general. Assuming that all other market participants enter into the auction at each hour does not contain any learning effects. Keeping this obvious limit in mind, the model nevertheless delivers interesting results.

2.6 Examples and Results

The model was run with different numbers of generators, different load patterns, and different marginal, fixed and switching costs. Most of the simulations behave like the first example printed in Appendix C. Centralized and decentralized unit commitment lead to the same generators scheduled at each hour and the same amounts of power traded at each hour. The interesting result of Example 2, however, violates the assumption that decentralized and centralized unit commitments result in the same outcome. We will later show that this result can be generalized to the unconstrained case, and that it is, therefore, not a result of the modeling process.

A qualitative explanation for the difference in example 2 is the fact that the given demand can be supplied at less overall cost if generator 3 is switched on during hours 13 and 14. Because of the resulting low prices, however, generator 3 would incur loss over the periods in which it is switched on. If it can choose, as in the decentralized case, generator 2 remains shut off and incurs no loss. Figure 2-1 shows the market supply curves for each of the generators and the market supply function if all generators are scheduled.

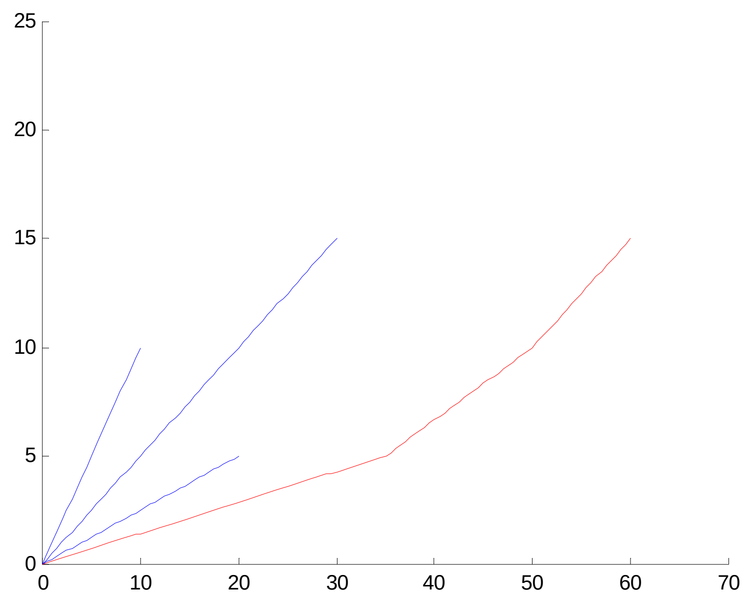

Figure 2-1. Example 2: individual and aggregate market supply functions

2.6.1 Analysis of Example 2

In order to compare the two results, we aggregate the market participants into groups and focus on key variables for the two markets. We regard as groups: consumers (welfare = -costs for electricity), generators 1+2 (profits and costs), generator 3 (profits and costs), and generator 1+2+3 (profits and costs). ISO and DC-markets differ only in hours 13 and 14. Table 2-3 shows the difference between centralized and decentralized optimization for these key variables:

W_{cons}	$G_{gen\ 1+2}$	$C_{gen\ 1+2}$	$G_{gen\ 3}$	$C_{gen\ 3}$	$G_{gen\ 1+2+3}$	$C_{gen\ 1+2+3}$
+144.3	-130.8	-70.82	-13.33	70.67	-144.1	-0.145

Table 2-3. Differences between centralized and decentralized outcomes, example 2

The consumers are obviously better off in the ISO-version with generator 3 switched on. More generators supplying demand results in a flatter supply function and hence in lower prices.

But the welfare of the consumers was not the primary target of ISO choosing this schedule. Lower prices result in lower costs for consumers, but also lower income for generators. The target of ISO is to minimize the costs necessary for supplying the demand. $C_{gen\ 1+2+3}$ is lower in the centralized version.

Generator 3 prefers not to be scheduled because it otherwise loses money. Generators 1+2 also prefer generator 3 to remain shut off, as they can then see their share in supply and their profit increased. An entity owning generators 1+2+3 also prefers generator 3 to remain shut off. If generators 2+3 belonged to the same entity, the owner would prefer generator 3 to be shut off even if the numbers were slightly different and generator 3 were making some profit when switched on. Another numerical example could easily be constructed.

2.7 Differences in the Outcomes Generalized

What follows is a mathematical investigation of when a centralized and a decentralized commitment process differ if intertemporal constraints are neglected. Demand is considered to be given, and we regard only one special hour. First, we again assume linear increasing marginal costs and fixed costs for being scheduled.

2.7.1 Aggregation into Two Generators

We will construct a situation in which an ISO would schedule a generator *g* for minimizing costs, but the generator would prefer not to be scheduled in order to prevent a loss. In the following derivation, subscript *g* stands for the special generator, *r* for the aggregation of other generators (rest), and *t* for the aggregation of all generators (total). Parameter *a* describes the coefficient for the linear marginal cost curves such as those shown in Figure 2-2. P_t and P_r are the market prices obtained respectively when generator g is scheduled or not. The slope of the total supply function a_t is calculated as

$$a_t = \frac{a_g a_r}{a_g + a_r}. \tag{2.27}$$

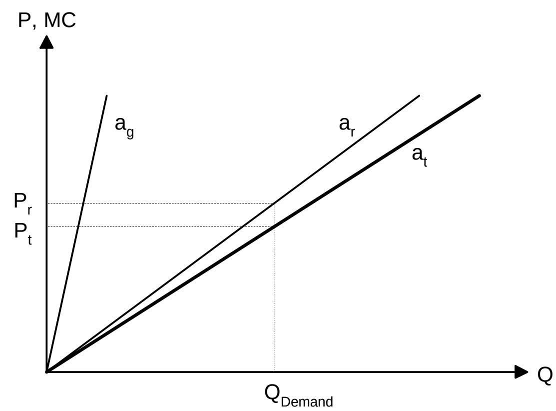

Figure 2-2. General example: market supply functions, aggregation in 2 generators

We introduce a new dimensionless parameter ν which describes the share of the power that is delivered by generator g in case it participates in the market. Basic economic theory holds that g will deliver energy proportionally to the inverse of its marginal cost slope:

$$\nu = \frac{\frac{1}{a_g}}{\frac{1}{a_t}} = \frac{a_r}{a_g + a_r}. \tag{2.28}$$

The situation we look for is created if two inequalities hold at the same time. First, generator g has to incur loss if scheduled, and second, the total cost necessary to supply given demand Q have to be smaller if generator g is scheduled. The first inequality holds if costs are larger than revenues for g:

$$\begin{gathered} C_{F,g} + a_g \frac{(Q\nu)^2}{2} > P_t(Q\nu) = (a_t Q)\cdot(Q\nu) \\ 2C_{F,g} > a_g (Q\nu)^2. \end{gathered} \tag{2.29}$$

The second inequality holds if the reduction in variable costs by scheduling *g* outweighs the additional fixed costs:

$$
\begin{aligned}
&C_{F,r} + a_r \frac{Q^2}{2} > C_{F,r} + C_{F,g} + a_t \frac{Q^2}{2} \\
&Q^2(a_r - a_t) > 2C_{F,g} \\
&Q^2(a_r - a_g \nu) > 2C_{F,g}.
\end{aligned}
\tag{2.30}
$$

Together these inequalities determine a possible range for the parameters:

$$
\begin{aligned}
&\nu^2 a_g < \frac{2C_{F,g}}{Q^2} < a_r - a_g \nu \\
&\boxed{\nu^2 < \frac{2C_{F,g}}{a_g Q^2} < \frac{\nu^2}{1-\nu}.}
\end{aligned}
\tag{2.31}
$$

The last inequality gives a straight-forward rule as to when the results of a decentralized and a centralized unit commitment process deviate from each other due to the differences in the optimization objectives of individual participants and the overall objectives. Neither reference to different levels of knowledge nor numerical rounding errors have to be used for explanation. However, the inequality also shows that the situation under scrutiny is a rather rare one. The parameter ν is an indicator as to what percentage of the total production capacity generator *g* holds. Under normal conditions, each generator should not hold more than a small share of the total market supply, which makes both the factor $(1-\nu)$ and hence, the range of values for which the effect can occur, rather small. Nevertheless, the above described discrepancy can happen and Table 2-1 shows several possible combinations of parameters which would lead to it.

ν	a_g	$C_{F,g}$	Q
1/11	0.9	0.01	1.45
1/11	0.9	1	19.2
1/11	2.9	180	145
1/11	3.09	0.1	14.5
1/3	4	0.01	0.4
1/3	1	1	4
1/3	1	100	40
1/3	0.6	9.1	4

Table 2-4. Numerical examples, aggregation in 2 generators

2.7.2 Aggregation into Three Generators

Next, we will construct a situation with 3 generators and more intricate issues.

- Generator 1 always wants to be on, independent of other generators' decisions to participate during the hour.
- Generators 2 and 3 do not want to be scheduled simultaneously. In other words, both lose money if the set of participating generators is {1,2,3}.
- Each of the two generators 2 and 3 would make profit during the hour, if the other one does not participate.
- ISO, pursuing its objective to minimize total costs necessary to supply given demand, would like to schedule all three generators.

Generator 1 can be seen as an aggregation of several generators that have small enough fixed costs to always want to be on for the given demand.

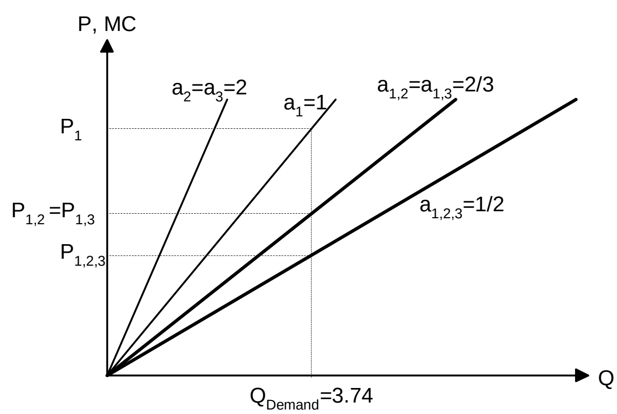

Figure 2-3. Market supply functions, aggregation in 3 generators

In order to establish the general case, four inequalities have to be fulfilled. We will gain insight and almost the same level of understanding by considering the special case in which generators 2 and 3 use the same technology, and hence have the same *MC* and C_F . In that case, the inequalities will reduce to two. We calculate:

$$
\begin{aligned}
a_3 &= a_2 \\
C_{F,3} &= C_{F,2} \\
a_{1,3} &= a_{1,2} = \frac{a_1 a_2}{a_1 + a_2} \\
a_{1,2,3} &= \frac{a_1 a_2}{2a_1 + a_2}.
\end{aligned}
\tag{2.32}
$$

ISO wants to schedule all three generators in order to minimize costs. Therefore, it must hold:

$$
C_{\{1,2,3\}} < C_{\{1,2\}}
\tag{2.33}
$$

and

$$
C_{\{1,2,3\}} < C_{\{1\}}.
\tag{2.34}
$$

The last inequality (2.34) can be shown to hold automatically if the previous one, (2.33), is fulfilled, which can be written as:

$$
\begin{aligned}
2C_{F,2} + \frac{a_1 a_2}{2a_1 + a_2}\frac{Q^2}{2} &< C_{F,2} + \frac{a_1 a_2}{a_1 + a_2}\frac{Q^2}{2} \\
\frac{2C_{F,2}}{Q^2} &< \frac{{a_1}^2 a_2}{(a_1 + a_2)(2a_1 + a_2)}.
\end{aligned}
\tag{2.35}
$$

Generators 2 and 3 lose money if all three generators are scheduled, but either generator 2 or 3 makes a profit, if the other one is not scheduled:

$$
\begin{aligned}
\pi_{\{1,2,3\},2} &< 0 \\
\pi_{\{1,2\},2} &> 0.
\end{aligned}
\tag{2.36}
$$

These two inequalities become respectively

$$\begin{aligned} \frac{1}{2a_2} Q^2 \frac{{a_1}^2 a_2}{\left(2a_1 + a_2\right)^2} &< C_{F,2} \\ \frac{{a_1}^2 a_2}{\left(2a_1 + a_2\right)^2} &< \frac{2C_{F,2}}{Q^2}, \end{aligned} \tag{2.37}$$

and

$$\begin{aligned} \frac{1}{2a_2} Q^2 \frac{{a_1}^2 a_2}{\left(a_1 + a_2\right)^2} &> C_{F,2} X_1, \ldots, X_n \\ \frac{{a_1}^2 a_2}{\left(a_1 + a_2\right)^2} &> \frac{2C_{F,2}}{Q^2}. \end{aligned} \tag{2.38}$$

The last inequality (2.38) can be shown to hold automatically if inequality (2.37) is fulfilled. We again introduce a dimensionless parameter ν describing the share of the power that is delivered by generator 2 in case it participates in the market:

$$\nu = \frac{\frac{1}{a_2}}{\frac{2a_1 + a_2}{a_1 a_2}} = \frac{a_1}{2a_1 + a_2}. \tag{2.39}$$

Inequalities (2.35) and (2.37) can now be combined to

$$\begin{aligned} \frac{{a_1}^2 a_2}{\left(2a_1 + a_2\right)^2} < \frac{2C_{F,2}}{Q^2} &< \frac{{a_1}^2 a_2}{\left(a_1 + a_2\right)\left(2a_1 + a_2\right)} \\ \nu a_2 < \frac{2C_{F,2}}{Q^2} &< \nu \frac{a_1}{a_1 + a_2} a_2. \end{aligned} \tag{2.40}$$

Together with

$$\frac{a_1}{a_1 + a_2} = \frac{\nu}{1 - \nu},$$

we retrieve the same formula as in the previous section:

$$\boxed{\nu^2 < \frac{2C_{F,2}}{a_2 Q^2} < \frac{\nu^2}{1-\nu}.} \tag{2.41}$$

Because of its definition, ν now has to be smaller than ½. Figure 2-4 and Table 2-5 show examples for numerical values for this situation.

ν	a_g	$C_{F,g}$	Q
1/11	0.9	0.01	1.45
1/11	0.9	1	19.2
1/11	2.9	180	145
1/11	3.09	0.1	14.5
1/3	4	0.01	0.4
1/3	1	1	4
1/3	1	100	40
1/3	0.6	9.1	4

Table 2-5. Numerical examples, aggregation into 3 generators

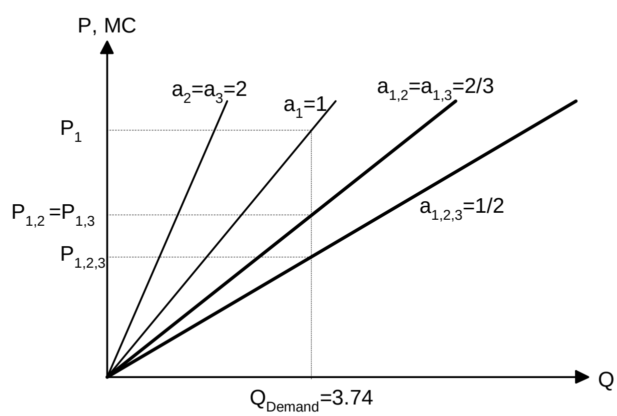

Figure 2-4. Numerical example: aggregation into 3 generators

2.7.3 Quadratic Cost Curves

In what follows we extend the previous results for two generators to the more realistic case with quadratic cost curves that include a linear term. We again consider fixed operating costs for being scheduled, but no intertemporal constraints. Demand is given for the hour. We will construct a situation with two generators in which the given demand can be satisfied at least cost with both generators scheduled, but in which one generator would prefer not to be scheduled because it would incur loss otherwise. A PoolCo-type market would schedule generator *2* to minimize costs and pay it the fixed operating costs to prevent it from loss. We assume generators bid true marginal cost curves.

2.7.3.1 Derivation

In the following derivation, subscript *2* stands for the special generator *g*, *1* for one or an aggregation of other generators. The total cost of each generator *i* is described by

$$C_i(Q_i) = a_i Q_i^2 + b_i Q_i + c_i. \tag{2.42}$$

The marginal costs are linear functions as shown in Figure 2-5.

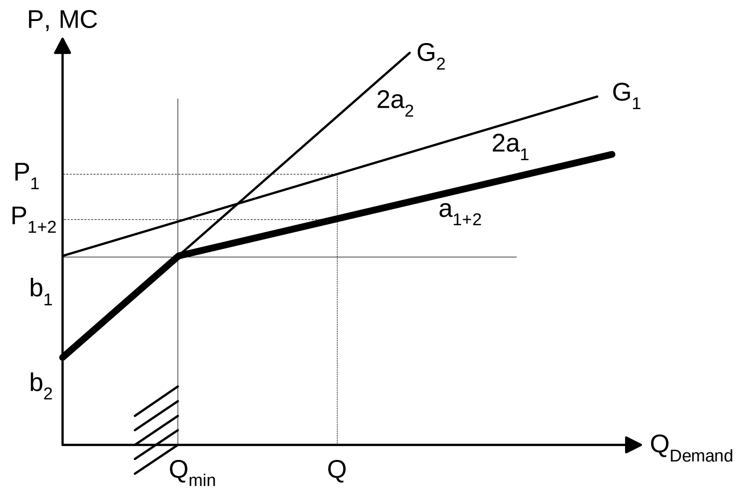

Figure 2-5. Generator supply functions

If both generators participate, the slope of the supply function becomes

$$a_{1+2} = \frac{2a_1 a_2}{a_1 + a_2}, \tag{2.43}$$

for

$$Q \geq Q_{\min} = \frac{b_1 - b_2}{2a_2}.$$

P_{1+2} and P_1 are the market prices obtained when generator *2* does or does not participate respectively with

$$
\begin{aligned}
P_{1+2} &= b_1 + a_{1+2}(Q - Q_{\min}) \\
&= \frac{a_1 b_2 + a_2 b_1}{a_1 + a_2} + \frac{2a_1 a_2}{a_1 + a_2} Q.
\end{aligned} \tag{2.44}
$$

Three conditions must hold simultaneously:

- Generator 1 always wants to be on, independent of generator 2's decisions to participate during the hour:

$$a_1 Q_1^{\ 2} + b_1 Q_1 + c_1 < P_{1+2} Q_1. \tag{2.45}$$

- Generator 2 incurs loss if it is scheduled and does not receive extra payment:

$$a_2 Q_2^{\ 2} + b_2 Q_2 + c_2 > P_{1+2} Q_2. \tag{2.46}$$

- The total cost for supplying the given demand is smaller if generators 1 and 2 are scheduled than if only generator 1 supplies electricity:

$$a_1 Q^2 + b_1 Q + c_1 > a_1 Q_1^{\ 2} + b_1 Q_1 + c_1 + a_2 Q_2^{\ 2} + b_2 Q_2 + c_2. \tag{2.47}$$

In addition, we know

$$Q_1 + Q_2 = Q, \tag{2.48}$$

and

$$2a_1 Q_1 + b_1 = 2a_2 Q_2 + b_2. \tag{2.49}$$

2.7.3.2 Generator 1

We introduce new dimensionless parameters $\nu_1 = \frac{Q_1}{Q}$ and $\nu_2 = \frac{Q_2}{Q}$. We write them as:

$$\nu_1 = \frac{Q_1}{Q} = \frac{P_{1+2} - b_1}{2a_1 Q}$$

$$= \frac{2a_2 Q - (b_1 - b_2)}{2(a_1 + a_2)Q} = \frac{a_2 - \dfrac{b_1 - b_2}{2Q}}{a_1 + a_2}, \tag{2.50}$$

and

$$\nu_2 = \frac{a_1 - \dfrac{b_2 - b_1}{2Q}}{a_1 + a_2}. \tag{2.51}$$

Using the formula for P_{1+2}, (2.44), inequality (2.45) can be written as

$$a_1 Q^2 \nu_1^{\,2} + b_1 Q \nu_1 + c_1 < \frac{a_1 b_2 + a_2 b_1}{a_1 + a_2} Q \nu_1 + \frac{2a_1 a_2}{a_1 + a_2} Q^2 \nu_1. \tag{2.52}$$

Further using the formula for ν_1, (2.50), we obtain after some transformations the condition

$$\boxed{Q > \frac{(a_1 + a_2)\sqrt{\dfrac{c_1}{a_1}} + \dfrac{b_1 - b_2}{2}}{a_2}.} \tag{2.53}$$

2.7.3.3 Generator 2

Using the formula for P_{1+2}, (2.44), we write inequality (2.46) as

$$a_2 Q^2 \nu_2^{\,2} + b_2 Q \nu_2 + c_2 > \frac{a_1 b_2 + a_2 b_1}{a_1 + a_2} Q \nu_2 + \frac{2a_1 a_2}{a_1 + a_2} Q^2 \nu_2, \tag{2.54}$$

and obtain after using the formula for ν_2 and some transformations:

$$
\boxed{Q < \frac{(a_1 + a_2)\sqrt{\frac{c_2}{a_2}} + \frac{b_2 - b_1}{2}}{a_1}.} \tag{2.55}
$$

2.7.3.4 Total Costs

We use the formula for v_1, (2.50), to rewrite inequality (2.47) as

$$
a_1 Q^2 + b_1 Q + c_1 > \left(a_1 v_1{}^2 + a_2(1 - v_1)^2\right) Q^2 + b_1 Q v_1 + b_2 Q(1 - v_1) + c_1 + c_2, \tag{2.56}
$$

and obtain after some transformations the inequality

$$
\boxed{Q > \frac{\sqrt{(a_1 + a_2)c_2} - \frac{b_1 - b_2}{2}}{a_1}.} \tag{2.57}
$$

2.7.3.5 Numerical Example

If all of the three above deduced inequalities hold, a centralized and a decentralized commitment would lead to different results. The following tables and graphs show a numerical example for this situation.

