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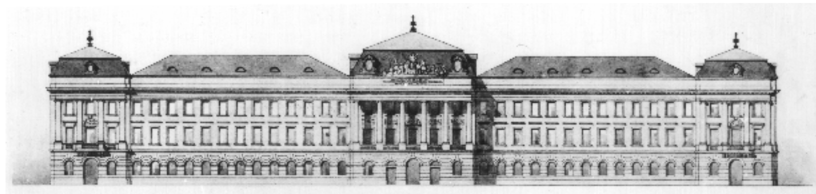

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# Comparing the Expressiveness of Argumentation Semantics

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# Comparing the Expressiveness of Argumentation Semantics

**Wolfgang Dvořák** [1] **Christof Spanring** [2]

**Abstract.** Understanding the expressiveness of a formalism is undoubtedly an important part of understanding its possibilities and limitations. Translations between different formalisms have proven to be valuable tools for understanding this very expressiveness.
In this work we complement recent investigations of the intertranslatability of argumentation semantics for Dung's abstract argumentation frameworks. As our focus is on the expressiveness of argumentation semantics we expand the area of interest beyond efficiently computable translations and also consider translations that might not (always) be efficiently computable. This allows us to provide translations between certain semantics, where under established complexity assumptions no efficiently computable translation exists. However, for some semantics we give strong translational impossibility results stating that even with arbitrary computational power we can not in all situations translate one to the other. Finally, this allows us to draw a hierarchy for the expressiveness of argumentation semantics.

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# 1 Introduction

As far as (nonmonotonic) reasoning is concerned the intertranslatability of different approaches has always been a vivid topic (see, e.g., [12, 22, 23, 24, 25]). Intertranslatability serves as a common method of motivation and justification and it is also considered to be important in order to understand the expressiveness of various formalisms. In particular in formal argumentation we have that generalizations of Dung's argumentation frameworks are often "flattened" to Dung argumentation frameworks by certain translations. Prominent examples are bipolar AFs [11], AFs with recursive attacks [2] and abstract dialectical frameworks [5]. However, the notion of intertranslatability is not only useful when comparing different kind of AFs, but can also be applied to different semantics of the same formalism. In [20] translations between different semantics for Dung's abstract argument frameworks are studied. We follow these lines and investigate the intertranslatability of abstract argumentation semantics, i.e., the question whether it is possible to modify an arbitrary argumentation framework such that the $\sigma$-extensions of the original framework are in a certain correspondence with the $\sigma'$-extensions of the modified framework ($\sigma$, $\sigma'$ being argumentation semantics).

In his seminal paper Dung [13] already proposed a broad range of argumentation semantics which since then was further broadened by the community (see, e.g., [1] for an extensive overview). When dealing with different semantics inevitably the question arises what kind of characteristics the difference affects. To this end basic properties [3] as well as the computational behavior [18] of semantics have been studied extensively in the literature. Studies on intertranslatability of semantics complement the perception of argumentation semantics by relating semantics w.r.t. their expressiveness. With being able to translate one semantics into another immediately we are also able to interlock implicit extensions and thus provide some sort of directed logical equivalence. On the other hand if one semantics cannot be translated into another we conclude that the first semantics provides certain expressiveness that cannot be simulated by the other semantics. Such investigations come into play in so called meta-level argumentation (e.g., [26]), where one wants to express certain semantics within another, for instance for the purpose of merging two frameworks with different corresponding semantics.

Intertranslatability results also affect more complex argumentation procedures. In this regards, we think about frameworks being instantiated from some (logical) knowledge-base, where the aim is to retrieve extensions satisfying specific rationality postulates w.r.t. the original knowledge-base (see, e.g., [7]). One is thus only interested in semantics ensuring that extensions satisfy the desired postulates. Given some translation from one semantics $\sigma$ to another semantics $\sigma'$ and an instantiation such that the conclusions provided by $\sigma$ satisfy the desired postulates one can build a similar instantiation for $\sigma'$ by concatenating the original instantiation and the translation.

Prior investigations for translating argumentation semantics are to be found in [20], where the ideas of intertranslatability are transferred to the area of abstract argumentation. Their work is motivated mainly by computational issues, e.g., generalizing existing solvers for application to various semantics, and thus focuses on translation functions that are efficiently computable. In contrast, the goal of our work is to get a better understanding of the expressiveness of argumentation semantics and thus we have to go beyond efficient translations. That is, this work also considers translations making use of arbitrary computational resources.

In this work we will consider the semantics proposed in Dung's seminal paper [13] as well as stage [28] and semi-stable [8, 28] semantics. We study two kinds of translations, exact translations, where the extensions of the original framework and the modified framework are identical, and faithful translations, where the extensions of the original framework and the translated framework agree on the original arguments but the extensions of the modified framework may contain additional arguments introduced by the translation. Depending on the concrete application, additional arguments in an extension might or might not be appropriate. As will be seen below the different notions of intertranslatability lead to different hierarchies of expressiveness.

The organization of the remainder of the paper and its main contributions are as follows.

- Section 2 on the one hand is dedicated to the necessary background information. We discuss abstract argumentation and recall the different notions for translations from [20]. On the other hand we also give a novel insight on the relation of the property of a translation being modular and the property of a translation being efficiently computable.
- Section 3 is all about translations. We deal with translations between 9 different semantics, which are conflict-free, naive, grounded, admissible, stable, complete, preferred, semi-stable, and stage. We complement existing results from [20] in several ways: (i) We go beyond efficiency and provide translations for semantics where no efficient translation is possible under standard complexity assumptions. Observe that throughout this work we assert the assumptions from [20] and thus consider all of their negative results to be true. (ii) We give an efficient translation from complete semantics to stage, stable, semi-stable and preferred semantics which improves over the translations given in [20]. (iii) We also consider conflict-free and naive semantics and relate them to the other semantics. Finally this will give further evidence that these two (often neglected) semantics indeed lack expressiveness.
- In Section 4 we present negative results. Namely certain translations are not possible at all. These results are stronger than those in [20]. In the sense that regardless of the available computational power it is impossible to give a translation while the typical impossibility result in [20] just claims that there is no efficiently computable translation function.
- In Section 5 we consider the properties of translations being monotone or embedding. The latter refers to translations that preserve the given structures in greatest detail. Impossibility of this property for the cases under consideration turns out to be related to the core differences between conflict-freeness and admissibility. Furthermore we typically seek for modular translation but in certain cases this is impossible. In that cases we want to at least maintain monotonicity. However several of the translations from Section 3 are not monotone. In this Section we put some extra effort to either make these translations monotone or to show that monotone translations are impossible.
- In Section 6, we summarize and reflect our results and put them into context. We provide a discussion of related work, in particular we discuss the relations to the work on so called signa-

tures of argumentation semantics [16], a different approach for characterizing expressiveness of argumentation semantics. Finally we discuss a list of open questions on intertranslatability of argumentation semantics and possible directions of future research.

# 2 Background

In this section we first present the concept of abstract argumentation frameworks together with the most prominent semantics and second recall the background on translations from [20]. We also give a new insight on the relation between two desirable properties for translations, namely modularity and efficient computability.

## 2.1 Abstract Argumentation

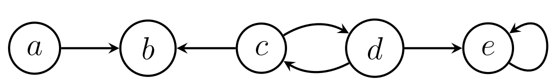
Abstract argumentation as formally introduced by Phan Minh Dung in [13] consists of structures, called (abstract) argumentation frameworks, and some form of evaluational meaning for these structures, called semantics. In the following we present formal definitions covering all the semantics we make use of in this work (see also [1], for an overview). Notice that for technical reasons in contrast to Dung's work we restrict ourselves to non-empty finite argumentation frameworks.[1]

**Definition 1.** *An* argumentation framework (AF) *is a pair $F = (A, R)$ where $A \neq \emptyset$ is a finite and non-empty set of arguments and $R \subseteq A \times A$ represents the attack relation. For a given AF $F = (A, R)$ we use $A_F$ to denote the set $A$ of its arguments and $R_F$ to denote its attack relation $R$. For the pair $(a, b) \in R$ we say that argument $a$ attacks argument $b$.*

*We sometimes use the notation $a \rightarrowtail^R b$ instead of $(a, b) \in R$. For $E \subseteq A$ and $a \in A$, we also write $E \rightarrowtail^R a$ (or $a \rightarrowtail^R E$) in case there exists an argument $b \in E$, such that $b \rightarrowtail^R a$ (or $a \rightarrowtail^R b$ resp.). In case no ambiguity arises, we use $\rightarrowtail$ instead of $\rightarrowtail^R$.*

An AF can naturally be represented as a directed graph.