	G_1	G_2
a	1	2
b	1	1.6
c	1.1	0.7
Q	2	

Table 2-6. Parameters for quadratic cost curves example

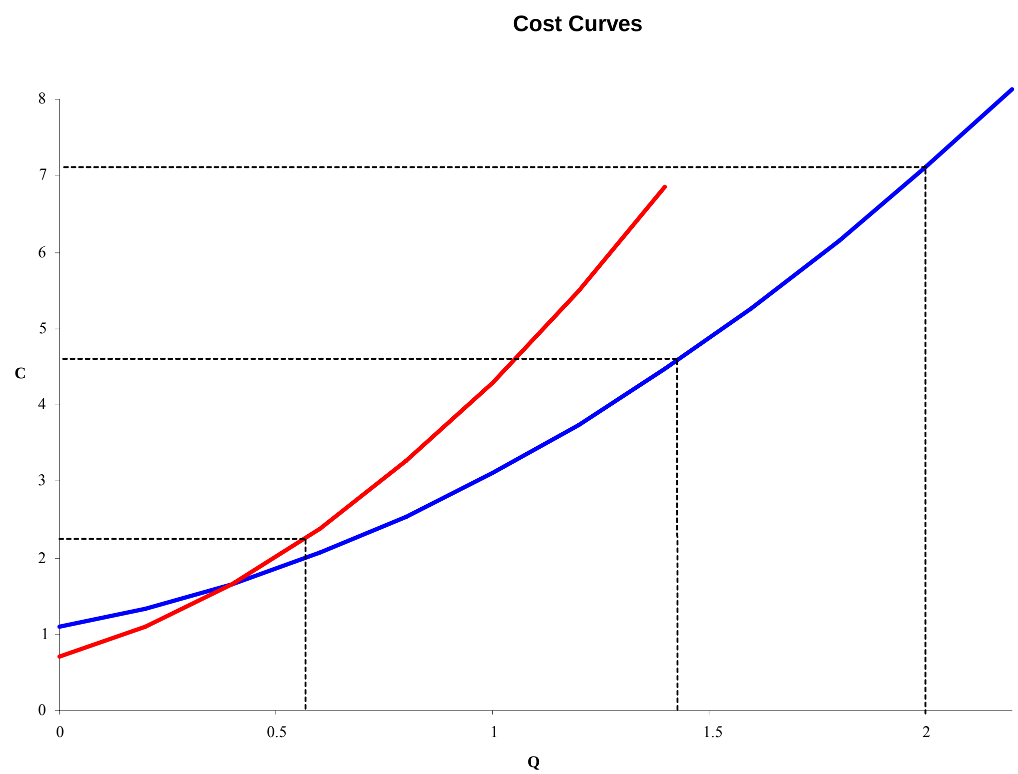

Figure 2-6. Quadratic cost curves of numerical example

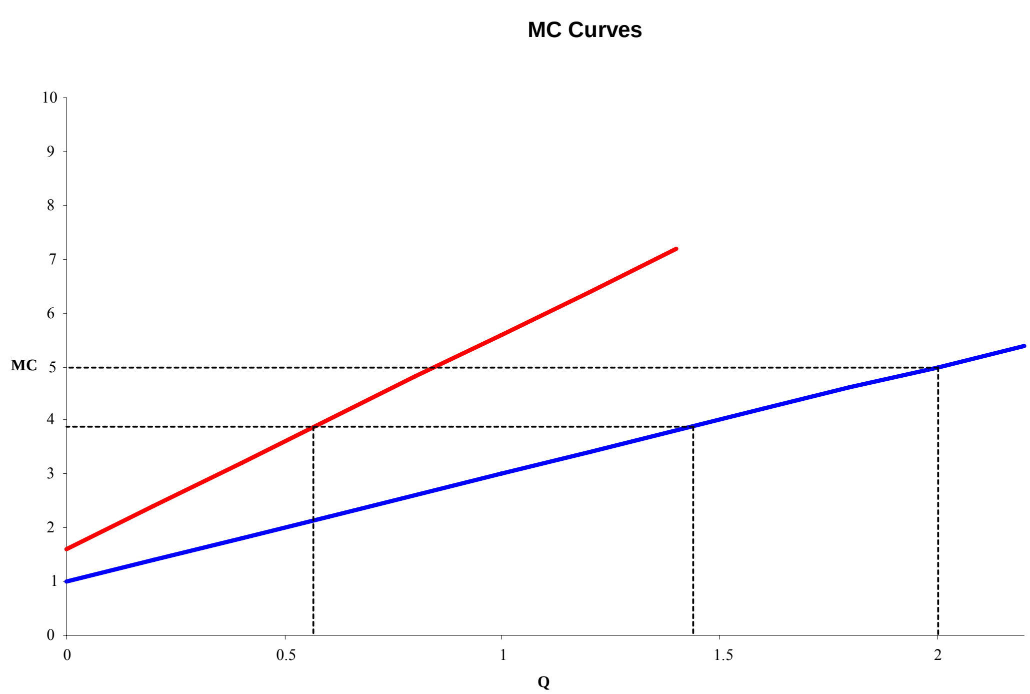

Figure 2-7. Market supply functions for quadratic cost curve example

	ver G_1	ver G_1 and G_2		
P	5	3.87		
	G_1	G_1+G_2	G_1	G_2
ν	100%	100%	72%	28%
C	7.1	6.84	4.59	2.25
Rev	10	7.73	5.54	2.19
π	2.9	0.9	0.95	-0.06

Table 2-7. Outcomes of quadratic cost curves example

2.8 Interpretation

The literature gives several examples of cases in which individual objective functions are not aligned with those of the overall societal welfare. The most famous example was given by Hardin in *The Tragedy of the Commons* [Hardin68]. Another one is Braess' article on traffic networks [Braess69] in which he gives an example in which drivers' attempt to minimize their transit times leads to increased congestion and increased traffic times for all participants. Braess' paradox has become an important issue in the context of queuing networks [Cohen90]. Wolpert and Tumer use the term COIN (Collective Intelligence) for their research in how to configure a system and assign private goals to individual agents, so that simple pursuit of them by the associated agents leads to a globally desirable solution [Wolpert99].

In power systems, however, the commonly held assumption is still that, at least in theory, a centralized and a decentralized commitment should lead to the same power quantities traded, and to the same optimal social welfare. The performance objectives of the individual market participants are considered equal to the one of minimizing total operating cost [Oren95], [Ilic98], [Christie01], [Day02].

The important implication of the examples given in this chapter is that, even in the absence of load uncertainties and intertemporal constraints, decentralized decision making, by which the market participants schedule themselves, need not necessarily lead to the same maximized welfare as centralized decision making. The reason is that, under certain circumstances, several generators can supply the load at a lower overall cost than the subset of generators that would make positive profits in a market setting if switched on during the hour.

In PJM, New York, the ISO offers a voluntary unit commitment service, based on three-part bids, allowing generators to bid actual operating costs more precisely and permitting a more efficient unit commitment. Generators may also self-schedule their own units, but they may also allow the ISO to determine the most economic unit commitment for their plants. Participating generators are guaranteed recovery of their start-up and minimum generation costs in the event they fail to recover these costs from the prices received in the ISO-coordinated markets [Chandley01], [O'Neill01]. This mechanism eliminates the uncertainty of whether a generator will be committed only to lose money, and it allows for a more efficient dispatch. The quadratic cost curve example shows how a PoolCo-type market would work more efficiently than a power exchange (for which the one-part bids result in some inefficiency).

The 3-generator example is slightly richer in modeling problems than the 2-generator one. From the perspective of generators 2 and 3, we can perceive the situation as that of a coordination game. If both schedule at the same time, they both lose money. If both refrain from scheduling, both lose the opportunity of making money, given the fact that the other generator did not participate. Trying to figure out an optimal strategy for each player, the reasoning process winds up in infinite regress as each of the players tries to anticipate the behavior of the other one, which itself tries to anticipate the first one's behavior and so on. Players have to play mixed strategies, randomize in particular ways (i.e., choose certain probabilities rather than others), and use outside information [Foss99], which frustrates any simple mathematical treatment and simulation of the problem. The "El Farol Bar" problem [Arthur94] has become a well-known example of how any model of such a problem that is shared by most of the agents is self-defeating. Recent simulations [Edmonds99] introduce heterogeneity into the models by using creative artificial agents. Evolutionary processes act upon a population of strategies inside each agent, so that after several runs, agents have taken on qualitatively different roles.

2.8.1 Arguments against Centralized Unit Commitment

It is important to state at this point, that the conclusions from this chapter focus on the short run, in that they do not take into account the long-term motivational effects of a decentralized commitment on investment decisions and the possible entry of new firms or generating plants. The literature gives several qualitative arguments why a decentralized commitment process might be preferable, despite the better overall efficiency of the centralized process.

According to [O'Neill01], a centralized unit commitment has the following problems in a competitive environment:

- Generators must submit start-up cost bids, as well as minimum run time, minimum down time, etc. This has lots of potential for strategic bidding, i.e. exercise of market power via technical means.

- The process lacks transparency, i.e. a generator cannot determine if the result of the unit commitment is "fair" with respect to the generator's bids.

- Because of the flat optimum in the unit commitment, major changes in the commitment pattern create only small changes in total cost. [Oren95]

- Because of the near-optimality in practical unit commitment solutions, two different programs, even with the same algorithm, but different "tuning" (e.g., how many states are retained), arrive at different solutions to the same problem. The objective function will have nearly the same value, but unit schedules can vary quite a bit. This can have major implications for the profitability of a generator.

According to [Tabors00], market participants complain of domination of decision-making by transmission owners or even ISO staff. As ISOs typically have weak or non-existent incentives to be responsive to market needs, embodying any system of checks and balances is often difficult, and ISOs have been accused of becoming self-perpetuating bureaucracies, with built-in incentives to block eventual evolution of the industry structure. Generation owners "argue that they can commit and dispatch their resources more efficiently than a central system with bidding rules." [Kirby99] states that because the fundamental goal of restructuring is to replace the highly regulated, centrally optimized and controlled system with one that is primarily market-based, generators can assume the responsibility (and associated financial risk) of performing the unit commitment function and deciding when to turn on and off.

Another argument against a centralized unit commitment is the computational complexity of solving a centralized unit commitment for a day-ahead market. The difference in computer power needed is shown by equations (2.25) and (2.26). According to [Tabors00], these complex systems depend on software programs whose development has in several cases taken years and cost hundreds of millions of dollars. Also, it is difficult to determine the actual optimum, since several nearly as efficient optima might exist.

Finally, the actual costs of generators are highly sensitive data. In order to fulfill its role, the PoolCo authority would have to know the exact cost structures of all power plants. Principal-agent problems might arise in this context, and it is still not clear how willing generators are to transfer this knowledge to a third entity and to give up decentralization in return for vast simplification.

3 Dynamic Programming Algorithms for Optimal Unit Scheduling

3.1 Overview

In a regulated market, a power generating utility solves the unit commitment problem by minimizing the production costs to supply the electricity demanded by its customers. The power generating utility has no option but to reliably supply the prevailing load. The price of electricity over this period is unchanging and, therefore, the decisions on the operation of the units have no effect on the firm's revenues.

Under deregulation, the unit commitment problem for an electric power producer includes the electricity market in the model, since the spot price of electricity is no longer predetermined but highly volatile. Modeling of the spot prices becomes very important as the expected revenue of the generator depends on the accuracy with which the stochastic nature of the price volatilities can be incorporated into the commitment decisions.

We investigate an electricity producer's problem of optimally scheduling its unit day-ahead for an hourly market, taking into account intertemporal operating constraints of the unit. We will use a day-ahead forecast of hourly prices and a probability density function (PDF) of forecast errors to model the market clearing price (MCP) as a random variable instead of a predicted value and, thereby, incorporate the MCP uncertainties and bidding risks into the optimization algorithm.

We develop price models and optimization algorithms that take into account the variance of forecast errors and correlation between successive hours. Using historical data from the ISO New England wholesale electricity market, we show how parameters can be statistically estimated, and then apply the algorithms to data previously not looked at.

Knittel and Roberts conclude in their empirical investigation of California electricity prices [Knittel01] that large values of higher-order moments relative to a Gaussian distribution render price models based on Normality assumptions of questionable use in representing price behavior. Relaxing the Normality assumption should aid in better representing the stochastic properties of prices. We will therefore compare models that use truncated Normal distributions with fat-tail models that assume that price deviations behave according to truncated Cauchy distributions.

A short overview of different approaches in the literature for stochastic unit commitment can be found in [Valenzuela01]: [Takriti97] introduces a stochastic model in which the uncertainty in the load and prices of fuel and electricity are modeled using a set of possible scenarios. [Allen99] proposes a stochastic model for unit commitment, assuming that the hourly prices at which electricity is sold are uncorrelated and normally distributed. [Tseng99] uses Ito processes to model the prices of electricity and fuel in the unit commitment formulation. [Valenzuela01] assumes the market-clearing price of electricity to be the variable cost of the marginal unit and models the aggregate load as a Gauss-Markov process.

3.2 Price Process

The ISO New England electricity wholesale market opened on May 1, 1999. Each evening, ISO New England posts an hourly price forecast for electricity prices of the next day on its web page [ISO-NE-www]. We assume these forecasts to be the best available to a generator before making a unit commitment decision for the next day.

A generator, which has to make a unit commitment decision about when to turn its unit on or off during the next day, will optimize its decision with regard to these forecasts. In what follows, we will improve the simple deterministic unit commitment by trying to model the dynamics between the historic hourly forecast prices $\widehat{p_k}$ and actual electricity prices p_k.

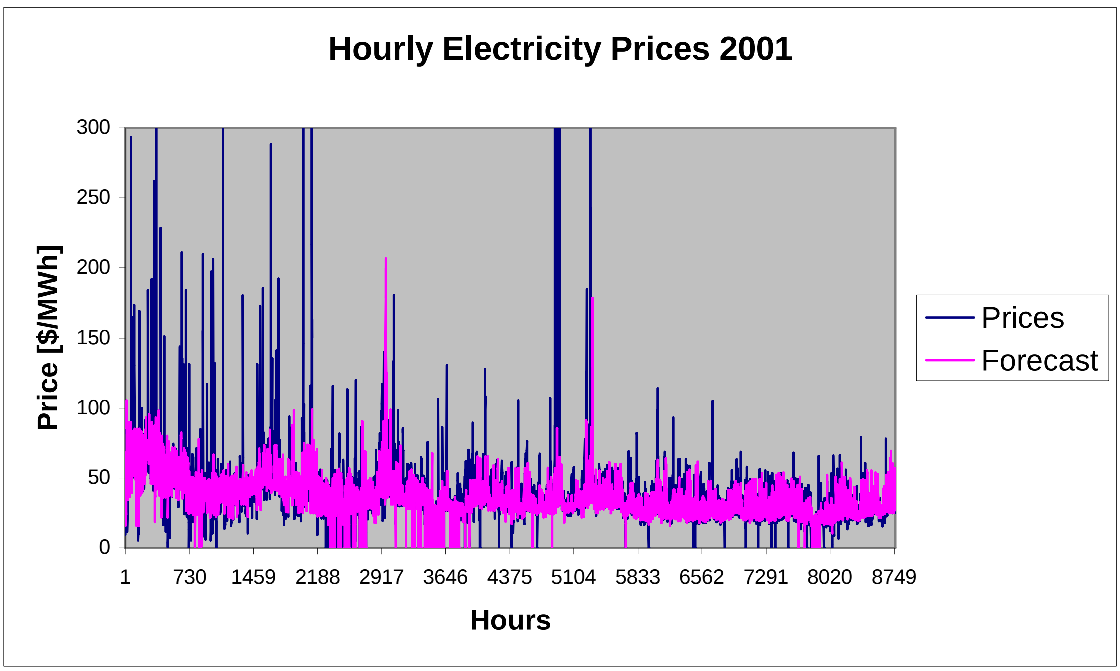

Figure 3-1. Prices and forecast prices: 2001

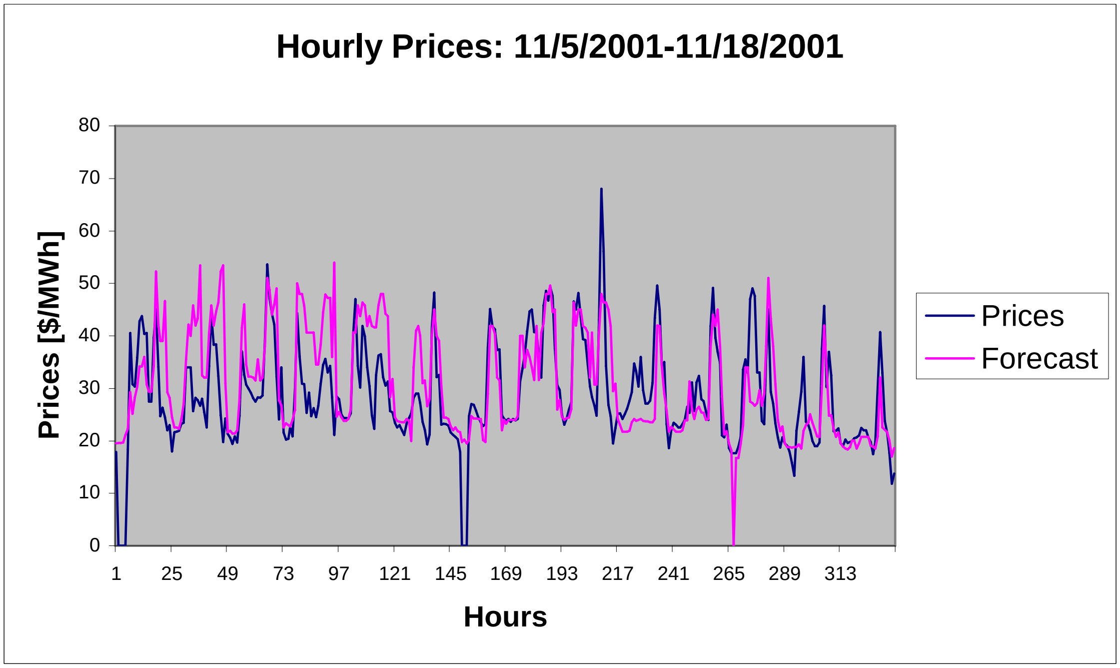

Figure 3-2. Prices and forecast prices: 2 weeks in Nov. 2001

3.2.1 Independent Random Variables

A logical first step is to assume that the hourly forecast errors behave like independent random variables. We can write

$$p_k = \widehat{p_k} + e_k \tag{3.1}$$

or

$$\Delta p_k = p_k - \widehat{p_k} = e_k, \tag{3.2}$$

with e_k being independent of e_u for any period $u \neq k$, and following an assumed probability distribution.

Sample Data	Mean $\bar{e}$	Median $\tilde{e}$	Variance $\hat{\sigma}^2$
01/01/2001-12/31/2001	2.94	0.25	1757
09/01/2001-12/31/2001	-0.243	-0.125	59.8
11/08/2001-11/26/2001	-2.10	-0.585	57.9

Table 3-1. Test statistics for forecast errors for sample periods

Figure 3-1 and Figure 3-2 show that, especially during the strong winter 2001, prices regularly reached unexpected levels and created the huge sample variance for the whole year in Table 3-1. With new power plants being built and the reserve margin becoming larger, we use the sample from September to December 2001 for finding appropriate distributions for future calculations.

3.2.1.1 Normal Distribution

Most commodity, asset and electricity price models use the normal probability distribution for simulations. It is defined as

$$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} \cdot e^{-\frac{x^2}{2\sigma^2}}, \tag{3.3}$$

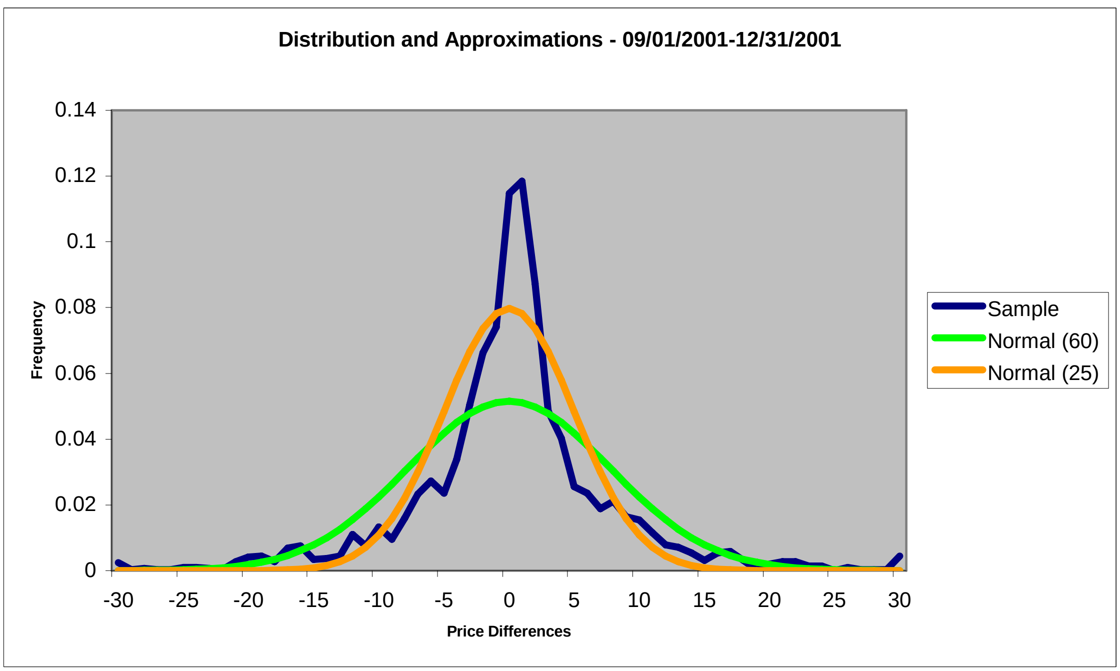

Figure 3-3. Gaussian approximations for the forecast error distribution

In order to run the algorithms explained in Section 3.4, the continuous distribution of Δp_k has to be discretized. The interval between two values Δp_i will be denoted as *d* and the number of different intervals or values *s*, with *s* being an odd number. The discretized probability distribution of Δp_k is then defined as $\mathrm{p_i} = \mathrm{p}\left(\left| \Delta p_i - \Delta p_k \right| < \frac{d}{2} \right)$, with *i* the index $\in \{1,2,...s\}$, and $\Delta p_i = d \cdot \left(i - \frac{s+1}{2} \right)$. Values outside the range $\left[-d \cdot \frac{s+1}{2},\ d \cdot \frac{s+1}{2} \right]$ are possible but considered to have negligible impact on the optimal decision.

For the discretized normal random distribution with variance σ we calculate

$$\mathrm{p_i} = \Phi\left(\frac{\left(\Delta p_i - \frac{d}{2} \right)}{\sigma} \right) - \Phi\left(\frac{\left(\Delta p_i + \frac{d}{2} \right)}{\sigma} \right) = \Phi\left(\frac{d\left(i - \frac{s}{2} \right)}{\sigma} \right) - \Phi\left(\frac{d\left(i - \frac{s+2}{2} \right)}{\sigma} \right) \quad \text{for i} \notin \{1,\mathrm{s}\}, \tag{3.4}$$

and

$$\begin{aligned} \mathrm{p_1} &= \Phi\left(\frac{\left(\Delta p_i - \frac{d}{2} \right)}{\sigma} \right) = \Phi\left(\frac{d\left(i - \frac{s}{2} \right)}{\sigma} \right) \\ \mathrm{p_s} &= 1 - \Phi\left(\frac{\left(\Delta p_s + \frac{d}{2} \right)}{\sigma} \right) = 1 - -\Phi\left(\frac{d\left(i - \frac{s+2}{2} \right)}{\sigma} \right). \end{aligned} \tag{3.5}$$

Table 3-2 shows typical numerical values, that we will use for the optimization in a later section.

Sample Data	d	s	$\hat{\sigma}^{2\prime}$	$\hat{\sigma}^{2\prime\prime}$
09/01/2001-12/31/2001	1	61	59.8	25

Table 3-2. Normal approximation for period 09/01/2001-12/31/2001

3.2.1.2 Cauchy Distribution

The superimposed Gaussian distributions do not seem to fit the sample well as they do not give enough credit to the heavy tails. Looking for a more representative distribution, we use a variant of the Cauchy distribution, which is generally defined as

$$f(x) = \frac{1}{m\pi} \cdot \frac{1}{1 + \left(\frac{x}{m}\right)^2}, \tag{3.6}$$

with *m* being the scale parameter. The expected value, variance and higher moments are undefined since the corresponding integrals diverge. We use a truncated distribution that is discretized in *s* values. In contrast to the Normal distribution, the integral of the densities outside of the specified range do represent a non-negligible part, which is why we have to normalize the probabilities:

$$p_i' = \frac{1}{1 + \left(\frac{x}{m}\right)^2}, \tag{3.7}$$

and normalizing factor *f*:

$$f = \frac{1}{\sum_i^s p_i'}. \tag{3.8}$$

The discrete probabilities then become:

$$p_i = f \cdot p_i'. \tag{3.9}$$

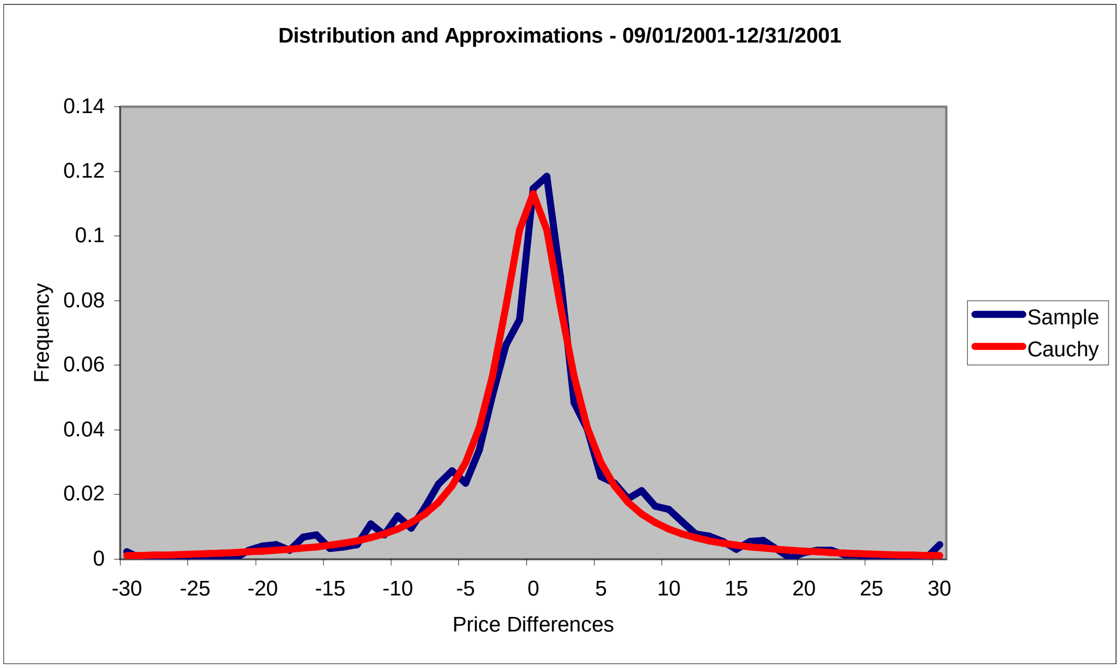

Figure 3-4. Cauchy approximation for the forecast error distribution

Figure 3-4 shows the approximation of the density function by the Cauchy distribution with parameters from Table 3-3. It seems to approximate the forecast error distribution to a better extend than the Gaussian distribution.