**Example 1.** *Consider the AF $F = (A, R)$, with $A = \{a, b, c, d, e\}$ and $R = \{(a, b), (c, b), (c, d), (d, c), (d, e), (e, e)\}$. The graph representation of $F$ is given as follows.*




So called extension-based semantics for argumentation frameworks are given via a mapping $\sigma$ which assigns to each AF $F = (A, R)$ a set $\sigma(F) \subseteq 2^A$ of extensions. In place of $\sigma$ we will consider the mappings $cf$, $naive$, $stb$, $adm$, $prf$, $com$, $grd$, $stage$, and $sem$ which stand for conflict-free, naive, stable, admissible, preferred, complete, grounded, stage, and respectively, semi-stable semantics. Observe that with the term semantics we might refer to some abstract concept as well as to a mapping or for specific AFs to a set of sets of arguments. In the latter case we also call the

[1] We discuss some aspects of translations for infinite AFs in Section 6.

elements of some semantics, extensions. By convention the elements of some simple semantics $\sigma$ are rather just called $\sigma$-sets. Before giving the actual definitions for these semantics, we require a few more, formal concepts.

**Definition 2.** *Given an AF $F = (A, R)$, an argument $a \in A$ is* defended *(in $F$) by a set $E \subseteq A$ where for each $b \in A$, such that $b \rightarrowtail a$, also $E \rightarrowtail b$ holds. Moreover, for a set $E \subseteq A$, we define the* range of $E$*, denoted as $E_R^+$, as the set $E \cup \{b \mid E \rightarrowtail b\}$.*

We continue with the definitions of the considered semantics. Observe that their common feature is the concept of conflict-freeness, i.e., arguments in an extension are not allowed to attack each other.

**Definition 3.** *Let $F = (A, R)$ be an AF. A set $E \subseteq A$ is* conflict-free (in $F$)*, denoted as $E \in cf(F)$, if there are no $a, b \in E$, such that $(a, b) \in R$.*

Another important concept for argumentation semantics is *defense*. A set of arguments $E$ is said to defend an argument $a$ if $E \rightarrowtail b$ for each argument $b$ with $b \rightarrowtail a$. We are now prepared to define the remaining semantics under consideration.

- The *naive sets* are the $\subseteq$-maximal conflict-free sets.
- The *stable extensions* are the conflict-free sets that attack all arguments not in the set, i.e., the range is the set of all arguments.
- The *admissible sets* are those conflict-free sets that defend all their arguments.
- The *preferred extensions* are the $\subseteq$-maximal admissible sets.
- The *complete extensions* are those admissible sets that also contain all the arguments they defend.
- The *grounded extension* is the unique $\subseteq$-minimal complete extension.
- The *stage extensions* are the conflict-free sets with $\subseteq$-maximal range.
- The *semi-stable extensions* are the admissible sets with $\subseteq$-maximal range.

The formal definitions of those semantics are given below.

**Definition 4.** *For $E \in cf(F)$, it holds that*

- *$E \in naive(F)$, if there is no $D \in cf(F)$ with $D \supset E$;*
- *$E \in stb(F)$, if for each $a \in A \setminus E$, $E \rightarrowtail a$, i.e., $E_R^+ = A$;*
- *$E \in adm(F)$, if for each $a \in A$ with $a \rightarrowtail E$ we have $E \rightarrowtail a$;*
- *$E \in prf(F)$, if $E \in adm(F)$ and there is no $D \in adm(F)$ with $D \supset E$;*

- $E \in com(F)$, *if* $E \in adm(F)$ *and for each* $a \in A$ *that is defended by* $E$, $a \in E$*;*
- $E \in grd(F)$, *if* $E \in com(F)$ *and there is no* $D \in com(F)$ *with* $D \subset E$*;*
- $E \in stage(F)$, *if there is no conflict-free set* $D$ *in* $F$*, such that* $D_R^+ \supset E_R^+$*;*
- $E \in sem(F)$, *if* $E \in adm(F)$ *and there is no* $D \in adm(F)$ *with* $D_R^+ \supset E_R^+$.

We recall some basic properties of these semantics, to be used frequently thereafter. First, for any AF $F$, the following chains of subset relations holds:

$$stb(F) \subseteq stage(F) \subseteq naive(F) \subseteq cf(F)$$

$$stb(F) \subseteq sem(F) \subseteq prf(F) \subseteq com(F) \subseteq adm(F) \subseteq cf(F)$$

Furthermore, for any of the considered semantics $\sigma$ except stable semantics we have that $\sigma(F) \neq \emptyset$ holds, i.e., these semantics always propose at least one extension. Grounded semantics always yields exactly one extension, thus we also say that grounded is a unique status semantics. Moreover if an AF has at least one stable extension then stable, semi-stable, and stage semantics coincide for this AF. Next we exemplify the semantics of the AF given in Example 1.