Sample Data	s	$\widehat{m}$	$\widehat{f}$	$\widehat{\sigma_c^2}$
09/01/2001-12/31/2001	61	3	0.883	53.1

Table 3-3. Cauchy approximation for period 09/01/2001-12/31/2001

An alternative approach for discretizing the Cauchy distribution is to use its cumulative density function

$$F(x) = 0.5 + \frac{1}{\pi} \cdot \arctan\left(\frac{x}{m}\right), \tag{3.10}$$

and assure the normalization by appropriately defining p_1 and p_s. We again define *d* as the interval between two values Δp_i, and *s* as the number of different intervals. For the discretized Cauchy distribution with parameter *s* we calculate

$$\mathrm{p_i} = F\left(\Delta p_i - \frac{d}{2}\right) - F\left(\Delta p_i + \frac{d}{2}\right) = F\left(d\left(i - \frac{s}{2}\right)\right) - F\left(d\left(i + \frac{s}{2}\right)\right) \text{ for i} \notin \{1,\mathrm{s}\}, \tag{3.11}$$

and

$$\begin{aligned} \mathrm{p_1} &= F\left(\Delta p_i - \frac{d}{2}\right) = F\left(d\left(i - \frac{s}{2}\right)\right) \\ \mathrm{p_s} &= 1 - F\left(\Delta p_i + \frac{d}{2}\right) = 1 - F\left(d\left(i + \frac{s}{2}\right)\right). \end{aligned} \tag{3.12}$$

3.2.2 Correlated Random Variables

It is reasonable to assume some correlation in the forecast errors between different hours of a day. Further making the assumption that the mean of the errors should actually be zero, the price process is characterized by a zero-mean-reverting process:

$$\Delta p_k = \alpha \cdot \Delta p_{k-1} + \varepsilon_k. \tag{3.13}$$

Under this assumption, the prices p_k behave according to a first-order autoregressive moving-average with exogenous input (ARMAX) discrete time model:

$$p_k = \widehat{p_k} + \alpha \cdot \left(p_{k-1} - \widehat{p_{k-1}}\right) + \varepsilon_k. \tag{3.14}$$

The coefficient α in the linear regression $\Delta p_k = \alpha \cdot \Delta p_{k-1} + \varepsilon_k$ for minimizing mean square error of ε_k is equal to the correlation between the forecast errors of different hours

$$\delta = \frac{Cov(e_k, e_{k-1})}{\sigma}. \tag{3.15}$$

Given a sample of *n* pairs of prices and price forecasts, the sample variance $\widehat{Cov}(e_k, e_{k-1})$ can be calculated as

$$\widehat{Cov}(e_k, e_{k-1}) = \frac{\sum_{i=2}^{n} e_i e_{i-1}}{n-2} \tag{3.16}$$

and the mean square error or residual errors of the model ε_k, not explained by correlation:

$$\sigma_\varepsilon^2 = \mathrm{E}\left\{\left(\Delta p_k - \delta \cdot \Delta p_{k-1}\right)^2\right\} = \sigma^2\left(1-\delta^2\right). \tag{3.17}$$

Sample Data	$\widehat{Cov}(e_k, e_{k-1})$	σ_ε^2
09/01/2001-12/31/2001	0.705	42.5

Table 3-4. Correlation parameters for period 09/01/2001-12/31/2001

Given the forecast deviation of a particular hour *k*, the model predicts expected future deviations according to:

$$\widehat{\Delta p_{k+n}} = \delta^n \cdot \Delta p_k. \tag{3.18}$$

Figure 3-5 shows the forecast deviations for 11/16/2001 and the superimposed expected deviations from the perspective of hours 3 and 12.

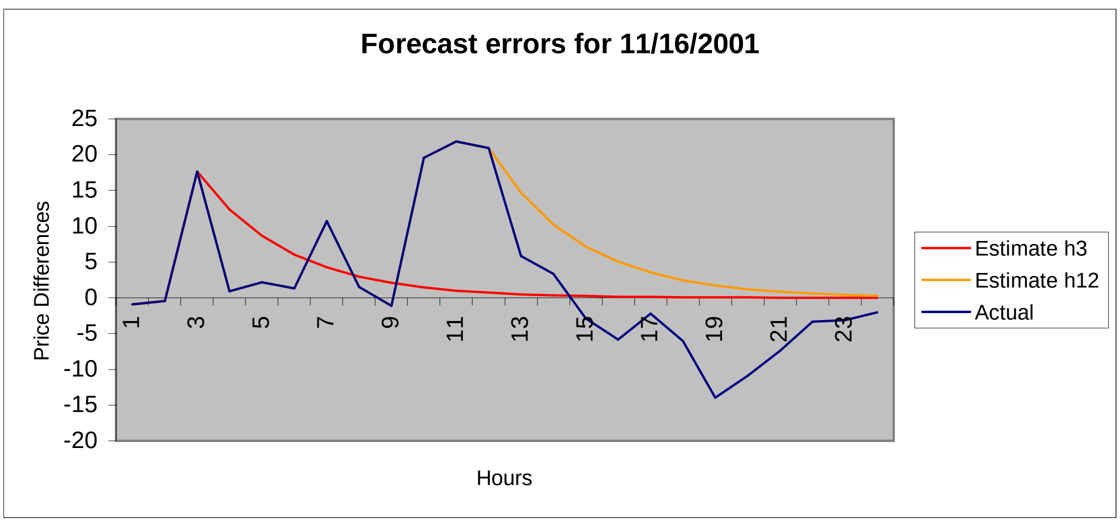

Figure 3-5. Intraday forecast errors

Like in Section 3.2.1, we will approximate the residual error with a random variable with either a Normal or a Cauchy Distribution.

3.2.2.1 *Normal Distribution*

Figure 3-6 shows the histogram for the residual forecast errors ε_k of our model and approximated normal random distribution with the parameters from Table 3-5.

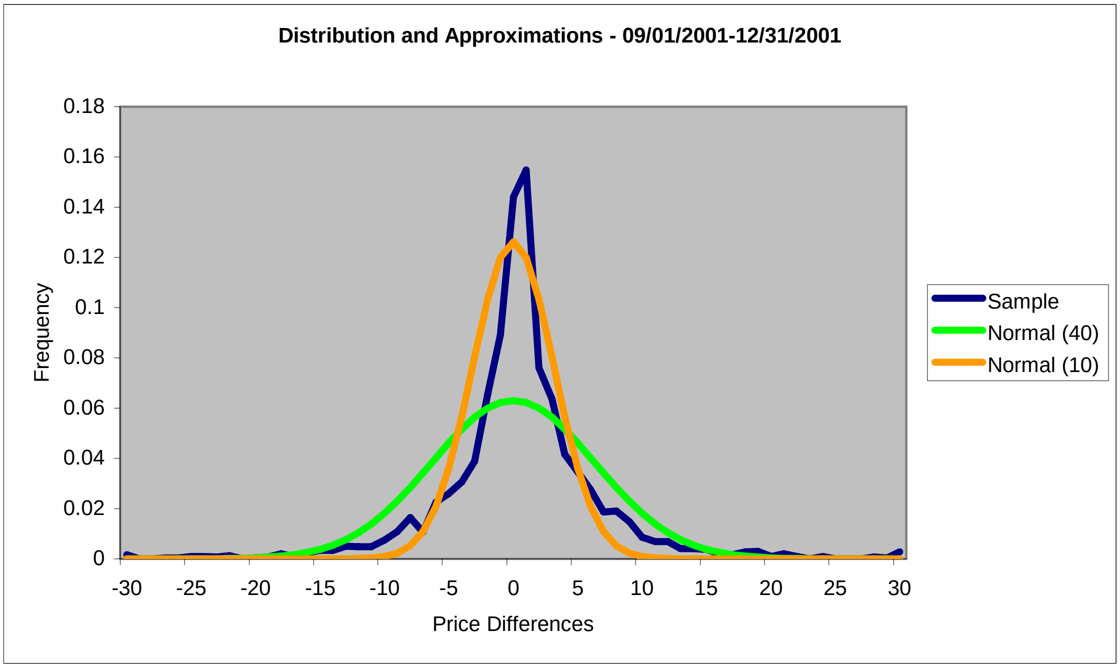

Figure 3-6. Gaussian approximations for the residual forecast error distribution

Sample Data	$\hat{\sigma}_\varepsilon^{2\prime}$	$\hat{\sigma}_\varepsilon^{2\prime\prime}$
09/01/2001-12/31/2001	42.5	10

Table 3-5. Normal approximation for period 09/01/2001-12/31/2001, correlation

We approximate the continuous probability distribution of $\varepsilon_k = \Delta p_k - \overline{\Delta p_k}$ with a normal distribution with 0 mean, and discretize with an odd number *s* intervals of size *d*. With $\overline{\Delta p_k} = \delta \cdot \Delta p_{k-1}$, we then write

$$\mathrm{p_{i,j}} = \mathrm{p}\left(\left| \Delta p_i - \Delta p_k \right| < \frac{d}{2} \;\middle|\; \left| \Delta p_j - \Delta p_{k-1} \right| < \frac{d}{2} \right) \tag{3.19}$$

for the conditional probability distribution of the forecast deviations for this hour based on the difference in the last hour. We have $\Delta p_i = d \cdot \left(i - \frac{s+1}{2} \right)$, and *i* and *j* as the indexes $\in \{1,2,...s\}$. Given the assumed variance σ_ε, we can write

$$
\begin{aligned}
\mathrm{p}_{\mathrm{i,j}} &= \Phi\left(\frac{\left(\Delta p_i - \frac{d}{2} \right) - \delta \cdot \Delta p_j}{\sigma} \right) - \Phi\left(\frac{\left(\Delta p_i + \frac{d}{2} \right) - \delta \cdot \Delta p_j}{\sigma} \right) \\
&= \Phi\left(\frac{d\left(i - \frac{s}{2} \right) - \delta \cdot d\left(j - \frac{s+1}{2} \right)}{\sigma} \right) - \Phi\left(\frac{d\left(i + \frac{s}{2} \right) - \delta \cdot d\left(j - \frac{s+1}{2} \right)}{\sigma} \right) \quad \text{, for i} \notin \{1,\mathrm{s}\}
\end{aligned}
\tag{3.20}
$$

and

$$
\mathrm{p}_{1,\mathrm{j}} = \Phi\left(\frac{\left(\Delta p_i - \frac{d}{2} \right) - \delta \cdot \Delta p_j}{\sigma} \right) = \Phi\left(\frac{d\left(i - \frac{s}{2} \right) - \delta \cdot d\left(j - \frac{s+1}{2} \right)}{\sigma} \right)
$$

$$
\mathrm{p}_{\mathrm{s,j}} = 1 - \Phi\left(\frac{\left(\Delta p_s + \frac{d}{2} \right) - \delta \cdot \Delta p_j}{\sigma} \right) = 1 - \Phi\left(\frac{d\left(i - \frac{s+2}{2} \right) - \delta \cdot d\left(j - \frac{s+1}{2} \right)}{\sigma} \right). \tag{3.21}
$$

3.2.2.2 Cauchy Distribution

We again formulate Δp_k evolving according to a conditional probability distribution approximated by a Cauchy distribution, before we discretize for the computer calculations in the next section.

Figure 3-7 shows the histogram for the residual forecast errors ε_k of our mean-reverting model and an approximated Cauchy distribution with the parameters from Table 3-6, as defined in Section 3.2.1.2.

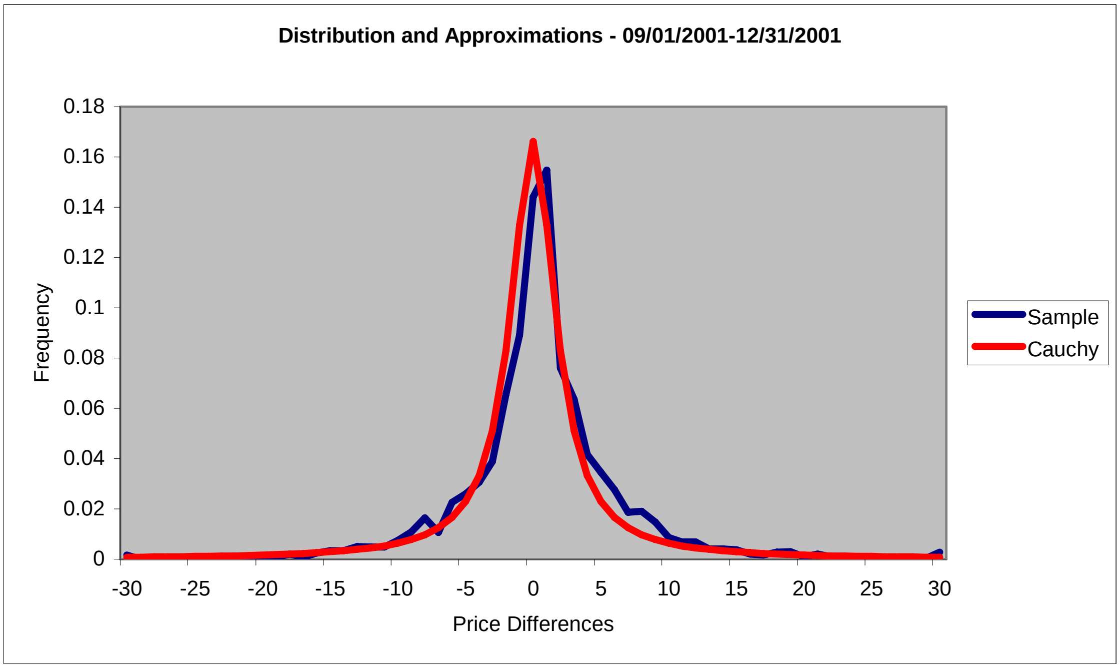

Figure 3-7. Cauchy approximations for the residual forecast error distribution

Sample Data	n	$\widehat{m_\varepsilon}$	$\widehat{f_\varepsilon}$	$\sigma^2_{\varepsilon,c}$
09/01/2001-12/31/2001	61	2	6.02	36.5

Table 3-6. Cauchy approximation for period 09/01/2001-12/31/2001, correlation

The cumulative Cauchy distribution for ε_k and Δp_k is:

$$F_{\varepsilon_k}(x) = 0.5 + \frac{1}{\pi} \cdot \arctan\left(\frac{x}{m}\right), \tag{3.22}$$

and becomes discretized:

$$F_{\Delta p_k}(x) = 0.5 + \frac{1}{\pi} \cdot \arctan\left(\frac{x - \overline{\Delta p_k}}{m}\right). \tag{3.23}$$

Using the definitions from Section 3.2.2.1, we can write:

$$
\begin{aligned}
\mathrm{p}_{\mathrm{i,j}} &= F\left(\left(\Delta p_i - \frac{d}{2}\right) - \delta \cdot \Delta p_j\right) - F\left(\left(\Delta p_i + \frac{d}{2}\right) - \delta \cdot \Delta p_j\right) \\
&= F\left(d\left(i - \frac{s}{2}\right) - \delta \cdot d\left(j - \frac{s+1}{2}\right)\right) - F\left(d\left(i + \frac{s}{2}\right) - \delta \cdot d\left(j - \frac{s+1}{2}\right)\right) \quad \text{, for i} \notin \{1,\mathrm{s}\}
\end{aligned} \tag{3.24}
$$

and

$$
\begin{aligned}
\mathrm{p}_{1,\mathrm{j}} &= F\left(\left(\Delta p_i - \frac{d}{2}\right) - \delta \cdot \Delta p_j\right) = F\left(d\left(i - \frac{s}{2}\right) - \delta \cdot d\left(j - \frac{s+1}{2}\right)\right) \\
\mathrm{p}_{\mathrm{s,j}} &= 1 - F\left(\left(\Delta p_i + \frac{d}{2}\right) - \delta \cdot \Delta p_j\right) = 1 - F\left(d\left(i + \frac{s}{2}\right) - \delta \cdot d\left(j - \frac{s+1}{2}\right)\right).
\end{aligned} \tag{3.25}
$$

3.3 The Model of the Generator

We assume as only intertemporal constraints on the generator the fact that once turned on/off, it has to remain turned on/ off for at least t_{up} / t_{down} hours. Each hour, the generator can therefore be in one of $t_{up} + t_{down}$ states. In a variant of the formulation in [Shaw95], we define the state in such a way, that it refers to the number of hours the unit has been on ($1 \le x_k \le t_{up}$) or off ($t_{up} + 1 \le x_k \le t_{up} + t_{down}$) at the beginning of time period *k* with the addition that the state does not change, once the unit has been on/ off for t_{up} / t_{down} hours:

$$
x_k = \begin{cases}
1 & \text{if } \textit{shut on for exactly } 1 \textit{ hour at the end of hour } k\text{-}1 \\
 & \vdots \\
t_{up} & \text{if } \textit{shut on for at least } t_{up} \textit{ hours at the end of hour } k\text{-}1 \\
t_{up}+1 & \text{if } \textit{shut off for exactly } 1 \textit{ hour at the end of hour } k\text{-}1 \\
 & \vdots \\
t_{up} + t_{down} & \text{if } \textit{shut off for at least } t_{down} \textit{ hours at the end of hour } k\text{-}1.
\end{cases} \tag{3.26}
$$

The on/off constraints on the ability to turn on or off for a specific hour *k* can be formulated as a constrained control space U_k at each stage [Bertsekas00]:

$$U_k(x_k) = \begin{cases} \{0\} & \text{if } t_{up} + 1 \le x_k < t_{up} + t_{down} \\ \{1\} & \text{if } 1 \le x_k < t_{up} \\ \{0,1\} & \text{if } x_k \in \{t_{up}, t_{up} + t_{down},\}. \end{cases} \tag{3.27}$$

The state of the generator evolves according to the state equation:

$$x_{k+1}=\mathrm{f}(x_k, u_k) = \begin{cases} 1 & \text{if } (x_k = t_{up} + t_{down} \wedge u_k = 1) \\ t_{up} + t_{down} & \text{if } (x_k = t_{up} + t_{down} \wedge u_k = 0) \\ t_{up} + 1 & \text{if } (x_k = t_{up} \wedge u_k = 0) \\ t_{up} & \text{if } (x_k = t_{up} \wedge u_k = 1) \\ x_k + 1 & \text{else.} \end{cases} \tag{3.28}$$

We use a quadratic function to model the cost function of the generator:

$$C(Q_k) = aQ_k^{\,2} + bQ_k + c. \tag{3.29}$$

If the generator is on during hour *k* with price p_k then the output variable Q_k is set to maximize the variable profit

$$\pi_k'(Q_k) = p_k Q_k - C(Q_k). \tag{3.30}$$

Taking the derivate of profit with respect to Q_k leads to

$$\frac{\delta \pi_k'(Q_k)}{\delta Q_k} = p_k - 2aQ_k - b. \tag{3.31}$$

Setting this derivative to zero, we find the value of Q_k which maximizes profits for stage *k*:

$$Q_k{}' = \frac{p_k - b}{2a}. \tag{3.32}$$

If the generator has minimum and maximum operating constraints $Q_{\min}$ and $Q_{\max}$, then the optimal amount of power produced is

$$Q_k'' = \min[\max[Q_k', Q_{\min}], Q_{\max}]. \tag{3.33}$$

Each time the generator starts up or shuts down, it incurs costs S or T:

$$C_{S,k}(\mathrm{x}_k, u_k) = \begin{cases} \mathrm{S} & \text{if } (x_k = t_{up} + t_{down} \wedge u_k = 1) \\ \mathrm{T} & \text{if } (x_k = t_{up} \wedge u_k = 0) \\ 0 & \text{else.} \end{cases} \tag{3.34}$$

Defining

$$\pi_k''(p_k, u_k) = \begin{cases} p_k Q_k' - C(Q_k') & \text{if } u_k = 1 \\ 0 & \text{if } u_k = 0, \end{cases} \tag{3.35}$$

we can write the total profit for each hour as:

$$\pi_k(\mathrm{x}_k, u_k, p_k) = \pi_k''(u_k, p_k) - C_{S,k}(\mathrm{x}_k, u_k). \tag{3.36}$$

3.4 Optimization Algorithms for Day-Ahead Decisions

The objective of this section is to formulate three approaches for calculating the optimal unit commitment decisions in a day-ahead market. The inputs are price forecasts and knowledge of the pattern by which price forecasts deviate from the actual price. In this formulation, the producer makes a unit commitment decision before deciding how much power to sell in the market.

The first algorithm just uses the price forecasts and optimizes the generator's scheduling strategy in a deterministic fashion. The second algorithm takes the observed past errors between forecasts and actual prices into account, and the third algorithm uses the observed correlations between subsequent hours in addition.

3.4.1 Deterministic DP - Forecast Prices

The first approach optimizes only with regard to the forecast prices $\widehat{p_k}$ for each hour. The prices are assumed to be the real-time prices and a deterministic version of dynamic programming can be formulated as

$$\begin{aligned} J_k(\mathrm{x}_k) &= \max_{u_k \in U_k} \left[\pi(\mathrm{x}_k, u_k, \widehat{p_k}) + J_{k+1}\left(\mathrm{f}(\mathrm{x}_k, u_k)\right) \right] \\ J_N(\mathrm{x}_N) &= 0, \end{aligned} \tag{3.37}$$

subject to the state equation (3.28).

3.4.2 Stochastic DP - Forecast Prices and Variance

In Section 3.2, we have observed that the deviations between forecast prices and real-time prices $\Delta p_k = p_k - \widehat{p_k}$ can be approximated by different probability distributions. Given this additional assumption, we can formulate a stochastic dynamic programming algorithm in the attempt to improve our expected daily profit:

$$\begin{aligned} J_k(\mathrm{x}_k) &= \max_{u_k \in U_k} \left[\mathop{\mathrm{E}}_{\Delta p_k} \left\{ \pi(\mathrm{x}_k, u_k, \widehat{p_k} + \Delta p_k) \right\} + J_{k+1}\left(\mathrm{f}(\mathrm{x}_k, u_k)\right) \right] \\ J_N(\mathrm{x}_N) &= 0. \end{aligned} \tag{3.38}$$

In order to use the algorithm, the continuous distribution of Δp_k has to be discretized. The interval between two values will be denoted as *d* and the number of different intervals or values Δp_k as *s*. Values outside this range are possible, but are considered to have negligible impact on the optimal decision. We can now write:

$$J_k(\mathrm{x}_k) = \max_{u_k \in U_k} \left[\left(\sum_{\Delta p_i} \mathrm{p}\left(\Delta p_k = \Delta p_i\right) \cdot \left(\pi(\mathrm{x}_k, u_k, \widehat{p_k} + \Delta p_i) \right) \right) + J_{k+1}\left(\mathrm{f}(\mathrm{x}_k, u_k)\right) \right], \tag{3.39}$$

or, with regard to the computer implementation,

$$J_k(\mathrm{x}_k) = \max_{u_k \in U_k} \left[\sum_i \mathrm{p_i} \cdot \left(\pi\left(\mathrm{x}_k, u_k, \widehat{p_k} + d \cdot (i - \frac{s+1}{2}) \right) + J_{k+1}\left(\mathrm{f}(\mathrm{x}_k, u_k), i\right) \right) \right], \tag{3.40}$$

with $\mathrm{p_i} = \mathrm{p}\left(\left|\Delta p_i - \Delta p_k\right| < \frac{d}{2} \right)$ being the discretized probability distribution of Δp_k, i the index $\in \{1,2,...s\}$, and $\Delta p_i = d \cdot \left(i - \frac{s+1}{2} \right)$. We will use the formulas of Section 3.2.1 for assuming either a Normal or a Cauchy distribution.