**Example 2.** *Consider the AF* $F = (A, R)$*, from Example 1. We have* $\{a, d\}$ *as the only stable extension and thus also as the only stage and only semi-stable extension of* $F$*, i.e.,* $stb(F) = stage(F) = sem(F) = \{\{a, d\}\}$*. Further we have the admissible sets* $adm(F) = \{\{\}, \{a\}, \{c\}, \{d\}, \{a, c\}, \{a, d\}\}$*, which leads us to the preferred extensions* $prf(F) = \{\{a, c\}, \{a, d\}\}$*. Moreover the conflict-free sets are given by* $cf(F) = adm(F) \cup \{\{b\}, \{b, d\}\}$*, and thus* $naive(F) = \{\{a, c\}, \{a, d\}, \{b, d\}\}$*. Finally the complete extensions of* $F$ *are* $\{a\}$*,* $\{a, c\}$ *and* $\{a, d\}$*, with* $\{a\}$ *being the grounded extension of* $F$*.* ◇

## 2.2 Translations

The concept of translations between extension-based argumentation semantics was introduced in [20], following a long history of intertranslatability studies in nonmonotonic reasoning (see, e.g., [12, 22, 23, 24, 25]). In what follows a translation $Tr$ will be a mapping between AFs, a function satisfying certain properties. In particular, we seek translations, such that for given semantics $\sigma, \sigma'$, for any AF $F$ the extensions $\sigma(F)$ are somehow interlocked with the extensions $\sigma'(Tr(F))$. Following [20], we consider a few additional properties which are desirable for such translations. To this end, for AFs $F = (A, R)$, $F' = (A', R')$, we define the union of AFs as $F \cup F' = (A \cup A', R \cup R')$, and inclusion as $F \subseteq F'$ iff jointly $A \subseteq A'$ and $R \subseteq R'$. Furthermore the size $|F|$ of some AF $F$ is given as the cardinality of its argument set, that is $|F| = |A_F|$.

**Definition 5.** *A translation* $Tr$ *is called*

- efficient *if for every AF* $F$*, the AF* $Tr(F)$ *can be computed using logarithmic space w.r.t. to* $|F|$*;*

- covering *if for every AF $F$, $F \subseteq Tr(F)$;*
- embedding *if for every AF $F$, $A_F \subseteq A_{Tr(F)}$ and $R_F = R_{Tr(F)} \cap (A_F \times A_F)$;*
- monotone *if for any AFs $F, F'$, $F \subseteq F'$ implies $Tr(F) \subseteq Tr(F')$;*
- modular *if for any AFs $F, F'$, $Tr(F) \cup Tr(F') = Tr(F \cup F')$.*

Efficient translations can be useful for practical purposes, as they allow to apply reasoning systems dedicated to a specific semantics also to other semantics. Consider one has a sophisticated system for a semantics $\sigma$ and an efficient translation from a semantics $\sigma'$ to $\sigma$. To reason with $\sigma'$ one can now also use this translation to transform the AF and then apply the existing system for $\sigma$. Anyway, as we are mainly interested in expressiveness, efficiency will not be our main concern. The properties covering and embedding deal with truth maintenance. Covering ensures that the translation does not hide some original arguments or conflicts. Embedding, in addition, ensures that no additional attacks between the original arguments are pretended. It is easy to see that each embedding translation is also covering. Monotonicity and modularity are interesting in case the source AF is extended after the translation. First, monotonicity prevents us from having to withdraw arguments in the translated AF. Second, when adding new arguments/attacks to a huge AF modularity allows us to update the translated AF by only considering the new arguments/attacks and thus might save resources. It is easy to see that a modular translation is also monotone. For a deeper discussion of these properties and their intuitive meaning the interested reader is kindly referred to [20].

Next, in accordance with [20], we give different notions of how extensions of the original AF and the modified AF correspond to each other. We describe the interlocking mechanisms we have in mind when talking about intertranslatability. First, for *exact translations* we have that extensions must be exactly the same. This is the strongest notion of correspondence and is very strict in nature. Second, a straightforward generalization for more flexibility, is the notion of *faithful translations*. Here we only require that the extensions of $F$ coincide with the projection of extensions of $Tr(F)$ to the original arguments. Finally, certain semantics always have the empty set as an extension while others do not. To add some flexibility in that direction the notion of *weakly exact/faithful translation* was introduced. It allows to exclude certain sets from being extensions. The formal definitions of these notions are given below.

**Definition 6.** *Let $\sigma, \sigma'$ be semantics. We call a translation $Tr$*

- exact *for $\sigma \Rightarrow \sigma'$ if for every AF $F$, $\sigma(F) = \sigma'(Tr(F))$;*
- faithful *for $\sigma \Rightarrow \sigma'$ if for every AF $F$, $\sigma(F) = \{E \cap A_F \mid E \in \sigma'(Tr(F))\}$ and $|\sigma(F)| = |\sigma'(Tr(F))|$.*
- weakly exact *for $\sigma \Rightarrow \sigma'$ if there exists a collection $\mathcal{S}$ of sets of arguments, such that for any AF $F$, $\sigma(F) = \sigma'(Tr(F)) \setminus \mathcal{S}$;*
- weakly *faithful for $\sigma \Rightarrow \sigma'$ if there exists a collection $\mathcal{S}$ of sets of arguments, such that for any AF $F$, $\sigma(F) = \{E \cap A_F \mid E \in \sigma'(Tr(F)) \setminus \mathcal{S}\}$ and $|\sigma(F)| = |\sigma'(Tr(F)) \setminus \mathcal{S}|$.*

Table 1 Results for (weakly) faithful/exact translations (state of the art).

| | $cf$ | $naive$ | $grd$ | $adm$ | $stb$ | $com$ | $prf$ | $sem$ | $stage$ |
|---|---|---|---|---|---|---|---|---|---|
| $cf$ | ✓ | | | | | | | | |
| $naive$ | | ✓ | | | | | | | |
| $grd$ | | | ✓ | ✓/ ? | ✓/ ? | ✓/ ? | ✓/ ? | ✓/ ? | ✓/ ? |
| $adm$ | | | – | ✓ | ✓/ - | ✓ | ✓/ - | ✓/ - | ✓/ - |
| $stb$ | | | – | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ |
| $com$ | | | – | ✓/ - | ✓/ - | ✓ | ✓/ - | ✓/ - | ✓/ - |
| $prf$ | | | – | | | | ✓ | ✓ | ? / - |
| $sem$ | | | – | | | | | ✓ | ? / - |
| $stage$ | | | – | | | | | ✓ | ✓ |

By definition every (weakly) exact translation is also a (weakly) faithful translation. The notion of "weakly" exact/faithful is due to the fact that for some semantics some AFs do not possess an extension, while other semantics always yield at least one extension, and further that for some (but not all) semantics the empty set is always an extension. We sometimes refer to the elements of $\mathcal{S}$ as remainder sets. Note that $\mathcal{S}$ depends only on the translation, but not on the input AF. Thus, by definition, each $S \in \mathcal{S}$ contains only arguments which never occur in AFs subject to translation. In other words, we reserve certain arguments for introduction in weak translations.

All the properties from Definition 5 as well as the properties of being exact, weakly exact and faithful are transitive, i.e., for two translations satisfying one of these properties, also the concatenation satisfies the respective property. However, this transitivity is not guaranteed for weakly faithful translations. Next we present a novel observation on the relation between the properties efficiency and modularity.

**Proposition 1.** *Any modular translation is already efficient.*

*Proof.* We look at an arbitrary AF $F = (A, R)$ and investigate some modular translation $Tr$. By the definition of modularity we have $Tr(F) = \bigcup_{G \subseteq F, |G| \leq 2} Tr(G)$. That is that the Translation is fully determined by its handling of AFs of size of at most 2, as attack is the only formal AF-relation and as such two-valued. Notice that naming of arguments is irrelevant and thus a modular translation is fully determined by its translations of all AFs with at most two arguments. This gives us a finite number of graph patterns of bounded size.

To translate an AF $F$, for each of these graph patterns we have to identify isomorphic subgraphs of $F$ and apply $Tr$ to these subgraphs. As the patterns are fixed searching for isomorphic subgraphs can be done in logarithmic space and as also the translation of the patterns is fixed also translations of isomorphic subgraphs can be handled with use of logarithmic space. □

In Table 1 we summarize previous results regarding the intertranslatability of semantics taken from [20] (recall that in contrast to [20] we do not require translations to be efficient). An entry in row $\sigma$ and column $\sigma'$ of Table 1 is to be read as follows: "✓" there is a (weakly) exact translation for $\sigma \Rightarrow \sigma'$; "✓/-" there is a (weakly) faithful translation, but there can not be any (weakly) exact translation, for $\sigma \Rightarrow \sigma'$; "✓/ ?" there is a (weakly) faithful translation, but it is not known whether there might be some (weakly) exact translation, for $\sigma \Rightarrow \sigma'$; " ? / -" there is no (weakly) exact translation, and it is not known whether there might be some (weakly) faithful translation, for $\sigma \Rightarrow \sigma'$; "–" there can not be any (weakly) faithful translation for $\sigma \Rightarrow \sigma'$.