3.5 Optimization Algorithm for Hour-Ahead Decision

Next, we assume that the generator does not have to commit its unit for each hour on the previous day, but can decide up until the actual hour whether to switch it on or not. Based on the current price, the original price forecasts and assumptions about the price behavior, the generator will make hourly commitment decisions.

If the generator assumes no correlation between hourly forecast errors, then the optimal bidding strategy will be same as either the one in Sections 3.4.1 or 3.4.2 because the knowledge of the current price does not provide additional knowledge to forecast prices in the future.

Now, we assume that the deviations between forecast and actual hourly prices behave according to the first-order autoregressive moving-average with exogenous input (ARMAX) discrete time model of Section 3.2.2. The additional assumption of a correlation δ in the optimization scheme should improve the average daily profit. At each hour *z*, the generator observes the actual forecast error Δp_z and will update the revised forecast prices for the next hours $k \in \{1,2,... n\}$ according to

$$\widehat{\Delta p_k}' = \delta^k \cdot \Delta p_z, \tag{3.41}$$

or

$$\widehat{p_k}' = \widehat{p_k} + \delta^k \cdot \Delta p_z. \tag{3.42}$$

The red line in Figure 3-5 shows the anticipated forecast deviations $\widehat{\Delta p_k}$ at hour z=3 of 11/16/2001.

The generator can now optimize its decision u_1 to turn on / off its unit for the next hour by using the algorithm:

$$\widetilde{J}_k(\mathrm{x}_k, \Delta p_{k-1}) = \max_{u_k \in U_k}\left[\mathop{\mathrm{E}}_{\Delta p_k}\left\{ \pi(\mathrm{x}_k, u_k, \widehat{p_k}' + \Delta p_k) + J_{k+1}\left(\mathrm{f}(\mathrm{x}_k, u_k), \Delta p_k\right) \mid \Delta p_{k-1} \right\}\right] \tag{3.43}$$
$$\widetilde{J}_n(\mathrm{x}_n, \Delta p_n) = 0,$$

and assigning 0 to Δp_0 and the current state to x_1. Δp_{k-1} has now become part of the state, and has to be discretized in order to perform the calculation. Parameters *d* and *s* are defined as in the previous sections. The iteration can be written as

$$J_k(\mathrm{x}_k, \Delta p_{k-1}) = \max_{u_k \in U_k}\left[\sum_{\Delta p_i} \mathrm{p}\left(\Delta p_k = \Delta p_i \mid \Delta p_{k-1}\right) \cdot \left(\pi(\mathrm{x}_k, u_k, \widehat{p_k}' + \Delta p_i) + J_{k+1}\left(\mathrm{f}(\mathrm{x}_k, u_k), \Delta p_i\right)\right)\right], \tag{3.44}$$

or for the implementation

$$J_k(\mathrm{x}_k, j) = \max_{u_k \in U_k}\left[\sum_i \mathrm{p}_{\mathrm{i,j}} \cdot \left(\pi\left(\mathrm{x}_k, u_k, \widehat{p_k}' + d \cdot (i - \frac{s+1}{2})\right) + J_{k+1}\left(\mathrm{f}(\mathrm{x}_k, u_k), i\right)\right)\right], \tag{3.45}$$

with

$$\mathrm{p}_{\mathrm{i,j}} = \mathrm{p}\left(\left|\Delta p_i - \Delta p_k\right| < \frac{d}{2} \;\middle|\; \left|\Delta p_j - \Delta p_{k-1}\right| < \frac{d}{2} \right) \tag{3.46}$$

being the conditional probability distribution of the forecast deviations for this hour, based on the difference in the last hour, *i* and *j* the indexes $\in \{1, 2, \ldots s\}$, and

$$\Delta p_i = d \cdot \left(i - \frac{s+1}{2} \right).$$

Table 3-7 shows an overview of the different approaches considered.

<table>
<tr><td>Deterministic Forecast (3.4.1)</td><td>1</td><td colspan="2">Deterministic Unit Commitment for given forecast prices</td></tr>
<tr><td>Stochastic DA (3.4.2) - Normal</td><td>2A</td><td rowspan="2">Stochastic Dynamic Programming in a Day-Ahead Market; Forecast errors behave accord. to:</td><td>Normal Distribution</td></tr>
<tr><td>Stochastic DA (3.4.2) - Cauchy</td><td>2B</td><td>Cauchy Distribution</td></tr>
<tr><td>Stochastic HA (3.5) - Normal</td><td>3A</td><td rowspan="2">Stochastic Dynamic Programming in an Hour-Ahead Market; Forecast errors behave according to:</td><td>Normal Distribution</td></tr>
<tr><td>Stochastic HA (3.5) - Cauchy</td><td>3B</td><td>Cauchy Distribution</td></tr>
<tr><td>Deterministic Real Prices (3.4.1)</td><td>4</td><td colspan="2">Optimal Unit Commitment if real prices were known</td></tr>
</table>

Table 3-7. Methods for calculating commitment decisions

3.6 Simulations and Results - 2001 Data

Table 3-8, Table 3-9 and Figure 3-8 show the outcomes of the proposed algorithms and price models for generators with the same parameters, except for the minimum up/ down-times, tested on the sample data from 09/01/2001-12/31/2001.

Based on the results, we make following observations:

- The day-ahead stochastic optimization (method 2) would have permitted both generators to improve their expected profits by around 3-8% as compared to the deterministic optimization (method 1).

- The more flexible generator 2 could have achieved a higher expected profit than generator 1. However, the difference is only in the order of 1% with method 1 and 2% with method 2.

- Deciding whether to switch one's unit at the last moment and using the hour-ahead algorithm could have increased the profits by around 17-18%.

- The Cauchy approximation has performed by 4% better the Normality approximation for method 2 and by 1% for method 3.

Calculations were performed on several machines in parallel. Due to the increased number of necessary calculations, method 3 is too costly to use the same fine discretization as for method 2. The algorithm for method 3B with s=11 and 6 periods looked into the future at each step took 3243 seconds on a Sun Sparc5 in Matlab.

Parameters	a	b	c	$Q_{\min}$	$Q_{\max}$	t_{up}	t_{down}	S	T
Values	2	20	18	1	10	3	2	1	1

Method	Profit (in k$)	
1	**28285**	
2A	29099 (s=31, d=1, σ_e^2=25),	**29099** (s=31, d=2, $\boldsymbol{\sigma_e^2}$**=25**),
	29099 (s=61, d=1, σ_e^2=25),	29645 (s=31, d=2, σ_e^2=85),
	30330 (s=31, d=1, σ_e^2=60),	30412 (s=31, d=2, σ_e^2=60),
	30550 (s=61, d=1, σ_e^2=60),	29412 (s=121, d=0.5, σ_e^2=60),
	31064 (s=31, d=2, σ_e^2=100)	
	31111 (s=31, d=2, σ_e^2=120),	30084 (s=31, d=2, σ_e^2=200),
	30722 (s=31, d=2, σ_e^2=140),	30943 (s=31, d=2, σ_e^2=110),
2B	28308 (s=5, d=1, m=3),	29099 (s=31, d=1, m=3),
	30182 (s=121, d=0.5, m=3),	**30192** (s=31, d=2, $\boldsymbol{m}$**=3**),
	30386 (s=61, d=1, m=3),	30134 (s=5, d=5, m=3),
	31012 (s=31, d=2, m=5),	31121 (s=31, d=2, m=6),
	30939 (s=31, d=2, m=7),	31098 (s=121, d=0.5, m=6),
	30318 (s=5, d=5, m=4),	
3A	31341 (s=5, d=2, σ_ϵ^2=60),	**33030** (s=11, d=3, $\boldsymbol{\sigma_\epsilon^2}$**=10**),
	32584 (s=11, d=3, σ_ϵ^2=40),	33139 (s=11, d=3, σ_ϵ^2=60),
	33139 (s=5, d=5, σ_ϵ^2=40),	33238 (s=5, d=5, σ_ϵ^2=60),
	33203 (s=5, d=5, σ_ϵ^2=80),	33196 (s=5, d=5, σ_ϵ^2=100),
	32980 (s=5, d=5, σ_ϵ^2=40, δ=0.65),	32968 (s=5, d=5, σ_ϵ^2=60, δ=0.65),
	33275 (s=5, d=5, σ_ϵ^2=40, δ=0.75),	33147 (s=5, d=5, σ_ϵ^2=60, δ=0.75),
3B	29989 (s=5, d=1, m_ϵ=3),	32212 (s=5, d=5, m_ϵ=1),
	33155 (s=5, d=5, m_ϵ=3),	33108 (s=5, d=5, m_ϵ=4),
	33227 (s=11, d=3, $\boldsymbol{m_\epsilon}$**=2**),	33096 (s=11, d=3, m_ϵ=3),
4	**38166**	

Table 3-8. Profits generator 1 during 09/01/2001-12/31/2001

<table>
<tr><td>Parameters</td><td>a</td><td>b</td><td>c</td><td>$Q_{\min}$</td><td>$Q_{\max}$</td><td>t_{up}</td><td>t_{down}</td><td>S</td><td>T</td></tr>
<tr><td>Values</td><td>2</td><td>20</td><td>18</td><td>1</td><td>10</td><td>2</td><td>1</td><td>1</td><td>1</td></tr>
<tr><td>Method</td><td colspan="9">Profit (in k$)</td></tr>
<tr><td>1</td><td colspan="9">28447</td></tr>
<tr><td>2A</td><td colspan="9">29597 (s=61, d=1, $\boldsymbol{\sigma_e^2}$=25), 30373 (s=61, d=1, σ_e^2=60),
30790 (s=31, d=2, σ_e^2=60), 29861 (s=31, d=2, σ_e^2=120),
29198 (s=31, d=2, σ_e^2=150),</td></tr>
<tr><td>2B</td><td colspan="9">30069 (s=61, d=1, m=3), 30878 (s=31, d=2, m=3),
30576 (s=31, d=2, m=4), 30348 (s=31, d=2, m=5),
29773 (s=31, d=2, m=7),</td></tr>
<tr><td>3A</td><td colspan="9">33352 (s=11, d=3, $\boldsymbol{\sigma_\epsilon^2}$=10), 33442 ($s$=11, d=3, σ_ϵ^2=40),
33888 (s=11, d=3, σ_ϵ^2=60), 33944 (s=11, d=3, σ_ϵ^2=80),
33662 (s=5, d=5, σ_ϵ^2=40, δ=0.65), 33785 (s=5, d=5, σ_ϵ^2=60, δ=0.65),
33635 (s=5, d=5, σ_ϵ^2=40, δ=0.75), 33691 (s=5, d=5, σ_ϵ^2=60, δ=0.75),</td></tr>
<tr><td>3B</td><td colspan="9">33734 (s=5, d=5, $\boldsymbol{m_\epsilon}$=2), 33717 (s=5, d=5, m_ϵ=3),
33746 (s=5, d=5, m_ϵ=4), 33638 (s=5, d=5, m_ϵ=1),
32492 (s=5, d=5, m_ϵ=2, δ=0.65), 32470 (s=5, d=5, m_ϵ=3, δ=0.65),
32560 (s=5, d=5, m_ϵ=2, δ=0.75), 32563 (s=5, d=5, m_ϵ=3, δ=0.75),</td></tr>
<tr><td>4</td><td colspan="9">38469</td></tr>
</table>

Table 3-9. Profits generator 2 during 09/01/2001-12/31/2001

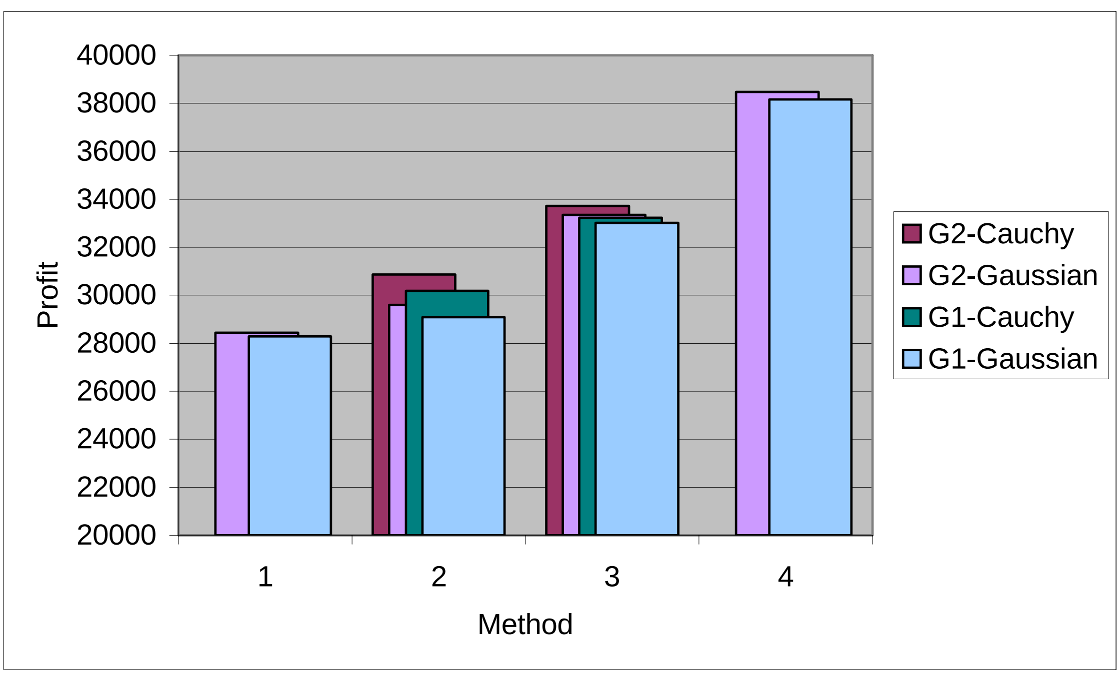

Figure 3-8. Profits under different assumptions, 09/01/2001-12/31/2001

3.7 Simulations and Results - 2002 Data

In Section 3.6, we estimated parameters for the stochastic optimization for data from 2001. We estimated the parameters for the same sample data on which we later ran the optimization algorithms, which means we used data we would not have had at the time of interest. Now, we assume that the underlying price mechanisms and forecast tools remain the same and try the same optimization methods for the first two months of 2002, using the parameters estimated for the 2002 prices. Table 3-10 and Figure 3-9 show the results.

Generator	G1	G2
Method	Profit (in k$)	
1	**4072**	**4033**
2A (s=31, d=1, σ_e^2=40)	3808	3751
(s=31, d=1, σ_e^2=25)	**3851**	**3880**
(s=31, d=1, σ_e^2=10)	4025	4016
2B (s=31, d=1, m=3)	**3851**	**3880**
(s=31, d=1, m=2)	3961	3945
(s=31, d=1, m=1)	4027	4016
3A (s=5, d=3, σ_e^2=10)	**4393**	**4451**
3B (s=5, d=3, m=2)	**4423**	**4492**
4	**6146**	**6283**

Table 3-10. Profits generator 1 and 2 during 01/01/2002-02/28/2002

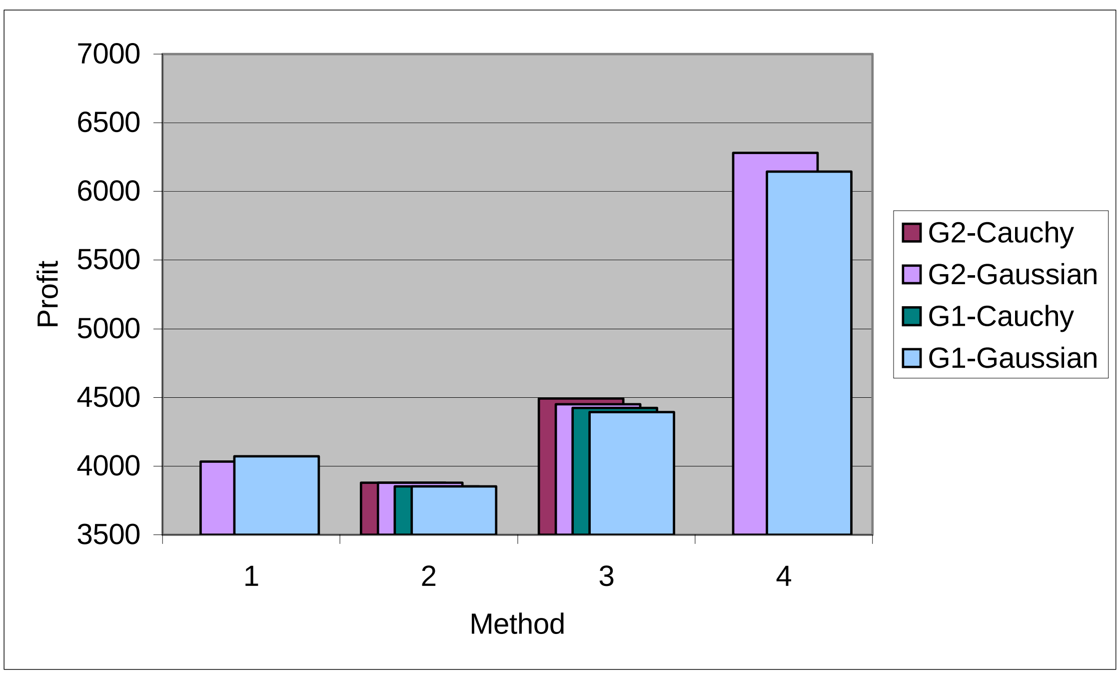

Figure 3-9. Profits under different assumptions, 01/01/2002-02/28/2002

The results were partly different from the ones of the previous section:

- Each of the day-ahead stochastic optimization algorithms (method 2) performed worse than the deterministic one on the forecasts (method 1).

- The more flexible generator 2 would have made less profit during the two months had it deterministically optimized with regard to the public price forecast (method 1).

- Committing one's unit only every hour by using the hour-ahead algorithm, could have again increased the profit by around 17-18%.

- The algorithm based on the Cauchy approximation would have given the same result as the one using the normality assumption for the day-ahead optimization and would have performed better for method 3 by around 1.5%.

To see why the stochastic optimization performed worse than the deterministic one, we examine the hours in which the scheduling sequences deviate from each other. For the stochastic algorithm from method 2B with (s=31, d=1, m=1), the bid sequences differ in only 2 periods, one of which is depicted in Figure 3-10.

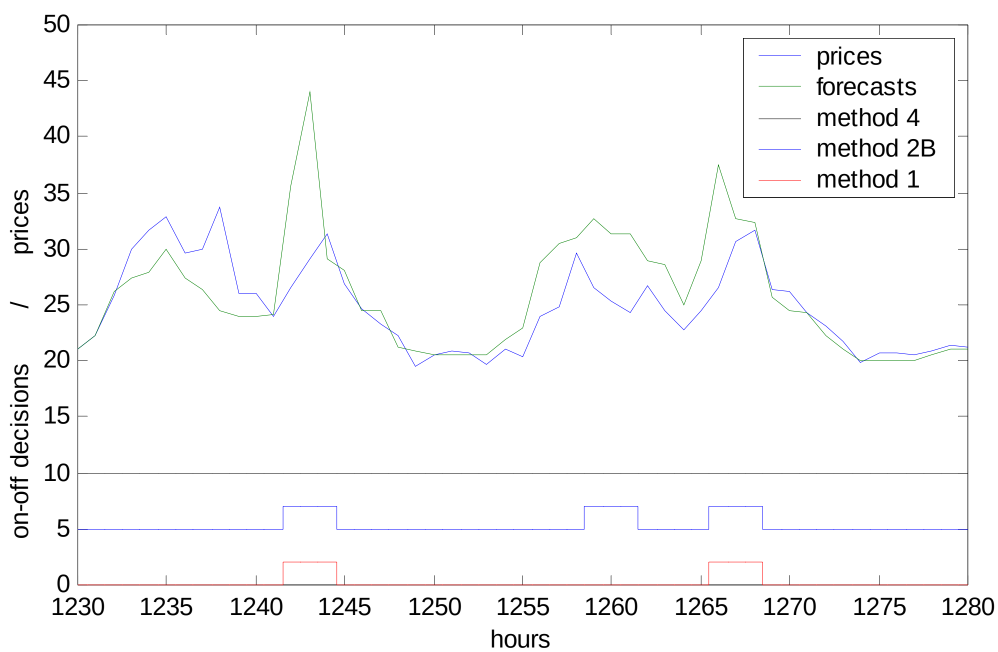

Figure 3-10. Difference between deterministic and stochastic DP

During hours 1259 to 1261, method 2B recommended the unit start-up (blue scheduling sequence) as compared to method 1 (red scheduling sequence). As the actual prices were lower than the forecast prices the unit would have lost around 48 K if switched on. If the actual prices had exactly been the same as the forecast prices, the stochastic scheduling sequence would have still led to a smaller profit by around 3.2 K. However, according to the statistical model, this stochastic sequence would lead to an average higher profit of around 1.2 K.

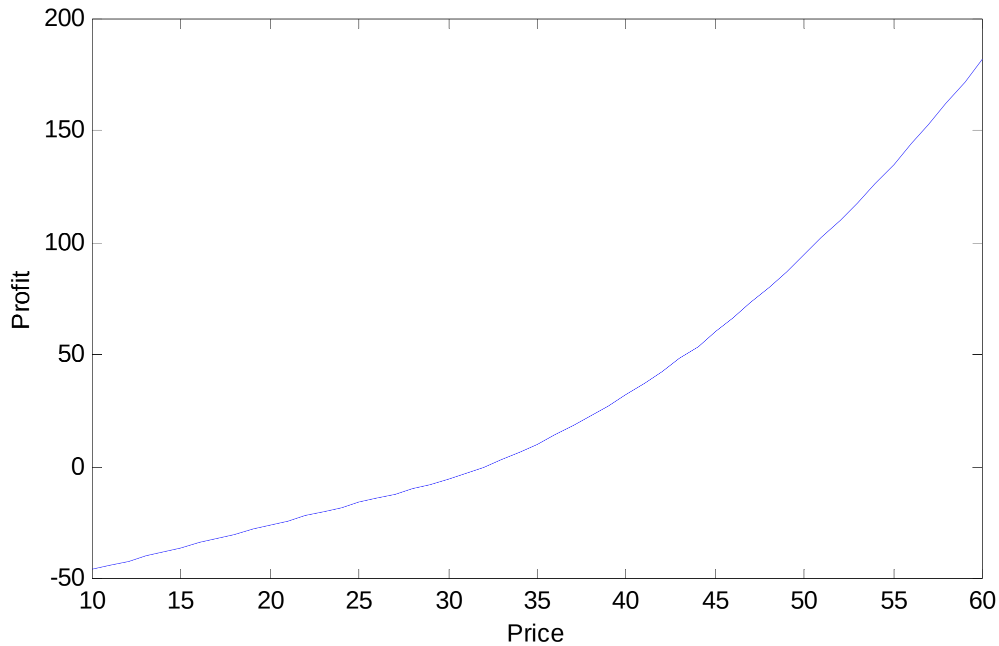

Figure 3-11. Hourly profit as function of price; start-up cost not considered

The reason comes from the unit's convex profit function (Figure 3-11). Assuming an unbiased probability distribution for deviations between forecast and actual prices, the expected profit would be higher for switching the unit on, because the potential increase in profits from a higher price is greater than the potential decrease from a lower price. The expected profit distribution is, therefore, skewed upwards (Figure 3-12).

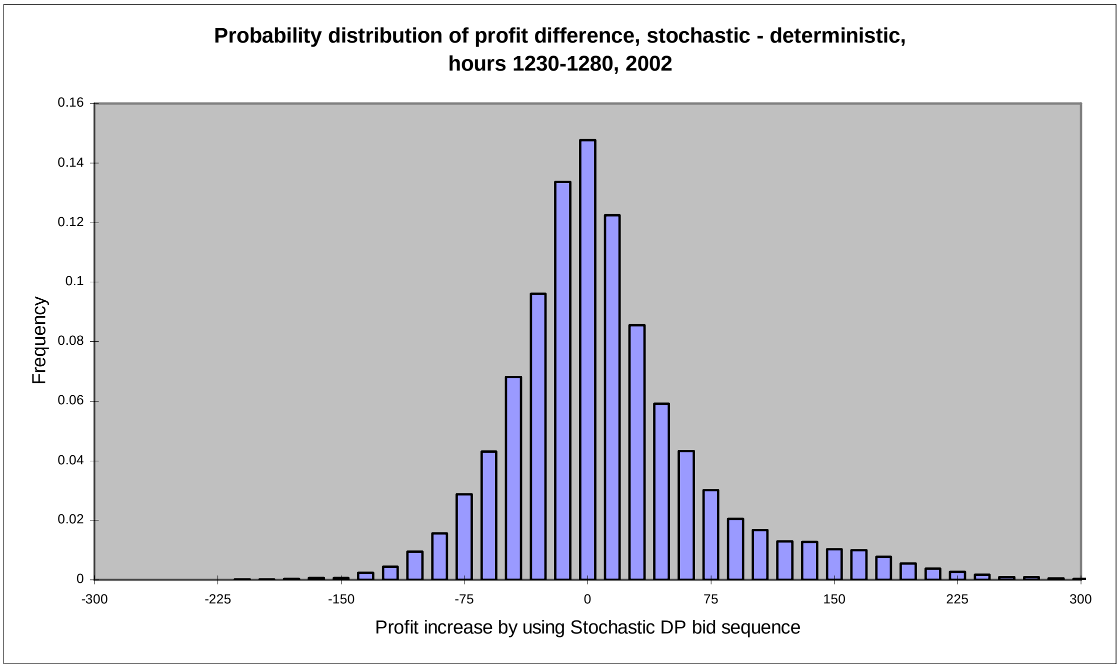

Figure 3-12. Probability distribution of profit difference, example 2002

3.8 Conclusions

We draw following conclusions:

- With concrete examples, we have shown how stochastic price models and stochastic dynamic programming algorithms can be used to improve the profits generators make on average when self-scheduling their units.
- We have also shown that the stochastic optimization does not necessarily lead to increased profits in all circumstances. The uncertain nature of the prices makes it also possible that a change in the optimal scheduling sequence actually decreases the profits.
- More flexible units with the same costs should on average be able better to optimize their schedules with regard to the uncertain prices. However, because of the uncertainties involved, it is possible as well that the seemingly optimal bid sequence for the more flexible unit actually leads to a lower profit.
- Assuming that a unit can postpone its decision to be on or off until only shortly before the hour, it can actually increase its expected profit by using the daily price forecast and observing the actual prices.
- We have found that a Cauchy distribution better represents the stochastic nature of the price deviations than a Normal distribution. The use of the Cauchy distribution in the bidding algorithms can on average improve the profits.
- Rougher discretization for the computer calculations does not notably decrease the efficiency of the algorithm.

3.9 Reservations and Open Questions

The price models described can be extended in several ways:

- Trying to fit a complicated statistical model that can describe the expected behavior of market prices accurately may require many years worth of data. Deregulated markets have been in operation for just a few years and not much data is yet available. In order to draw solid conclusions, more data would have to be used.

- During the transition phase, the variability of prices and accuracy of price forecasts might change drastically. We observe huge price spikes in the New England market at the beginning of 2001, and the patterns for the price forecast deviations at the beginning of 2002 seem to deviate from the ones at the end of 2001.

- In the optimization for September to December 2001, we estimated the parameters on the same sample data on which we later ran the optimization. We, therefore, used information we would not have had at the time of commitment. For an actual implementation of this strategies, we would have to use online-estimation, e.g. a sliding horizon. We assume that seasonal volatility depends on exogenous factors, that can be observed by the unit on regular basis and used in separate models not treated in this thesis, e.g. cold winter 2001/2002.

- Our model assumes homoskedasticity for the price variations, which means that the volatility was assumed to be constant during each day. As the price forecast is updated once per day, and the prediction accuracy decreases with the time difference between the forecast and the respective hour, the volatilities actually vary with the time of the day. The variance of price forecast deviations ranges from 10 to 114 for different hours of the day in the period of 09/01/2001-12/31/2001.

- The accuracy of the algorithms can be possibly improved by using intervals of different sizes for the discretization.

4 Market Power and Optimal Bidding Strategies

It is a fundamental concept in economic theory that the exercise of market power by suppliers of goods decreases social welfare by reducing economic efficiency and causing inefficient transfer of wealth from the consumers to the producers. With the main goal of deregulation being to decrease the cost of energy and, hence, to increase the overall consumer welfare, much attention has been given to the topic of market power in the process of deregulating energy markets in the past few years.

Methods for evaluating market power often compare the prices in the market to a competitive price level, and much of the literature uses the marginal costs of producing another unit of output as the competitive price. This chapter shows why equating the competitive price level to marginal production costs is wrong in the context of power markets, which neglects the peculiarities of the electricity industry.

4.1 Economic and Legal definitions of Market Power

Market power in economic terms is generally considered the ability of a market participant to change the price of a good:

- “Market Power: The ability of a seller or buyer to affect the price of a good.” [Pyndick01]
- “The ability to alter profitably prices away from competitive levels.” [Mas-Colell95]
- “Market power to a seller is the ability profitably to maintain prices above competitive levels for a significant period of time.” [DOJ-www]

Both last definitions refer to the competitive level, which is implicitly assumed to be the price level if both the supply and demand side of the market behaved competitively. Marginal analysis of profit maximization in economics states that competition drives price to marginal costs if there are many producers and consumers. For electricity, this means that competitive prices for generation services would be based on the costs of producing the last kilowatt hour of electricity. The application of marginal costs as the basis of prices assumes that no supplier or consumer exercises market power. [DOE97] states that “Market power exists when a supplier or consumer influences prices by virtue of size or control over important aspects of the market, such as access to transmission lines. If suppliers exercise market power, prices could be higher than marginal costs.”

In the context of the antitrust analysis of joint ventures, the regulatory body of the US, uses the following three definitions of market power according to [McFalls97]:

- The Supreme Court has defined market power as the ability to raise prices "above the levels that would be charged in a competitive market."

- The US Government agencies have defined market power as "the ability to maintain prices above competitive levels for a significant period of time."

- The Intellectual Property Guidelines have made explicit that the price increase must be profitable.

Thus, three questions are commonly answered during the market power inquiry: (1) Could a firm increase prices by restricting its output; (2) Would increasing prices be profitable for that firm; (3) Could the prices be maintained above competitive levels for a significant period of time.

4.2 Measures for Market Power

The three most common proxies used by courts and agencies to determine whether a firm (or group of firms) has the ability and incentive to raise or maintain prices above competitive levels are [McFalls97]:

- the Lerner Index, which measures the extent to which the price exceeds marginal cost;

- market share, which is the percentage of sales or capacity a firm controls in a relevant market; and

- the Herfindahl-Hirschman Index (HHI), which turns market shares into a measure of market concentration.

Courts and agencies most typically employ market concentration analysis (market share and HHI), due to the simplicity of its use, whereas economists prefer to analyze the actual price levels (Lerner Index and similar price markup ratios).

Next, we introduce standard economic monopoly and oligopoly pricing models, calculate the Lerner Index, and explain the HHI.

4.2.1 Monopoly Pricing and Lerner Index

According to general economic theory, the equilibrium between demand and supply of a good leads to prices equaling the marginal production cost of the last unit of output (Figure 4-1).

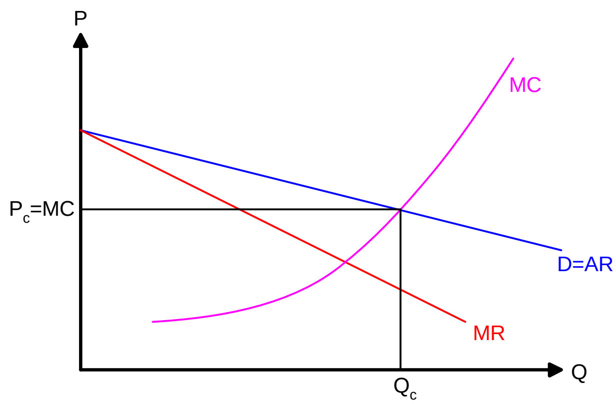

Figure 4-1. Competitive pricing

Like any competitive supplier, a monopolist rationally wants to maximize its profit and realizes that in comparison with a competitive price taker, it can affect the price by cutting back on the supply. The monopolist's short-run profit function is

$$\pi = P(Q)Q - C(Q)\,. \tag{4.1}$$

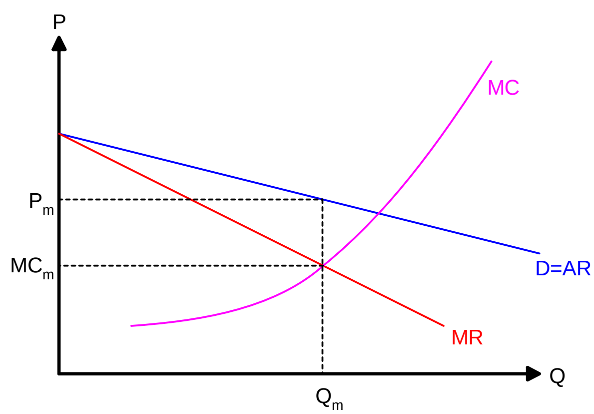

Figure 4-2. Monopoly pricing

Q represents market supply, which is the monopolist's output, and equals market demand in equilibrium. The objective of profit maximization for a monopolistic supplier facing competitive buyers leads to the following pricing rule of thumb:

$$\frac{d\pi}{dQ} = \frac{dP}{dQ}Q + P - \frac{dC}{dQ} = 0$$

$$\frac{dP}{dQ}\frac{Q}{P}P = MC - P \tag{4.2}$$

$$\frac{1}{E_D} = \frac{MC}{P} - 1, \text{ and}$$

$$P = \frac{MC}{1 + \frac{1}{E_D}}. \tag{4.3}$$

The price responsiveness or elasticity of demand E_D measures the percentage by which demand increases when price increases by 1%. It is usually a negative number and is defined as

$$E_D = \frac{\frac{dQ}{Q}}{\frac{dP}{P}} = \frac{dQ}{dP}\frac{P}{Q}. \tag{4.4}$$

One measure for monopoly power is the extent to which the profit-maximizing price of the monopolist exceeds marginal cost by using the markup ratio of price minus marginal cost to price. This index, introduced by economist Abba Lerner in 1934, is called the Lerner Index of Monopoly Power and is defined as

$$L = \frac{P - MC}{P}. \tag{4.5}$$

Using the above defined elasticity of demand, L can be written as

$$L = \frac{P - MC}{P} = -\frac{1}{E_D}. \tag{4.6}$$

In order to maximize its profit, the monopolist withholds output, causing a price-cost markup equal to the reciprocal of the demand elasticity.

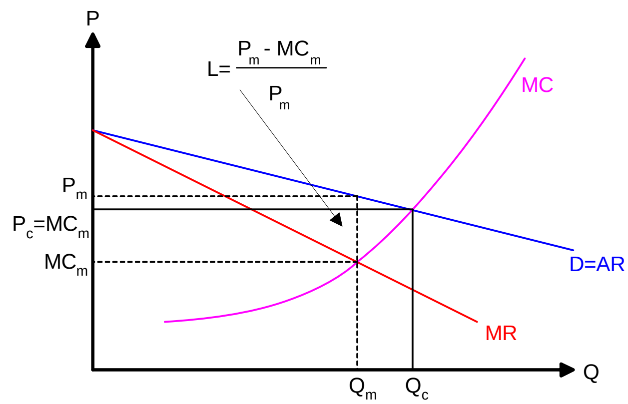

Figure 4-3. Pricing and Lerner Index

Monopoly pricing is restrained by the response of customers. The more they respond to a price increase by cutting back their demand, the less profitable it becomes for a monopolist to raise prices. Because the demand in power markets shows almost no short-term price responsiveness, suppliers in power markets can have enormous market power [Stoft02], [Pyndick01].

A similar criterion for the estimation of market power is the Price-Cost Margin Index (PCMI) which is defined as

$$PCMI = \frac{actual\ price - perfectly\ competitive\ price}{perfectly\ competitive\ price}. \tag{4.7}$$

Here, the perfect competitive equilibrium price is the baseline from which the degree of market power abuse can be evaluated. [Kumar01]

4.2.2 Oligopoly and Cournot Equilibrium

The Cournot model describes the exercise of market power by a group of non-colluding suppliers and explains much about the role of market share in the determination of market power. The essence of the Cournot model is that each firm treats the output level of its competitors as fixed and then decides how much to produce. Price is determined by total supply and the consumers' demand curve. The model assumes that all suppliers have the same complete information about the market demand function and the production cost functions of all other suppliers. It is a strong assumption, but can be justified by the fact that suppliers are long-term players in the market and learn a great deal about their competitors.

What the model tells us about pricing in oligopolistic markets is that prices can be expected to fall between the extremes of a perfectly competitive market at the low end and an unregulated monopoly market at the high end.

The Lerner Index, above defined for a monopoly, can be generalized to a Cournot oligopoly by distinguishing the supply of an individual supplier Q_i from the market supply Q and defining a supplier's market share as

$$s_i = \frac{Q_i}{Q}. \tag{4.8}$$

The markup of the oligopolistic suppliers is not only determined by demand elasticity, but also by its market share. The Lerner Index for supplier *i* for the Cournot Oligopoly is then defined as

$$L_i = \frac{s_i}{E_D}. \tag{4.9}$$

Profit maximization for one supplier leads to

$$\frac{d\pi_i}{dQ_i} = \frac{dP}{dQ_i} Q_i + P - \frac{dC_i}{dQ_i} = 0, \tag{4.10}$$

with $dQ_i = dQ$. We calculate

$$\begin{aligned} &\frac{dP}{dQ}\frac{Q}{P}\frac{Q_i}{Q} P = MC_i - P \\ &\frac{1}{E_D} s_i = \frac{MC_i - P}{P}, \text{ and} \end{aligned} \tag{4.11}$$

$$L_i = \frac{MC_i - P}{P} = \frac{s_i}{E_D}. \tag{4.12}$$

Whereas in the Cournot solution each firm is assumed to treat all of the other firm's decisions as fixed in determining its own pricing strategy, more complex "conjectural variations" models involve strategic assumptions about how the other firms' behavior will change [Stoft02].

4.2.3 Hirschman-Herfindahl Index (HHI)

The HHI is a way of measuring the concentration of market share held by particular suppliers in a market. It is the sum of squares of the market shares held by the firms in a market. Market shares are usually measured in percentage. If there is a monopoly – one firm with all sales – the HHI is 100^2=10,000. If there is perfect competition, with an infinite number of firms with near-zero market share each, the HHI is approximately zero. Other industry structures will have HHIs between zero and 10000. Jean Tirole's version is bounded between zero and one because each of the market shares is not measured in percentage, but described between zero and one. [Tirole88].

$$HHI = \sum (s_i)^2 \tag{4.13}$$

The relevance of HHI as a measure for market power and its connection to the Learner Index of the Cournot model is described in [Stoft02]. For Cournot suppliers, we can write

$$\begin{aligned} HHI &= \sum (s_i)^2 = \sum s_i L_i E_D = E_D \sum s_i L_i \\ HHI &= E_D \overline{L}, \end{aligned} \tag{4.14}$$

with $\overline{L}$ being the average Learner Index. Put in another way, the average price-cost markup (weighted by market share) in a Cournot oligopoly is equal to the HHI divided by the demand elasticity at the equilibrium price and output level. Therefore, the level of concentration in the industry by itself does not determine market power. Demand elasticity is equally important and two markets that have the same HHI may well have different levels of market power.

Another fact that diminishes HHI as a measure for market power is the underlying assumption of Cournot competition as described before. For this reason, economists tend not to rely on HHI as an indicator of market power. Despite its shortcomings, its relatively simple calculation makes it popular with regulators. DOJ, and even more heavily, FERC use it in evaluating the effect of mergers on market power.

4.3 The Exercise of Market Power in the Electricity Industry

Market power in the electricity industry can be generally classified into two categories: vertical and horizontal. Vertical market power exists when a competitor has the ability to favor its own generation due to joint control of production and transmission or joint control of production and the utility purchasing function. Because the electricity industry has historically been dominated by vertically integrated regulated monopolies, policymakers and regulators have primarily been concerned with vertical market power. Therefore, the major policy initiatives with regards to electricity restructuring in the US have focused on providing transmission access to potential entrants in the generation sector (FERC Order No. 888).

With the formation of ISOs, which are charged with operating the transmission networks in a non-discriminatory manner, the focus has now shifted to the analysis of horizontal market power, which exists when a competitor has the ability to influence production prices due to the concentration of generation ownership.

In the regulatory area, heavy emphasis has been placed on concentration measures, such as those mentioned in the previous section. Unfortunately, several characteristics of the electricity industry make concentration measures a poor indicator for the potential of market power according to [Borenstein99]. Factors beyond the number and size of firms in a market that impact the degree of competition within an industry include:

- The price-responsiveness (elasticity) of demand: In markets where customers can easily choose not to consume a product, or to consume a substitute instead, producers cannot raise prices far above costs without significantly reducing sales. Conversely, a producer that knows that its product is absolutely needed can profitably raise prices to very high levels.

- The potential for expansion of output by competitors and potential competitors: Just as a producer with very price responsive customers cannot exercise much market power, neither can a producer faced with many price-responsive competitors. Transmission capacity into a region and available competitive generation capacity are the main factors in determining the potential for short-run competitive entry.

- The storage cost: If a commodity can be easily stored and released, than a tertiary market can develop that takes on the role of temporary buffer for the periods of temporary unbalance between demand and production. Energy cannot be stored and released as easily as other commodities and, therefore, has to be produced at the same time as demanded.

Even though one firm may have a relatively small market share at a given demand level, it may be the case that if that firm reduces output, no other firm will be able to replace that supply because of cost, capacity or transmission constraints. This withholding of output can be accomplished by either bidding a high price, or not bidding at all.

FERC Discussion Paper E-47 [FERC01] states as follows: “Anticompetitive behavior or exercises of market power include behavior that raises the market price through physical or economic withholding of supplies [...] Physical withholding occurs when a supplier fails to offer its output to the market during periods when the market price exceeds the supplier’s full incremental costs [...] Economic withholding occurs when a supplier offers output to the market at a price that is above both its full incremental costs and the market price (and thus, the output is not sold).”

Harvey and Hogan define market power in the electricity industry as “the ability to withhold production on some units in order to increase market prices and profit more from production on other units” [Harvey01a], and extend this definition to “reduce profits from production on some units in order to change market prices and profit more from production on other units” [Harvey01b]. This latter definition encompasses the fact that, in the presence of constrained electricity networks or during shortage conditions, power producers can strategically opt to inject more electricity at certain points of the network, and thereby change locational marginal prices in a way favorable to them.

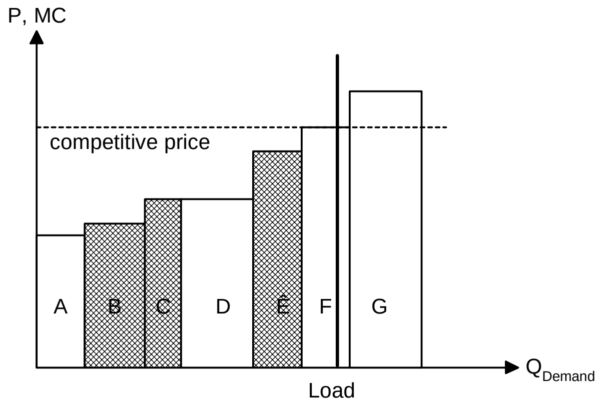

Figure 4-4. Competitive price

Figure 4-4 illustrates how a utility company owning units B, C and E can exercise economic withholding. According to general economic theory, every power generator in the system bids its actual cost until the total load is satisfied. In this case, the market clearing price is the marginal cost of unit F, and unit G is pushed out of operation because of its high cost and bidding price. However, this outcome is not the best for the utility owning units B, C and E. In Figure 4-5, unit E raises its bidding price so that unit G sets the margin. Despite unit E being pushed out of operation in this scenario, the utility has more than proportionally increased its revenue from units B and C. According to Yang and Jordan, this demonstrated ability of one utility to increase market price profitably for a sustained period of time is a sign of market power [Yang00].

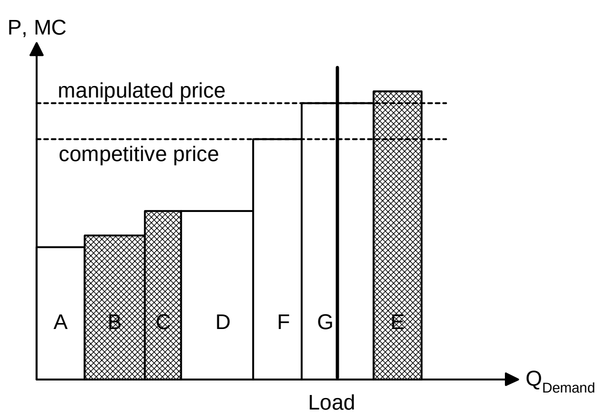

Figure 4-5. Manipulated price through withholding

A comprehensive review of market power related issues in electricity markets can be found in [Kumar01]. Visudhiphan, Ilic and Mladjan propose to compare the actual prices to benchmark prices, that take into account several factors unique to electricity markets, when assessing market power [Visduhiphan02].

4.4 Optimum Bidding Strategies for Single Auction Markets

This chapter addresses the optimization problem that generators with intertemporal constraints face when bidding into wholesale markets that require the generator to internalize start-up costs. Prices are assumed to be exogenous random variables with known probability distributions. The concepts are described by means of a concrete theoretical situation. Three different approaches for calculating an optimal bidding strategy for the generator are proposed and their effectiveness compared under two different assumptions. It is shown that the optimal bidding strategy is to bid higher than marginal costs despite the generator's lacking market power, and that different assumptions of price correlation and the time of commitment change the optimal bidding behavior.

4.4.1 Example

We consider a generator whose marginal costs (MCs) are constant over the output range. The owner can offer his electricity by submitting a bid to a centralized market for each hour and is scheduled if the bid price turns out to be lower than or equal to the market price. We neglect the case of the generator being the marginal unit and scheduled for less than full output. Because of the constant MC, the most efficient way to operate the generator is to either produce full output or nothing, and to use a flat bid curve.

In addition to variable costs, the generator incurs hourly fixed cost (HFC) for every hour of operation regardless of whether it is producing electricity or not, and also start-up (SU) and shut-down (SD) cost. Once the generator is switched on, it has to remain in that state for at least 2 hours, during which it incurs the HFC. If the generator gets scheduled for one hour, but not for the other, it still incurs the HFC for the second hour as well. Hence, the generator has to internalize these intricacies when it is bidding into an hourly market as described in Section 2.3.3.

The generator does not know the market prices when bidding, but has some knowledge about the probability distribution of the prices, which are considered to be exogenous variables, not influenced by the behavior of the generator (Figure 4-6).

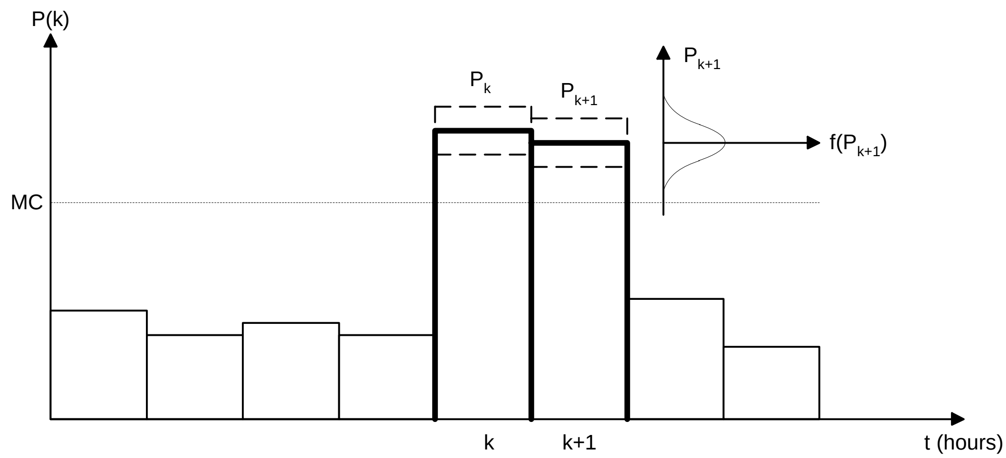

Figure 4-6. Marginal cost and hourly predicted prices for the next day

We now consider the specific situation in which only 2 successive hours have price distributions above MC. The problem of finding the optimal bids is drastically simplified and can be solved in a closed form.

In this special example, the costs of SU, SD, and 2 hours of HFC can be united into one constant term FOC (fixed operating cost) which will be incurred once the generator starts up. This aggregation does not change the optimal strategy, but simplifies the formulation. Fixed costs, such as capital costs, which are incurred regardless of the generator producing output or not during one hour, do not affect the optimal decision. For the numerical calculation, we assume that prices can have only a limited number of discrete values during the two hours (Figure 4-7).

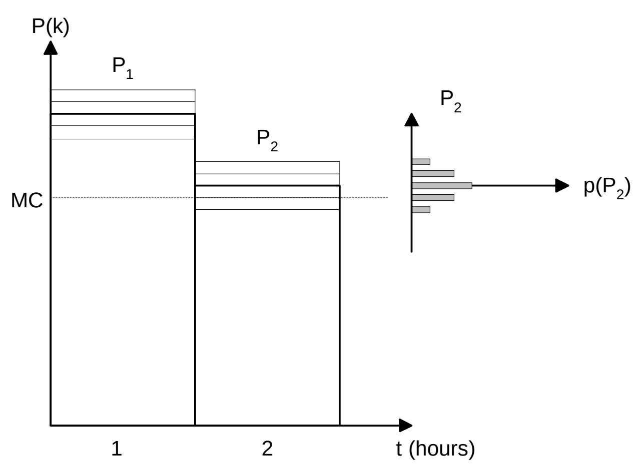

Figure 4-7. Assumed price distribution of two relevant hours

We consider 3 different basic approaches to optimizing the generator's bids for maximizing profits.

- Variant 1: For each hour, the generator either bids MC or not at all. It optimizes with regard to the hours in which to participate in the market.
- Variant 2: The generator also first decides on the hours in which it wants to participate, but then bids low or high enough to be reliably scheduled or not.
- Variant 3: The generator optimizes with regard to the height of the bid-price.

We will compare these three different approaches with regard to two different assumptions

4.4.1.1 Simultaneous / Sequential

In the simultaneous variant (A), the generator submits bids for both hours at the same time, which corresponds to a day-ahead market, in which the generator has to decide on bids for several hours simultaneously.

If it does not get scheduled in any of the two hours, the generator will not start up. We assume that if it gets scheduled in only one of the two hours, it nevertheless has to provide the energy and will incur the total FOC, which incorporates SU, SD and HFC for running two hours. It can, however, sell its energy only during one of the two hours in which it makes positive revenue if the accepted bid was above its MC.

In the sequential variant (B), the generator submits a bid only for the first hour. After learning about the prices and the fact of being scheduled for hour 1, it submits the bid for the second hour. Even if it has not started up for hour 1, it can still decide to submit a bid for the second hour. If accepted then, it runs and incurs costs during hours 2 and 3, but delivers energy only in hour 2.

4.4.1.2 Uncorrelated / Correlated Prices

In the basic variant, we assume that the probability distribution of the prices of the second hour does not change with the additional information of the first hour's price. Hence, the two hours are uncorrelated.

In version c, the hourly prices are correlated and knowledge of the first hour’s prices changes the probability distribution for hour 2 (Figure 4-7). In order to compare the results, the unconditional probability distribution of the second hour is the same as in the uncorrelated variant. Table 4-1 shows the matrix of different approaches.

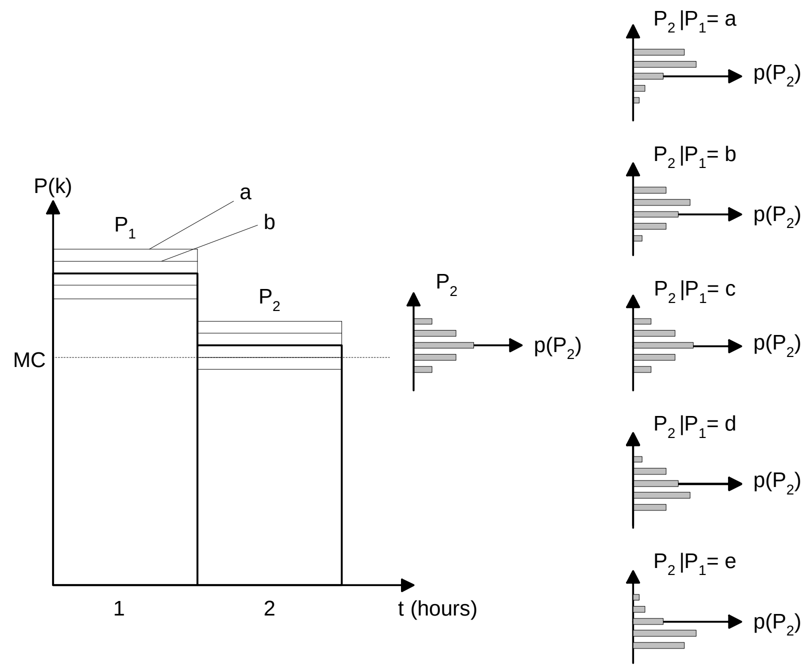

Figure 4-8. Version c: prices between periods are correlated

		Simultaneous	Sequential
Prices independent	On/ Off decision; Bids MC	1A	1B
	On/ Off decision; Bids to participate	2A	2B
	Bid height; Optimal bid	3A	3B
Prices correlated	On/ Off decision; Bids MC	1Ac	1Bc
	On/ Off decision; Bids to participate	2Ac	2Bc
	Bid height; Optimal bid	3Ac	3Bc

Table 4-1. Approach matrix

4.5 Mathematical Formulation

In Chapter 3, we assumed that the generator was able to self-schedule its unit. The problem of optimizing the bid decisions for different hours could then be solved by using Dynamic Programming as long as we did not include the correlation between hourly prices in the algorithm. This tool is used to solve problems that can be structured in a sequential way. DP cannot be used for our formulations 1A and 3A, since the decision making process is not a sequential one. At the time when the optimal bid for one hour is considered, the generator's state of the previous hour (whether or not it got scheduled during and, therefore, started up) is not yet known. As a result, the optimum solution has to take all different possibilities of combinations into account and the problem does not increase linearly with the numbers of hours considered as in DP, but exponentially.

Variant 1A: Generator bids MC, simultaneously, prices independent

The bids for the maximum expected profit are the ones that maximize profits among the 4 possible bid combinations $\left\{(u_1,u_2)\middle| u_1,u_2 \in \{0,1\}\right\}$:

$$J = \max_{u_1,u_2}\left[J(u_1,u_2)\right]. \tag{4.15}$$

The expected profit for each bid sequence is calculated either by:

$$\begin{aligned}
J(u_1,u_2) = & \sum_{(P_i|P_i \geq MC)}\sum_{(P_i|P_i \geq MC)} \mathrm{p}(P_1 = P_i)\cdot \mathrm{p}(P_2 = P_j)\cdot \begin{pmatrix} \left((P_i - MC)u_1 + (P_j - MC)u_2\right)Q \\ -FOC\cdot \max[u_1,u_2] \end{pmatrix} \\
& + \sum_{(P_i|P_i \geq MC)}\sum_{(P_i|P_i < MC)} \mathrm{p}(P_1 = P_i)\cdot \mathrm{p}(P_2 = P_j)\cdot \left((P_i - MC)Q - FOC\right)u_1 \\
& + \sum_{(P_i|P_i < MC)}\sum_{(P_i|P_i \geq MC)} \mathrm{p}(P_1 = P_i)\cdot \mathrm{p}(P_2 = P_j)\cdot \left((P_j - MC)Q - FOC\right)u_2,
\end{aligned} \tag{4.16}$$

or by

$$\begin{aligned}
J(u_1,u_2) = & \ \mathrm{p}(P_1 \geq \mathrm{M}C)\cdot \mathrm{p}(P_2 \geq \mathrm{M}C)\cdot \begin{pmatrix} \left(\mathrm{E}\{P_1 \mid P_1 \geq MC\} - MC\right)\cdot Q\cdot u_1 + \\ \left(\mathrm{E}\{P_2 \mid P_2 \geq MC\} - MC\right)\cdot Q\cdot u_2 - \\ FOC\cdot \max[u_1,u_2] \end{pmatrix} \\
& + \mathrm{p}(P_1 \geq \mathrm{M}C)\cdot \mathrm{p}(P_2 < \mathrm{M}C)\cdot \left(\mathrm{E}\{P_1 \mid P_1 \geq MC\} - FOC\right)\cdot u_1 \\
& + \mathrm{p}(P_1 < \mathrm{M}C)\cdot \mathrm{p}(P_2 \geq \mathrm{M}C)\cdot \left(\mathrm{E}\{P_2 \mid P_2 \geq MC\} - FOC\right)\cdot u_2.
\end{aligned} \tag{4.17}$$

Variant 2A: Generator bids to participate, simultaneously, prices independent

This is the standard situation for which Dynamic Programming would be used. In the case that the generator had linear increasing MC in such a way that bidding its true costs would always lead it to be scheduled, the states (on/off) for each period become certain for a certain sequence of biddings. That is why the decision process can be structured sequentially even though the bidding process occurring simultaneously. Therefore, the mathematical formulation is either the same as in 1A with $P_{\min}$ replacing MC, or in the form of a DP-algorithm:

$$J_1 = \max_{u_1} \begin{bmatrix} \left(\sum_{P_i} \mathrm{p}(P_1 = P_i) \cdot \left(P_i - MC \right) Q \right) - FOC + J_2(on), \\ J_2(off) \end{bmatrix}, \tag{4.18}$$

and

$$\begin{aligned} J_2(on) &= \max_{u_2} \sum_{(P_j)} \mathrm{p}(P_2 = P_j) \cdot \left(\left(P_j - MC \right) Q \right) \cdot u_2 \\ J_2(off) &= \max_{u_2} \left(-FOC + \sum_{(P_j)} \mathrm{p}(P_2 = P_j) \cdot \left(\left(P_j - MC \right) Q \right) \right) \cdot u_2. \end{aligned} \tag{4.19}$$

Variant 3A - Generator bids optimally, simultaneously, prices independent

In order to find the optimal bidding behavior, the profits for all possible combinations of bid heights have to be compared:

$$J = \max_{b_1, b_2} \left[J(b_1, b_2) \right], \tag{4.20}$$

with $\left\{ (b_1, b_2) \middle| (b_1, b_2) = (P_i, P_j) \right\}$ and (P_i, P_j) being possible prices for the respective hours. In order to calculate the expected profit for a bid combinations, all possible price outcomes have to be compared:

$$\begin{aligned} J(b_1, b_2) = & \sum_{(P_i | P_i \geq b_1)} \sum_{(P_j | P_j \geq b_2)} \mathrm{p}(P_1 = P_i) \cdot \mathrm{p}(P_2 = P_j) \cdot \left(\left(P_i + P_j - 2MC \right) Q - FOC \right) \\ & + \sum_{(P_i | P_i \geq b_1)} \sum_{(P_j | P_j < b_2)} \mathrm{p}(P_1 = P_i) \cdot \mathrm{p}(P_2 = P_j) \cdot \left(\left(P_i - MC \right) Q - FOC \right) \\ & + \sum_{(P_i | P_i < b_1)} \sum_{(P_j | P_j \geq b_2)} \mathrm{p}(P_1 = P_i) \cdot \mathrm{p}(P_2 = P_j) \cdot \left(\left(P_j - MC \right) Q - FOC \right). \end{aligned} \tag{4.21}$$

An alternative formulation is:

$$
\begin{aligned}
J(b_1, b_2) = {} & \mathrm{p}(P_1 \geq b_1) \cdot \mathrm{p}(P_2 \geq b_2) \cdot \Big(\big(\mathrm{E}\{P_1 \mid P_1 \geq b_1\} + \mathrm{E}\{P_2 \mid P_2 \geq b_2\} - 2MC \big) Q - FOC \Big) \\
& + \mathrm{p}(P_1 \geq b_1) \cdot \mathrm{p}(P_2 < b_2) \cdot \Big(\big(\mathrm{E}\{P_1 \mid P_1 \geq b_1\} - MC \big) Q - FOC \Big) \\
& + \mathrm{p}(P_1 < b_1) \cdot \mathrm{p}(P_2 \geq b_2) \cdot \Big(\big(\mathrm{E}\{P_2 \mid P_2 \geq b_2\} - MC \big) Q - FOC \Big).
\end{aligned}
\tag{4.22}
$$

Whereas finding the optimal bidding sequence in our example is still possible, the same task becomes computationally infeasible when optimizing for more periods. The time for calculation increases exponentially with the number of periods.

Variant 1B: Generator bids MC, sequentially, prices independent

All problems for sequential decision making can be formulated using a DP-algorithm. The generator learns at each hour whether it was scheduled, and only then decides on whether to start up or not.

Variant 1B uses:

$$
J_1 = \max_{u_1} \begin{bmatrix} \left(\sum\limits_{(P_i \mid P_i \geq MC)} \mathrm{p}(P_1 = P_i) \cdot \Big(\big(P_i - MC \big) Q - FOC \Big) + J_2(on) \right) \\ + \sum\limits_{(P_i \mid P_i < MC)} \mathrm{p}(P_1 = P_i) \cdot J_2(off), \\ J_2(off) \end{bmatrix}, \tag{4.23}
$$

and

$$
\begin{aligned}
J_2(on) &= \max_{u_2} \begin{bmatrix} \sum\limits_{(P_j \mid P_j \geq MC)} \mathrm{p}(P_2 = P_j) \cdot \big(P_j - MC \big) Q, \\ 0 \end{bmatrix} \\
J_2(off) &= \max_{u_2} \begin{bmatrix} \left(\sum\limits_{(P_j \mid P_j \geq MC)} \mathrm{p}(P_2 = P_j) \cdot \big(P_j - MC \big) Q \right) - FOC, \\ 0 \end{bmatrix}.
\end{aligned}
\tag{4.24}
$$

Variant 2B: Generator bids to participate, sequentially, prices independent

The formulation is the same as the one for variant 2A.

Variant 3B: Generator bids optimally, sequentially, prices independent

The generator decides on the optimal bid height at each hour. We write:

$$\begin{aligned} J_1 = \max_{b_1} & \sum_{(P_i | P_i \geq b_1)} \mathrm{p}(P_1 = P_i) \cdot \Big(\big(P_i - MC \big) Q - FOC + J_2(on) \Big) \\ & + \sum_{(P_i | P_i < b_1)} \mathrm{p}(P_1 = P_i) \cdot J_2(off), \end{aligned} \tag{4.25}$$

and

$$\begin{aligned} J_2(on) &= \max_{b_2} \sum_{(P_j | P_j \geq b_2)} \mathrm{p}(P_2 = P_j) \cdot \Big(\big(P_j - MC \big) Q \Big) \\ J_2(off) &= \max_{b_2} \sum_{(P_j | P_j \geq b_2)} \mathrm{p}(P_2 = P_j) \cdot \Big(\big(P_j - MC \big) Q - FOC \Big). \end{aligned} \tag{4.26}$$

Variant 1Ac: Generator bids MC, simultaneously, prices dependent

The formulation for this variant is the same as the one for variant 1A with the only difference being the conditional probability distributions of the prices for the second hour, $\mathrm{p}(P_2 = P_j \mid P_1 = P_i)$:

$$J = \max_{u_1, u_2} \left[J(u_1, u_2) \right], \tag{4.27}$$

and

$$J(u_1,u_2) = \sum_{(P_i|P_i \geq MC)} \sum_{(P_j|P_j \geq MC)} \mathrm{p}(P_1 = P_i) \cdot \mathrm{p}(P_2 = P_j \mid P_1 = P_i) \cdot \begin{pmatrix} \big((P_i - MC)u_1 + (P_j - MC)u_2\big)Q \\ -FOC \cdot \max[u_1,u_2] \end{pmatrix}$$

$$+ \sum_{(P_i|P_i \geq MC)} \sum_{(P_j|P_j < MC)} \mathrm{p}(P_1 = P_i) \cdot \mathrm{p}(P_2 = P_j \mid P_1 = P_i) \cdot \big((P_i - MC)Q - FOC\big)u_1 \qquad (4.28)$$

$$+ \sum_{(P_i|P_i < MC)} \sum_{(P_j|P_j \geq MC)} \mathrm{p}(P_1 = P_i) \cdot \mathrm{p}(P_2 = P_j \mid P_1 = P_i) \cdot \big((P_j - MC)Q - FOC\big)u_2.$$

Variant 2Ac: Generator bids to participate, simultaneously, prices dependent

This variant uses the same formulation as variant 2A, because the unconditional price distributions for each hour are assumed to be the same.

Variant 3Ac: Generator bids optimally, simultaneously, prices dependent

Like with variant 1Ac, the formulation for this variant is the same as for variant 3A with the only difference being the conditional probability distributions for the second hour:

$$J = \max_{b_1, b_2} \left[J(b_1, b_2) \right], \qquad (4.29)$$

and

$$J(b_1,b_2) = \sum_{(P_i|P_i \geq b_1)} \sum_{(P_j|P_j \geq b_2)} \mathrm{p}(P_1 = P_i) \cdot \mathrm{p}(P_2 = P_j \mid P_1 = P_i) \cdot \Big(\big(P_i + P_j - 2MC\big)Q - FOC\Big)$$

$$+ \sum_{(P_i|P_i \geq b_1)} \sum_{(P_j|P_j < b_2)} \mathrm{p}(P_1 = P_i) \cdot \mathrm{p}(P_2 = P_j \mid P_1 = P_i) \cdot \Big(\big(P_i - MC\big)Q - FOC\Big) \qquad (4.30)$$

$$+ \sum_{(P_i|P_i < b_1)} \sum_{(P_j|P_j \geq b_2)} \mathrm{p}(P_1 = P_i) \cdot \mathrm{p}(P_2 = P_j \mid P_1 = P_i) \cdot \Big(\big(P_j - MC\big)Q - FOC\Big).$$

Variant 1Bc: Generator bids MC, sequentially, prices dependent

The formulations for variants 1Bc to 3Bc differ from 1B to 3B only through the usage of conditional probability distributions. At each hour, the generator observes the current prices and, thereby, concludes the probability distribution for prices in the next hour. In 1Bc, the generator decides at each hour whether to bid its MC for the next hour or not:

$$J_1 = \max_{u_1} \begin{bmatrix} \left(\sum_{(P_i | P_i \geq MC)} \mathrm{p}(P_1 = P_i) \cdot \left(\left(P_i - MC \right) Q - FOC \right) + J_2(on, P_i) \right) \\ + \sum_{(P_i | P_i < MC)} \mathrm{p}(P_1 = P_i) \cdot J_2(off, P_i), \\ J_2(off, P_i) \\ \end{bmatrix}, \tag{4.31}$$

and

$$\begin{aligned} J_2(on, P_1) &= \max_{u_2} \begin{bmatrix} \sum_{(P_j | P_j \geq MC)} \mathrm{p}(P_2 = P_j \mid P_1) \cdot \left(P_j - MC \right) Q, \\ 0 \end{bmatrix} \\ J_2(off, P_1) &= \max_{u_2} \begin{bmatrix} \left(\sum_{(P_j | P_j \geq MC)} \mathrm{p}(P_2 = P_j \mid P_1) \cdot \left(P_j - MC \right) Q \right) - FOC, \\ 0 \end{bmatrix}. \end{aligned} \tag{4.32}$$

Variant 2Bc: Generator bids to participate, sequentially, prices dependent

The generator decides at each hour whether it wants to be scheduled at the next hour or not, and either bids a very low price or does not bid at all:

$$J_1 = \max_{u_1} \begin{bmatrix} \left(\sum_{P_i} \mathrm{p}(P_1 = P_i) \cdot \left(P_i - MC \right) Q \right) - FOC + J_2(on, P_i), \\ J_2(off, P_i) \end{bmatrix}, \tag{4.33}$$

and

$$\begin{aligned} J_2(on, P_1) &= \max_{u_2} \sum_{P_j} \mathrm{p}(P_2 = P_j \mid P_1) \cdot \left(\left(P_j - MC \right) Q \right) \cdot u_2 \\ J_2(off, P_1) &= \max_{u_2} \left(-FOC + \sum_{P_j} \mathrm{p}(P_2 = P_j \mid P_1) \cdot \left(\left(P_j - MC \right) Q \right) \right) \cdot u_2. \end{aligned} \tag{4.34}$$

Variant 3Bc: Generator bids optimally, sequentially, prices dependent

The generator optimizes the bid height for the next hour so as to maximize the expected profit:

$$\begin{aligned} J_1 = \max_{b_1} & \sum_{(P_i \mid P_i \geq b_1)} \mathrm{p}(P_1 = P_i) \cdot \big((P_i - MC)Q - FOC + J_2(on, P_i) \big) \\ & + \sum_{(P_i \mid P_i < b_1)} \mathrm{p}(P_1 = P_i) \cdot J_2(off, P_i), \end{aligned} \tag{4.35}$$

and

$$\begin{aligned} J_2(on, P_1) &= \max_{b_2} \sum_{(P_j \mid P_j \geq b_2)} \mathrm{p}(P_2 = P_j \mid P_1) \cdot \big((P_j - MC)Q \big) \\ J_2(off, P_1) &= \max_{b_2} \sum_{(P_j \mid P_j \geq b_2)} \mathrm{p}(P_2 = P_j \mid P_1) \cdot \big((P_j - MC)Q - FOC \big). \end{aligned} \tag{4.36}$$

4.6 Numerical Example

We compare the expected profits of different optimization methods for different numerical values of the parameters. We use one example to illustrate our conclusions. Parameters are listed in Appendix E E; Table 4-2 contains the numerical results. Bold numbers are expected profits for each of the optimization methods and assumed dependencies. The values in brackets are the optimal bid decisions (on/off for variants 1 and 2, bid heights for variant 3). For the sequential decision-making variants, the symbols [x] signify that the decision of the second period depends on the prices from the first period.

		Simultaneous	Sequential
Prices independent	On/ Off decision; Bids MC	(on, on) **1.0798**	(on,[on]) **1.0798**
	On/ Off decision; Bids to participate	(on, on) or (off, off) **0**	(on,[x]) or (off,[x]) **0**
	Bid height; Optimal bid	(58,52) or (60,54) **1.1720**	(60, [x]) **1.7804**
Prices correlated	On/ Off decision; Bids MC	(on, on) **1.0798**	(on,[x]) **1.0798**
	On/ Off decision; Bids to participate	(on, on) or (off, off) **0**	(on,[x]) **0.5399**
	Bid height; Optimal bid	(60,52) **1.7650**	(60, [x]) **2.0503**

Table 4-2. Optimal bids and expected profits, numerical example

4.7 Interpretations

This simple numerical example allows us to draw several conclusions:

4.7.1 Market Power

Many of the recent papers on assumed market power abuse in deregulated electricity markets assume that market participants bid their true marginal costs in a competitive market if no market power is exerted. However, in the context of bidding decisions of power plants, which not only incurs MC, but also start-up, shut-down costs and minimum commitment constraints, these assumptions lose their basis. Generators bid higher than MC not because they can exercise market power, but because of intertemporal constraints and uncertainties about prices of consecutive hours.

The literature disagrees as to what exactly constitutes market power, but generally agrees that it has to do with actively raising the prices at which one is willing to sell output (one's price offer) above MC in order to change the market price [DOE97]. MC include both the variable costs due to fuel and the other variable operating and maintenance costs. E.g., [Borenstein00] states that "Offering power at a price significantly above marginal production (or opportunity) cost, or failing to generate power that has production costs below the market price, is an indication of the exercise of market power [...] the offer price of a competitive firm, one without market power, will always be its marginal cost, which will be the greater of marginal production cost or its opportunity cost of selling the power elsewhere."

In the formulation of this paper, the power producer is modeled as price taker. He has assumptions about the probability distributions of prices for certain hours. Its bidding decision does not affect the prices and, hence, it has no market power. Nevertheless, its optimum bids deviate from MC. It is, therefore, not market power that creates prices above MC, but the necessity to incorporate start-up and shut-down constraints in the presence of uncertain prices. The generator in this example responds to the simple economic incentive of maximizing profits given uncertain prices. As a result, the competitive price does not equal marginal cost at peak periods under competition, and therefore simple price-cost margin studies cannot necessarily confirm the exercise of market power.

We state as a conclusion that above MC bids of generators do not indicate the exercise of market power. Especially in times when prices are very volatile, generators have to bid above marginal costs in order to take account of the possibility of being scheduled for one hour and not the following one. The thesis, therefore, comes to the same conclusion as [Visduhiphan02].

4.7.2 Knowledge about Correlation

If prices have the same unconditional probability distribution, but correlation between successive hours exists, then the optimal bid decisions are different. In our numerical example, the optimal bid sequence in the independent case is either (58,52) or (60,54), whereas it is (60,52) in the case that the prices of each hour are correlated (Table 4-3). Figure 4-9 shows the difference in expected profits for possible bids under the assumption of uncorrelated prices (solid lines) or correlated prices (dashed lines). Applying either of the optimal bidding sequences to the other variant leads to suboptimal profit maximization. In order to calculate the most effective bidding strategy, it is therefore important to take the price correlation between different hours of the day into account.

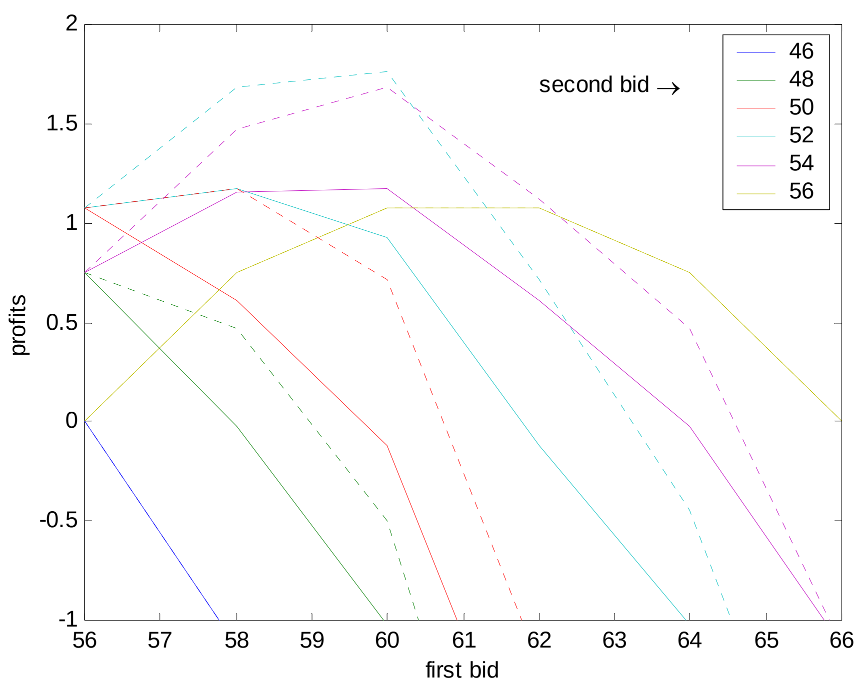

Figure 4-9. Expected profits for correlated and uncorrelated prices

	Bid Sequence	Expected Profit
Prices independent variant 3A	(58,52), (60,54)	**1.1720**
	(58,54)	**1.1538**
	(56,50),(56,52), (60,56),(62,56)	**1.0798**
	(60,52)	**0.9266**
Prices correlated variant 3Ac	(60,52)	**1.7650**
	(58,52)	**1.6838**
	(60,54)	**1.6834**

Table 4-3. Optimal bids for independent or correlated prices, numerical example

4.7.3 Dynamic Programming for Optimization

In Section 3.4 on the DP optimization for the day-ahead market we have assumed that the power producer self-schedules its generator by deciding on which hours to turn on the unit. The DP formulation permits to calculate an optimal schedule, assuming that the prices of each hour are uncorrelated, so that the unit commitment decision can be made before deciding how much power to sell [Allen99]. However, the DP formulation does not permit to include the derived correlation between hourly prices into account, as the state space can only include random variables for a problem in which the decisions for all stages have to be made at once.

In this section we assumed that the generator submits hourly bids into a centralized auction market and does not know in advance the hours in which it gets scheduled. DP cannot be used anymore in this formulation as the state of the generator in each hour is a random variable. The mathematical formulation for calculating the expected profit given a certain bid sequence $\mathrm{b} = (b_1, b_2, \ldots, b_{24})$, becomes

$$J(\mathrm{b}) = \sum_{(P_1 = P_i)} \sum_{(P_2 = P_j)} \ldots \sum_{(P_2 = P_z)} \mathrm{p}(P_1 = P_i) \cdot \mathrm{p}(P_2 = P_j) \cdot \ldots \cdot \mathrm{p}(P_2 = P_z) \cdot G(\mathrm{b}|\mathrm{P}) \qquad (4.37)$$

with $\mathrm{P} = (P_i, P_j, \ldots, P_z)$ being the random price sequence, and $G(\mathrm{b}|\mathrm{P})$ the profit of the generator given b and P.

4.7.4 Simultaneous versus Sequential Bidding

The expected profit is higher if a generator does not have to decide on all 24 hours in advance, but can instead base his bidding decision for one hour on whether his unit was scheduled for the previous hour or not, and in the correlated version on the price as well. This result is coherent with the results from Chapter 3.

The New England Market is a Single-Settlement Market, in which, the day-ahead bids are used for scheduling, but prices are determined ex post based on real-time dispatch. Those power producers that fail to perform as originally scheduled are charged compliance penalties in varying magnitude. Therefore, this persistent correlations of forecast deviations between hours can exist, and are not arbitraged by generators with excess capacity.

Another proposed method to structure a wholesale market is to determine different prices for different times of commitment [Crampton98]. In such a multi-settlement market, a unit can commit part of its output in the forward market and sell the rest in the real-time market. The actual dispatch determines real-time spot prices, which are used to price only the deviations from the day-ahead schedules (second settlement). According to Crampton and Wilson, such a settlement system is favorable to a single-settlement system, in which bidders can take advantage of short-term inelasticities in supply and demand schedules to reap excess profits. Knowing how to do this is complex, and can be best exploited by large market participants with sufficient size to the efforts worthwhile. On the long run, this added complexity and risk tend to discourage entry and participation by small bidders whose net revenues might be whipsawed by price volatility in the real-time market. Figure 4-10 shows the pattern of price and load volatility of a typical day in October 2001. The price differences between forecast and actual price of 100% cannot be explained in any way by load changes, and are therefore due to changes in the supply schedules.

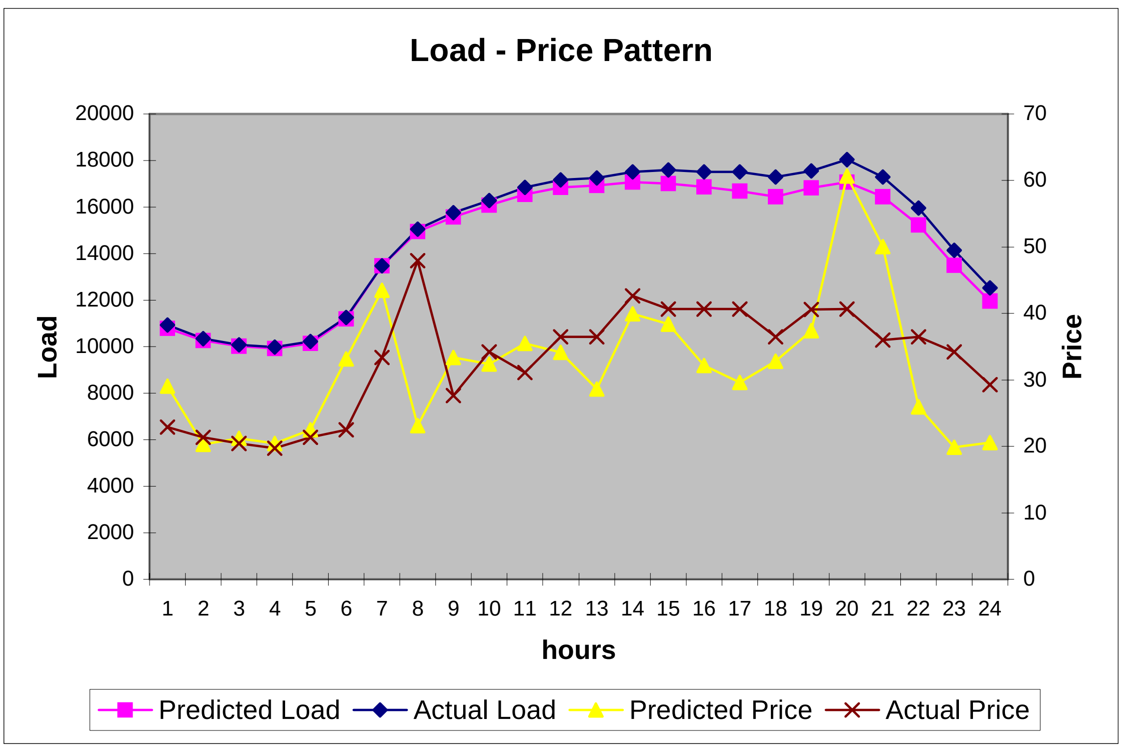

Figure 4-10. Price and Load volatility on 10/04/2001

In a multi-settlement market, the price for a certain hour in the day-ahead market should be an unbiased estimator for the real-time price in the same way as the forecast price is an unbiased estimator for the actual price in the single settlement market, assuming risk-neutrality of market participants. Because market participants can observe the real time price at each hour and react to a higher price in one hour by bidding an additional amount of energy in the following hour, a possible correlation factor between ahead- and real-time-prices of subsequent hours has to be lower than the in the single-settlement market, e.g. $\delta < 0.7$ in New England. This is a consequence of the observed possibility that we can increase the average profits when we observe correlation between hours and actually are able to take advantage of this dynamics (Chapter 3.5).

5 Conclusions and Open Questions

5.1 Conclusions

We have shown that decentralized and centralized commitment do not lead to the same amounts of power traded, even in the theoretical case of absence of uncertainties. Neglecting the influence of long-term capacity effects, we could show that the performance objectives of the individual market participants are not equal to the one of minimizing total operating cost: a centralized unit commitment can achieve an overall higher economic efficiency in the short run. We draw the analogy to *The Tragedy of the Commons* [Hardin68], which became famous for exemplifying how individual objective functions are not necessarily aligned with those of the overall societal welfare.

Second, we proved that market power in electric power auctions cannot be measured by referring to the marginal production cost as the baseline of competitive prices. In order to incorporate intertemporal constraints dominating the operation of electric power plants, generation owners have to bid higher than a simple marginal cost analysis would predict. Market power measures like the Lerner Index are, therefore, not able to measure level of market power exerted in electric power auctions.

Third, we showed how a generator owner can improve his scheduling tactics and increase expected profits by using stochastic dynamic programming and observing past price volatility. We relaxed the Normality assumption commonly used in price models and found that a Cauchy error distribution of forecast prices more accurately represents their stochastic nature permitting to more profitably schedule a unit.

5.2 Future Research

We have listed several qualitative arguments of the literature against a centralized market: lack of transparency, difficulty to find the efficient optimum, huge computational complexity, potential for strategic bidding, unresponsiveness to market needs, increased indirect and bureaucratic costs, sensitive price data. To the knowledge of the author, no quantitative study or simulation has been done that compared the long-run effects on economic efficiency of centralized and decentralized auction markets.

The detection of abused market power plays an especially important role in a deregulated electric power industry. With marginal costs not being a reliable estimate of the competitive price level, criterions have to be used that take the specificities of power production into account.

With more price data becoming available over time, stochastic models more accurately representing of the underlying price dynamics can be developed. As we showed, the profit of individual market participants will rise and fall with the quality of the algorithms used for scheduling units, and we expect that much effort will be concentrated in the future to improve such models. We have listed several possible directions and believe that this field will make challenging and possibly lucrative future research.

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Appendix A - The Dynamic Programming Algorithm

Mathematically, commitment decision problems can be expressed as dynamic programming (DP) problems, including control inputs, system states, and uncertain quantities [Shaw95], [Allen99]. Time is broken down into a series of stages, and a control decision is made at the beginning of each stage. The system can be described by the following equation [Bertsekas00]:

$$x_{k+1} = f(x_k, u_k, w_k) \tag{2.1}$$

where $k = 0, 1, \ldots$ is the time index, x_k is the state vector at time k, u_k is the control input at time k, and w_k is a random disturbance. The control u_k is constrained to be in the set of admissible controls $U_k(x_k)$ and is usually chosen as a function of x_k:

$$u_k = \mu_k(x_k) \tag{2.2}$$

A set of functions $\mu_k(x_k)$ for all k is defined as a control policy. At each stage, there is a cost to be paid. This cost may be negative, meaning that a reward is received. The problem is to determine a control policy that minimizes the cost (or maximizes the reward). The exact definition of minimal cost depends on whether the planning horizon is finite or infinite.

The problems covered in this thesis are all of finite horizon, meaning that the total cost over a specified number of stages is to be minimized. The number of stages is denoted by N. At each stage k, a cost $G_k(x_k, u_k, w_k)$ is incurred. Additionally, there is a terminal cost $G_N(x_N)$ which depends on the final value of the state vector. The object of the problem is to find the control policy that minimizes the total expected cost over N stages; this is known as the optimal policy. Dynamic programming is an algorithm to find the optimal policy; the algorithm is expressed mathematically as:

$$\begin{aligned} J_k(x_k) &= \min_{u_k \in U_k(x_k)} \ \mathop{E}_{w_k} \left\{ G_k(x_k, u_k, w_k) + J_{k+1}\left(f_k(x_k, u_k) \right) \right\} \\ J_N(x_N) &= G_N(x_N) \end{aligned} \tag{2.3}$$

where $\mathop{E}_{w_k}$ denotes the expected value operator with respect to the random variables w_k, and $J_k(x_k)$ the optimal expected cost when beginning at stage k. The DP algorithm begins by finding the optimal cost-to-go for the last stage and then iterates backwards in time to calculate $J_k(x_k)$ having already calculated $J_{k+1}(x_{k+1})$, and the iteration continues until stage 0 (the current stage) is reached. An optimal policy is obtained as a set of functions $\mu_k^*(x_k)$ such that $u_k^*(x_k) = \mu_k^*(x_k)$ attains the minimization in equation (2.3) for each x_k and k. The optimal policy need not be unique.

Appendix B - Code for Chapter 2.4

```
%MainDC.m
%given Q -> calculates assumed PPP
%given PPP -> calculates states and decisions [x,u] of generators
%given u -> calculates the actual prices P
%-----------
%output: array of Variables for each stage:
%[Q PPP P x1 u1 x2 u2 x3 u3]

clear;

Parameters

for k=1:length(Q)
    PPP(k)=PP(MCm(L),Qm(L),a(L),Q(k));
end;                                          %L=list of generators, k=stage

stage
for j=1:length(L)
   [x(:,j),u(:,j)]=DecisionG(PPP,Qm(j),a(j),CF(j),CSon(j),CSoff(j));
end;

for k=1:length(Q)
   LL{k}=[];
   for j=1:length(L)
      if u(k,j)==1
         LL{k}=[LL{k} j];
      end;
   end;
   %LL{k}
end;


%calculates P - actual price, given the list of participating generators at each
hour

for k=1:length(Q)
    P(k)=PP(MCm(LL{k}),Qm(LL{k}),a(LL{k}),Q(k));     %P market price depends on
which generators are on
end;

for j=1:length(L)
 [Gg(:,j),Qg(:,j)]=OutcomeG(x(:,j),u(:,j),P,Qm(j),a(j), CF(j),CSon(j),CSoff(j));
end;

for k=1:24
  Ct(k)=Cost(x(k,:),u(k,:), P(k),Qm(1:l),a(1:l),CF(1:l),CSoff(1:l), CSon(1:l));
end;

fid = fopen('D:\matlab\MainDC.txt','w');
fprintf(fid,'MainDC\n');

fprintf(fid,'   k    Q     P^     P     C_tot');
for j=1:l
   fprintf(fid,' x(%1.0f) u(%1.0f)  G(%1.0f)   Q(%1.0f)',[j,j,j,j]);
end;
fprintf(fid,'\n');

for k=1:24
   fprintf(fid,'%4.0f %6.2f %6.2f %6.3f %6.3f',[k, Q(k), PPP(k), P(k), Ct(k)]);
   for j=1:l
      fprintf(fid,'%4.0f %4.0f %6.3f %6.3f',[x(k,j),u(k,j),Gg(k,j),Qg(k,j)]);
   end;
   fprintf(fid,'\n');
end;

fclose(fid);
```

```
%MainISO.m
%given Q -> calculates central dispatch for states and decisions [x,u] of
generators

clear;

Parameters

J_opt = zeros(25,4^l);     %J*(k,x)
      J_opt(1:24,:)=10000;       %initial
u_opt = zeros(24,4^l);           %u*(k,x)

for i=1:4^l
   X{i}=[];
   for j=1:l
             y=  floor(   mod((i-1)/4^(j-1),4)   ) +1;
      X{i}=[X{i}, y];
   end;
end;

for i=1:2^l-1
   U{i}=[];
   U_list{i}=[];
   for j=1:l
      y=  floor(  mod(i/2^(j-1),2)  );
      U{i}=[U{i}, y];
      if y == 1   U_list{i}=[U_list{i}, L(j)];     end;
   end;
end;

for k=24:-1:1                                                   %stage
   for x=1:4^l                                                        %state
      for u=1:2^l-1                                             %possible decisions

         if sum(Qm(U_list{u})) >= Q(k)

            P_temp = PP(MCm(U_list{u}),Qm(U_list{u}),a(U_list{u}),Q(k));
            C_temp = Cost(X{x},U{u},P_temp,Qm, a,CF,CSoff,CSon     );
            x_next = x_new(X{x},U{u});
            J_temp = C_temp + J_opt(k+1, x_new(X{x},U{u}));

            %[X{x},U{u}, C_temp, J_temp, J_opt(k,x)]

            if J_temp < J_opt(k,x)
               J_opt(k,x) = J_temp;
               u_opt(k,x) = u;
            end;

         end;
      end;
   end;
   fprintf(1,'%4.0f',k);
end;
fprintf(1,'\n');

x_k = zeros(25,1);
u_k = zeros(24,1);
x_k(1) = 4^l;                          %initial condition is all shut off
for k=1:24
   u_k(k)=u_opt(k,x_k(k));
   x_k(k+1)=x_new(X{x_k(k)},U{u_k(k)});
end;
x_k=x_k(1:24);


for k=1:24
   x_kj(k,:)=X{x_k(k)};
   u_kj(k,:)=U{u_k(k)};
end;

for k=1:24
   P_k(k)=PP(MCm(U_list{u_k(k)}),Qm(U_list{u_k(k)}), a(U_list{u_k(k)}),Q(k));
```

```
end;

for j=1:l
 [Gg(:,j),Qg(:,j)]=OutcomeG(x_kj(:,j),u_kj(:,j),
P_k,Qm(j),a(j),CF(j),CSon(j),CSoff(j));
end;

%calculates actual total costs

for k=1:24
   C_tot(k) = Cost(X{x_k(k)},U{u_k(k)},P_k(k),Qm, a,CF,CSoff,CSon );
end;


fid = fopen('D:\matlab\MainISO.txt','w');
fprintf(fid,'MainISO\n');

fprintf(fid,'   k    Q            P     C_tot');
for j=1:l
   fprintf(fid,' x(%1.0f) u(%1.0f)  G(%1.0f)   Q(%1.0f)',[j,j,j,j]);
end;
fprintf(fid,'\n');

for k=1:24
   fprintf(fid,'%4.0f %6.2f %13.3f %6.3f',[k, Q(k), P_k(k), C_tot(k)]);
   for j=1:l
      fprintf(fid,'%4.0f %4.0f %6.3f %6.3f',[x_kj(k,j),u_kj(k,j),
Gg(k,j),Qg(k,j)]);
   end;
   fprintf(fid,'\n');
end;

fclose(fid);
```

%Parameters.m

```
%DEMAND - known deterministic
Q=[25 25 25 25 25 25 15 15 15 15 15 15 26 27 25 25 15 15 25 15 15 15 15 15]';

%PARAMETERS known by generators; ordered with increasing MC_max for function
PP.m !!
L=[1,2,3];                       %LIST of all considered generators;
MCm=[5,10,15];       %max. MC at max. output
Qm=[20,10,30];       %max. OUTPUT Q of generators
a=MCm./Qm;           %slope of MC with Q, =2a in typical total cost function

%secrete PARAMETERS
CF = [1,0,20];              %FIXED costs if on in respective hour
CSon = [1,1,1];             %SWITCHING costs off -> on
CSoff = [1,1,1];            %SWITCHING costs on  -> off

l=length(L);
```

function P=PP(MC_m,Q_m,A,Q)

```
%determines market price for demand Q
%arrays of max MC, max Q, and slopes a of generators
%generators have to be ordered with increasing MC_m

if Q>sum(Q_m)               %Q must be smaller than total capacity
   P=[];                           %P = empty if violated
   %P=3*max(MC_m);          %P = big if violated
   return
end;

if Q==sum(Q_m)                     %special case
   P=max(MC_m)      ;
   return
end;
```

```
P=0;
while 2>1
   Q_b= min(MC_m) * sum(1./A);   % next "breakpoint"
   if Q_b>=Q
      P=P + Q/sum(1./A);
      return
   else
      P=P + min(MC_m);
      Q=Q-Q_b;

      Q_m=Q_m-min(MC_m)./A;             %reduce remaining capacities
      MC_m=MC_m-min(MC_m);

      A(1)=[];                          %clear the first entries
      MC_m(1)=[];
      Q_m(1)=[];

      end;
end;
```

```
function [GG,QQ] = OutcomeG(x,u,P,Qm,a,CF,CSon,CSoff)
%hourly profits GG(k) and QQ(k) for one generator
%given x(k), u(k), P(k), and parameters

GG=[]; QQ=[];
L=length(u);

for i=1:L
   [GG(i),QQ(i)]=GQ(x(i),u(i),P(i),Qm,a,CF,CSon,CSoff);
end;

GG=GG'; QQ=QQ';
```

```
function [x_k,u_k] =DecisionG(Pk,Qm,a,CF,CSon,CSoff)
% calculates optimal decision for given prices and parameters of a single
generator

J_k_x = zeros(25,4);

for k=24:-1:1                       %stage
   x=1; %u=1;
   J_k_x(k,x)= G(x,1,Pk(k),Qm,a,CF,CSon,CSoff)+J_k_x(k+1, f(x,1));

   x=2; %u=[0,1];
   J_k_x(k,x)= max( G(x,0,Pk(k),Qm,a,CF,CSon,CSoff) +  J_k_x(k+1, f(x,0)) ,
...
                             G(x,1,Pk(k),Qm,a,CF,CSon,CSoff) +  J_k_x(k+1,
f(x,1)) );

   x=3; %u=0;
   J_k_x(k,x)= G(x,0,Pk(k),Qm,a,CF,CSon,CSoff)+J_k_x(k+1, f(x,0));

   x=4; %u=[0,1];
   J_k_x(k,x)= max( G(x,0,Pk(k),Qm,a,CF,CSon,CSoff) +  J_k_x(k+1, f(x,0)) ,  ...
                             G(x,1,Pk(k),Qm,a,CF,CSon,CSoff) +  J_k_x(k+1,
f(x,1)) );
end;


x_k = zeros(25,1);
u_k = zeros(24,1);
x_k(1) = 4;                         %initial condition is shut off

for k=1:24
   if x_k(k) == 1
      u_k(k) = 1;
      x_k(k+1) = 2;
   else
      if x_k(k) == 3
```

```
         u_k(k)=0;
         x_k(k+1) = 4;
      else
         if J_k_x(k,f(x,0)) > J_k_x(k,f(x,1))
            u_k(k)= 0;
         else
            u_k(k)= 1;
         end;
         x_k(k+1)=f(x_k(k),u_k(k));
      end;
   end;
end;

x_k=x_k(1:24);
```

```
function [f] =f(x,u)
%calculates next state as f(x,u)
%assumes x in {1,2,3,4}, u in {0,1}

%not meaningful:
% - and(x==1, u==0)
% - and(x==3, u==1)

if x==4 & u==1
   f=1;
end;

if x==1 | and(x==2, u==1)
   f=2;
end;

if x==2 & u==0
   f=3;
end;

if x==3 | and(x==4, u==0)
   f=4;
end;
```

```
function Cost = Cost(x,u,P,Qm,a,CF,CSoff,CSon)
%total cost for all generators during one period
%given states, decisions, parameters of generators

l=length(x);
Cost=0;
for i=1:l
   if or(x(i)==1 & u(i)==0, x(i)==3 & u(i)==1)
      Cost = 20000;               %penalty for breaking of decision space
constraints
      break;
   else
      Cost = Cost + CostInd(x(i),u(i),P,Qm(i),a(i),CF(i),CSoff(i),CSon(i));
   end;
end;

%------------SUBFUNCTION-----

function CostInd = CostInd(x,u,P,Qm,a,CF,CSoff,CSon)
%cost in period
%or(x==1 & u==0, x==3 & u==1) not allowed

%FIXED costs if turned on
if u==1   C_F = CF;
else      C_F = 0;    end;

%SWITCHING costs if state changed
if (x==2 & u==0)       C_S = CSoff;
else
   if (x==4 & u==1)    C_S = CSon;
```

```
   else                C_S = 0;   end;end;

%Output Q
if u==1             Q=min(Qm,P/a);
else                Q=0;                          end;

%"VARIABLE" costs
C_V=a/2*Q^2;

CostInd = C_V + C_S + C_F;
```

Appendix C - Examples 1 and 2 for Chapter 2.4

Example 1 - 2 generators, ISO and DC the same

```
%DEMAND - known deterministic
Q=[25 25 25 25 25 25 15 15 15 15 15 15 26 27 25 25 15 15 25 15 15 15 15 15]';

%PARAMETERS known by generators; ordered with increasing MC_max for function
PP.m !!

L=[1,2];                    %LIST of all considered generators;
MCm=[5,10];                 %max. MC at max. output
Qm=[20,10];                 %max. OUTPUT Q of generators
a=MCm./Qm;                  %slope of MC with Q, =2a in typical total cost
function

%secrete PARAMETERS
CF = [1,0];                 %FIXED costs if on in respective hour
CSon = [1,1];        %SWITCHING costs for off -> on
CSoff = [1,1];              %SWITCHING costs for on  -> off
```

MainDC

k	Q	P^	P	C_tot	x(1)	u(1)	G(1)	Q(1)	x(2)	u(2)	G(2)	Q(2)
1	25.00	5.00	5.00	65.50	4	1	48.00	20.00	4	1	11.50	5.00
2	25.00	5.00	5.00	63.50	1	1	49.00	20.00	1	1	12.50	5.00
3	25.00	5.00	5.00	63.50	2	1	49.00	20.00	2	1	12.50	5.00
4	25.00	5.00	5.00	63.50	2	1	49.00	20.00	2	1	12.50	5.00
5	25.00	5.00	5.00	63.50	2	1	49.00	20.00	2	1	12.50	5.00
6	25.00	5.00	5.00	63.50	2	1	49.00	20.00	2	1	12.50	5.00
7	15.00	3.00	3.00	23.50	2	1	17.00	12.00	2	1	4.50	3.00
8	15.00	3.00	3.00	23.50	2	1	17.00	12.00	2	1	4.50	3.00
9	15.00	3.00	3.00	23.50	2	1	17.00	12.00	2	1	4.50	3.00
10	15.00	3.00	3.00	23.50	2	1	17.00	12.00	2	1	4.50	3.00
11	15.00	3.00	3.00	23.50	2	1	17.00	12.00	2	1	4.50	3.00
12	15.00	3.00	3.00	23.50	2	1	17.00	12.00	2	1	4.50	3.00
13	26.00	6.00	6.00	69.00	2	1	69.00	20.00	2	1	18.00	6.00
14	27.00	7.00	7.00	75.50	2	1	89.00	20.00	2	1	24.50	7.00
15	25.00	5.00	5.00	63.50	2	1	49.00	20.00	2	1	12.50	5.00
16	25.00	5.00	5.00	63.50	2	1	49.00	20.00	2	1	12.50	5.00
17	15.00	3.00	3.00	23.50	2	1	17.00	12.00	2	1	4.50	3.00
18	15.00	3.00	3.00	23.50	2	1	17.00	12.00	2	1	4.50	3.00
19	25.00	5.00	5.00	63.50	2	1	49.00	20.00	2	1	12.50	5.00
20	15.00	3.00	3.00	23.50	2	1	17.00	12.00	2	1	4.50	3.00
21	15.00	3.00	3.00	23.50	2	1	17.00	12.00	2	1	4.50	3.00
22	15.00	3.00	3.00	23.50	2	1	17.00	12.00	2	1	4.50	3.00
23	15.00	3.00	3.00	23.50	2	1	17.00	12.00	2	1	4.50	3.00
24	15.00	3.00	3.00	23.50	2	1	17.00	12.00	2	1	4.50	3.00

MainISO

k	Q	P	C_tot	x(1)	u(1)	G(1)	Q(1)	x(2)	u(2)	G(2)	Q(2)
1	25.00	5.00	65.50	4	1	48.00	20.00	4	1	11.50	5.00
2	25.00	5.00	63.50	1	1	49.00	20.00	1	1	12.50	5.00
3	25.00	5.00	63.50	2	1	49.00	20.00	2	1	12.50	5.00
4	25.00	5.00	63.50	2	1	49.00	20.00	2	1	12.50	5.00
5	25.00	5.00	63.50	2	1	49.00	20.00	2	1	12.50	5.00
6	25.00	5.00	63.50	2	1	49.00	20.00	2	1	12.50	5.00
7	15.00	3.00	23.50	2	1	17.00	12.00	2	1	4.50	3.00
8	15.00	3.00	23.50	2	1	17.00	12.00	2	1	4.50	3.00
9	15.00	3.00	23.50	2	1	17.00	12.00	2	1	4.50	3.00
10	15.00	3.00	23.50	2	1	17.00	12.00	2	1	4.50	3.00
11	15.00	3.00	23.50	2	1	17.00	12.00	2	1	4.50	3.00
12	15.00	3.00	23.50	2	1	17.00	12.00	2	1	4.50	3.00
13	26.00	6.00	69.00	2	1	69.00	20.00	2	1	18.00	6.00
14	27.00	7.00	75.50	2	1	89.00	20.00	2	1	24.50	7.00
15	25.00	5.00	63.50	2	1	49.00	20.00	2	1	12.50	5.00
16	25.00	5.00	63.50	2	1	49.00	20.00	2	1	12.50	5.00
17	15.00	3.00	23.50	2	1	17.00	12.00	2	1	4.50	3.00
18	15.00	3.00	23.50	2	1	17.00	12.00	2	1	4.50	3.00
19	25.00	5.00	63.50	2	1	49.00	20.00	2	1	12.50	5.00
20	15.00	3.00	23.50	2	1	17.00	12.00	2	1	4.50	3.00

21	15.00		3.00	23.50	2	1	17.00	12.00	2	1	4.50	3.00
22	15.00		3.00	23.50	2	1	17.00	12.00	2	1	4.50	3.00
23	15.00		3.00	23.50	2	1	17.00	12.00	2	1	4.50	3.00
24	15.00		3.00	23.50	2	1	17.00	12.00	2	1	4.50	3.00

Example 2 - 3 generators, ISO and DC different

```
%DEMAND - known deterministic
Q=[25 25 25 25 25 25 15 15 15 15 15 15 26 27 25 25 15 15 25 15 15 15 15 15]';

%PARAMETERS known by generators; ordered with increasing MC_max for function
PP.m !!

L=[1,2,3];                          %LIST of all considered generators;
MCm=[5,10,15];                      %max. MC at max. output
Qm=[20,10,30];                      %max. OUTPUT Q of generators
a=MCm./Qm;                          %slope of MC with Q, =2a in typical total cost
function

%secrete PARAMETERS
CF = [1,0,20];              %FIXED costs if on in respective hour
CSon = [1,1,1];             %SWITCHING costs for off -> on
CSoff = [1,1,1];            %SWITCHING costs for on  -> off
```

MainDC

k	Q	P^	P	C_tot	x(1)	u(1)	G(1)	Q(1)	x(2)	u(2)	G(2)	Q(2)
1	25.00	3.57	5.00	65.50	4	1	48.00	20.00	4	1	11.50	5.00
2	25.00	3.57	5.00	63.50	1	1	49.00	20.00	1	1	12.50	5.00
3	25.00	3.57	5.00	63.50	2	1	49.00	20.00	2	1	12.50	5.00
4	25.00	3.57	5.00	63.50	2	1	49.00	20.00	2	1	12.50	5.00
5	25.00	3.57	5.00	63.50	2	1	49.00	20.00	2	1	12.50	5.00
6	25.00	3.57	5.00	63.50	2	1	49.00	20.00	2	1	12.50	5.00
7	15.00	2.14	3.00	23.50	2	1	17.00	12.00	2	1	4.50	3.00
8	15.00	2.14	3.00	23.50	2	1	17.00	12.00	2	1	4.50	3.00
9	15.00	2.14	3.00	23.50	2	1	17.00	12.00	2	1	4.50	3.00
10	15.00	2.14	3.00	23.50	2	1	17.00	12.00	2	1	4.50	3.00
11	15.00	2.14	3.00	23.50	2	1	17.00	12.00	2	1	4.50	3.00
12	15.00	2.14	3.00	23.50	2	1	17.00	12.00	2	1	4.50	3.00
13	26.00	3.71	6.00	69.00	2	1	69.00	20.00	2	1	18.00	6.00
14	27.00	3.86	7.00	75.50	2	1	89.00	20.00	2	1	24.50	7.00
15	25.00	3.57	5.00	63.50	2	1	49.00	20.00	2	1	12.50	5.00
16	25.00	3.57	5.00	63.50	2	1	49.00	20.00	2	1	12.50	5.00
17	15.00	2.14	3.00	23.50	2	1	17.00	12.00	2	1	4.50	3.00
18	15.00	2.14	3.00	23.50	2	1	17.00	12.00	2	1	4.50	3.00
19	25.00	3.57	5.00	63.50	2	1	49.00	20.00	2	1	12.50	5.00
20	15.00	2.14	3.00	23.50	2	1	17.00	12.00	2	1	4.50	3.00
21	15.00	2.14	3.00	23.50	2	1	17.00	12.00	2	1	4.50	3.00
22	15.00	2.14	3.00	23.50	2	1	17.00	12.00	2	1	4.50	3.00
23	15.00	2.14	3.00	23.50	2	1	17.00	12.00	2	1	4.50	3.00
24	15.00	2.14	3.00	23.50	2	1	17.00	12.00	2	1	4.50	3.00

k	x(3)	u(3)	G(3)	Q(3)
1	4	0	0.00	0.00
2	4	0	0.00	0.00
3	4	0	0.00	0.00
4	4	0	0.00	0.00
5	4	0	0.00	0.00
6	4	0	0.00	0.00
7	4	0	0.00	0.00
8	4	0	0.00	0.00
9	4	0	0.00	0.00
10	4	0	0.00	0.00
11	4	0	0.00	0.00
12	4	0	0.00	0.00
13	4	0	0.00	0.00
14	4	0	0.00	0.00
15	4	0	0.00	0.00

16	4	0	0.00	0.00
17	4	0	0.00	0.00
18	4	0	0.00	0.00
19	4	0	0.00	0.00
20	4	0	0.00	0.00
21	4	0	0.00	0.00
22	4	0	0.00	0.00
23	4	0	0.00	0.00
24	4	0	0.00	0.00

MainISO

k	Q	P	C_tot	x(1)	u(1)	G(1)	Q(1)	x(2)	u(2)	G(2)	Q(2)
1	25.00	5.00	65.50	4	1	48.00	20.00	4	1	11.50	5.00
2	25.00	5.00	63.50	1	1	49.00	20.00	1	1	12.50	5.00
3	25.00	5.00	63.50	2	1	49.00	20.00	2	1	12.50	5.00
4	25.00	5.00	63.50	2	1	49.00	20.00	2	1	12.50	5.00
5	25.00	5.00	63.50	2	1	49.00	20.00	2	1	12.50	5.00
6	25.00	5.00	63.50	2	1	49.00	20.00	2	1	12.50	5.00
7	15.00	3.00	23.50	2	1	17.00	12.00	2	1	4.50	3.00
8	15.00	3.00	23.50	2	1	17.00	12.00	2	1	4.50	3.00
9	15.00	3.00	23.50	2	1	17.00	12.00	2	1	4.50	3.00
10	15.00	3.00	23.50	2	1	17.00	12.00	2	1	4.50	3.00
11	15.00	3.00	23.50	2	1	17.00	12.00	2	1	4.50	3.00
12	15.00	3.00	23.50	2	1	17.00	12.00	2	1	4.50	3.00
13	**26.00**	**3.71**	**70.29**	**2**	**1**	**26.59**	**14.86**	**2**	**1**	**6.90**	**3.71**
14	**27.00**	**3.86**	**73.07**	**2**	**1**	**28.76**	**15.43**	**2**	**1**	**7.44**	**3.86**
15	25.00	5.00	64.50	2	1	49.00	20.00	2	1	12.50	5.00
16	25.00	5.00	63.50	2	1	49.00	20.00	2	1	12.50	5.00
17	15.00	3.00	23.50	2	1	17.00	12.00	2	1	4.50	3.00
18	15.00	3.00	23.50	2	1	17.00	12.00	2	1	4.50	3.00
19	25.00	5.00	63.50	2	1	49.00	20.00	2	1	12.50	5.00
20	15.00	3.00	23.50	2	1	17.00	12.00	2	1	4.50	3.00
21	15.00	3.00	23.50	2	1	17.00	12.00	2	1	4.50	3.00
22	15.00	3.00	23.50	2	1	17.00	12.00	2	1	4.50	3.00
23	15.00	3.00	23.50	2	1	17.00	12.00	2	1	4.50	3.00
24	15.00	3.00	23.50	2	1	17.00	12.00	2	1	4.50	3.00

k	x(3)	u(3)	G(3)	Q(3)
1	4	0	0.00	0.00
2	4	0	0.00	0.00
3	4	0	0.00	0.00
4	4	0	0.00	0.00
5	4	0	0.00	0.00
6	4	0	0.00	0.00
7	4	0	0.00	0.00
8	4	0	0.00	0.00
9	4	0	0.00	0.00
10	4	0	0.00	0.00
11	4	0	0.00	0.00
12	4	0	0.00	0.00
13	**4**	**1**	**-7.20**	**7.43**
14	**1**	**1**	**-5.12**	**7.71**
15	2	0	-1.00	0.00
16	3	0	0.00	0.00
17	4	0	0.00	0.00
18	4	0	0.00	0.00
19	4	0	0.00	0.00
20	4	0	0.00	0.00
21	4	0	0.00	0.00
22	4	0	0.00	0.00
23	4	0	0.00	0.00
24	4	0	0.00	0.00

Appendix D - Code for Chapter 3.4 and 3.5

```
%Algorithm3.m
% INPUT
% generator a, b, c, Qmin, Qmax, tup, tdown, S, T
% prices P_, P, n
% discretization d(=Interval-Size), s(=Number of Intervalls), q(=p_ij),
DP(=Delta P)
% initial values x0, j0
% LOCAL
% J,U,k,x,j,i,J0,J1
% OUTPUT
% decision u for first hour, next state x

%---OPTIMAL COST-TO-GO---

J = zeros(n+1,tup+tdown,s);
U = zeros(n,tup+tdown,s);

for k=n:-1:1                            %stage
   for x=1:tup+tdown                    %state x
      for j=1:s                         %state j (Delta P)
         J0=0;J1=0;

         if Possible(x,0,tup,tdown)==0
            J0=-10000;
         else
            for i=1:s
               J0=J0 + q(i,j) *
(G(x,0,P_(k)+DP(i),tup,tdown,Qmin,Qmax,a,b,c,S,T) + J(k+1,f(x,0,tup,tdown),i));
            end;
         end;

         if Possible(x,1,tup,tdown)==0
            J1=-10000;
         else
            for i=1:s
               J1=J1 + q(i,j) *
(G(x,1,P_(k)+DP(i),tup,tdown,Qmin,Qmax,a,b,c,S,T) + J(k+1,f(x,1,tup,tdown),i));
            end;
         end;

         if J0>J1
            J(k,x,j)=J0;
            U(k,x,j)=0;
         else
            J(k,x,j)=J1;
            U(k,x,j)=1;
         end;

      end;
   end;
end;


%---OPTIMAL DECISION + STATES---

u=U(1,x0,(j0+(s+1)/2));
x=f(x0,u,tup,tdown);


%Algorithm2.m
% INPUT
```

```
% generator a, b, c, Qmin, Qmax, tup, tdown, S, T
% prices P_, n
% s(=Number of Intervalls), p(=p_i), DP(=Delta P)
% initial values x0
% OUTPUT
% decisions u, states x

%---OPTIMAL COST-TO-GO---

J = zeros(n+1,tup+tdown);
U = zeros(n,tup+tdown);

for k=n:-1:1                            %stage
   if mod(k,5)==0
      fprintf(1,'%4.0f',k); end;
   for x=1:tup+tdown                    %state x

         J0=0;J1=0;

         if Possible(x,0,tup,tdown)==0
            J0=-10000;
         else
            for i=1:s
               J0=J0 + p(i) * (G(x,0,P_(k)+DP(i),tup,tdown,Qmin,Qmax,a,b,c,S,T)
+ J(k+1,f(x,0,tup,tdown)) );
            end;
         end;

         if Possible(x,1,tup,tdown)==0
            J1=-10000;
         else
            for i=1:s
               J1=J1 + p(i) * (G(x,1,P_(k)+DP(i),tup,tdown,Qmin,Qmax,a,b,c,S,T)
+ J(k+1,f(x,1,tup,tdown)) );
            end;
         end;

         if J0>J1
            J(k,x)=J0;
            U(k,x)=0;
         else
            J(k,x)=J1;
            U(k,x)=1;
         end;

   end;
end;

fprintf(1,'\n',k);

%---OPTIMAL DECISION + STATES---

u=zeros(n,1);
x=zeros(n+1,1);

x(1)=x0;
for k=1:n
   u(k)=U(k,x(k));
   x(k+1)=f(x(k),u(k),tup,tdown);
end;

x=x(1:n);
```

```
function [G] = G(x,u,p,tup,tdown,Qmin,Qmax,a,b,c,S,T)
%profit in period
%state 1..tup+tdown

%if Possible(x,u,tup,tdown)==0
%   G=-10000
%else

   if u==1
      Q=(p-b)/2/a;
      Q=min(max(Q,Qmin),Qmax);
      G=p*Q-a*Q^2-b*Q-c;
   else
      G=0;
   end;

   if (x==tup & u==0)
      G=G-T; end;
   if (x==tup+tdown & u==1)
      G=G-S; end;
%end;


function [f] = f(x,u,tup,tdown)
%state equation

if Possible(x,u,tup,tdown)==0
   error('impossible state transition');end;

if x<tup
   f=x+1; end;

if x==tup
   if u==0
      f=tup+1;
   else
      f=x;end;end;

if (x>tup & x<tup+tdown)
   f=x+1; end;

if x==tup+tdown
   if u==0
      f=x;
   else
      f=1;end;end;


% Initialize1.m
% a, b, c, tup, tdown, Qmin, Qmax, S, T
% P, P_, n
% s, d, DP
% x0, j0
% Corr

clear;

a=2;
b=20;
c=18;
tup=3;
tdown=2;
Qmin=1;
Qmax=10;
```

```
S=1;
T=1;

Import_Excel_Forecasts;
Import_Excel_Prices;
%P=Prices(1:300);
%P_=Forecasts(1:300);
P_=Forecasts;
P=Prices;
n=length(P);

s=31;
d=2;
for i=1:s
   DP(i)=d*(i-(s+1)/2);end;                    %discrete values, median in
intervall i

x0=tup+tdown;
j0=(s+1)/2;

Corr=0.705;
```

% Initialize2B.m

```
% Cauchy

% INPUT
% s, d, DP
% LOCAL
% m, ff
% OUTPUT
% p,q

clear p q;

m=3;

for i=1:s
   p(i)=1/(1+(DP(i)/m)^2);end;

ff=sum(p);
p=p/ff;


%plot(DP,p);

for j=1:s
   for i=1:s
      q(i,j)= 1/(1+(  (DP(i)- Corr* DP(j))  /m)^2);end;end;

for j=1:s                                      %normalize
   q(:,j)=q(:,j)/sum(q(:,j));
end;

%figure(100)
%plot(q);
```

%Main3B.m

```
%uses Algorithm 3 for stochastic HA-market

%LOCAL
%df (=delta future), steps looked ahead
```

```
%dP,dj

%OUTPUT
%SaveU2B, SaveX2B

Initialize1
Initialize2B
df=6;

tP_=P_;
tP=P;
%tn=50;
tn=n;
tx(1)=x0;
tj(1)=j0;


for tk=1:tn

   fprintf(1,'%4.0f',tk);

   n=min(df,tn-tk+1);
   P_=tP_(tk:(tk+n-1));
   for ti=1:n
      P_(ti)=P_(ti)+Corr^ti*DP(tj(tk));end;

   x0=tx(tk);
   j0=0;
   Algorithm3;
   tu(tk)=u;
   tx(tk+1)=x;

   dP=tP(tk)-tP_(tk);
   dP=min(max(dP,-(s-1)/2*d),(s-1)/2*d);
   tj(tk+1)=round(dP/d)+(s+1)/2;

end;


P_=tP_;
P=tP;
u=tu;
x=tx(1:tn);

fprintf(1,'\n',tk);

%---OUTPUT---

%figure(3);
%plot([P,P_]); hold on;
%ZeichneCD(10*u,'red',0.5); hold off;
%axis([0, tn,0,80]);

%---SAVE---

R=fopen('D:\matlab\u3.txt','w');
fprintf(R,'%5.0f',u);
fclose(R);

R=fopen('D:\matlab\x3.txt','w');
fprintf(R,'%5.0f',x);
fclose(R);
```

Appendix E - Example 1 for Chapter 4.6

Example1:

MC=50;	$P_1 \in \{56,58,60,62,64,66\}$
Q=1;	$P_2 \in \{46,48,50,52,54,56\}$
FC=10;	

With $p_i = p(P_1 = P_{1i}) = p(P_2 = P_{2i})$ and $p_{i|j} = p(P_2 = P_{2j} | P_1 = P_{1i})$:

$p_1=0.1888$	$p_{1\|1}=0.45$	$p_{1\|2}=0.20$	$p_{j\|3}=p_j$
$p_2=0.1624$	$p_{2\|1}=0.20$	$p_{2\|2}=0.32$	
$p_3=0.2978$	$p_{3\|1}=0.27$	$p_{3\|2}=0.33$	$p_{j\|4}=p_{5-j\|2}$
$p_4=0.1624$	$p_{4\|1}=0.06$	$p_{4\|2}=0.08$	
$p_5=0.1888$	$p_{5\|1}=0.02$	$p_{5\|2}=0.08$	$p_{j\|3}=p_{5-j\|1}$