In the next sections we fill the gaps in Table 1 and provide definite answers to the questions marks. Finally, we provide a complete picture in Table 2.

# 3 Translations between Semantics

This Section is all about translations. For the semantics under our consideration we give translations whenever possible. To this end, we will use the following notational convention. We consider one specific AF $F = (A, R)$ as given and define a translation $Tr$ as a mapping $Tr(F) = Tr(A, R) = F' = (A', R')$. Furthermore, given the set of arguments $A$, we write $A^*$ (or $\bar{A}$) to denote the set of new arguments $\{a^* \mid a \in A\}$ (or $\{\bar{a} \mid a \in A\}$ resp.). We consider all arguments $a^*$ (or $\bar{a}$ resp.) to be fresh arguments not contained in the original AF.

In the following we will give several (weakly) exact/faithful translations. As far as notation is concerned for each translation we first state which semantics are affected, then give the formal definition, then state what kind of translation it is, and then give a proof. Observe that as far as the semantics are concerned, for $\sigma \Rightarrow \sigma'$ both $\sigma$ and $\sigma'$ might consist of more than one semantics; observe furthermore that for some translations the semantics $\sigma$ will be part of $\sigma'$ and for others not, i.e., some but not all translations will result in a framework the original semantics still yields the same or comparable extensions.

## 3.1 Efficient Translations

We start with a translation mapping complete semantics to stage, stable, semi-stable and preferred semantics. While the intertranslatability of these semantics was already shown in [20], the given translation improves existing results in two directions. First, it gives an explicit translation for $com \Rightarrow prf$, whereas the equivalent from [20] relies on transitive concatenation of other translations and second it provides significantly smaller target frameworks, which might be crucial when applying translations for the matter of computation.

The intuition behind this translation is to emphasize attacks. Complete semantics is a very cautious semantics; by definition attacked arguments are not considered unless properly defended, however if an argument is defended it has to be considered. We elaborate on this observation by adding attacking arguments to ensure a stable extension, and adding defenses against this arguments that again follow the principles of the characteristic function. It turns out that it suffices to consider one argument for each source and target of attacks.

**Translation 1** ($com \Rightarrow (stb|prf|stage|sem)$)**.** *The transformation* $Tr(A, R) = (A', R')$*, with* $A' = A \cup A^*$ *and*

$$R' = R \cup \{(a, b^*), (a^*, b) \mid (a, b) \in R\}$$

*is an embedding modular faithful translation for* $com \Rightarrow (stb|prf|stage|sem)$.

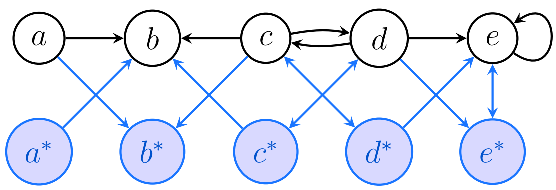



Figure 1 Illustration of Translation 1 ($com \Rightarrow (stb|prf|stage|sem)$).

*Proof.* We take an AF $F = (A, R)$ as given and investigate $Tr(F)$. Observe that for any set $E \subseteq A_F$ we have that $E$ attacks $a^*$ in $Tr(F)$ iff $E$ attacks $a$ in $F$ and therefore $E$ defends some argument $a$ (and $a^*$) in $Tr(F)$ iff $E$ defends $a$ in $F$.

In the following we are going to first show that complete extensions of the original framework $F$ are linked to stable extensions of the translated framework $F'$, and second that preferred extensions of $F'$ are linked to complete extensions of $F$. We will proceed with remarks on the equal cardinality of the semantics for $F$ and $F'$ and well-known subset relations, concluding the original translational statement.

*From complete to stable semantics*, $E \in com(F) \Rightarrow E' = E \cup \{a^* \mid E \not\rightarrowtail^R a\} \in stb(Tr(F))$, we augment a given complete extension $E$ of $F$ with as many as possible arguments from $A^*$. Now observe that $A^*$ itself and thus by definition $E'$ are conflict-free. Furthermore, $E'$ contains the $*$-image of $E$, we have that $\{a^* \mid a \in E\} \subseteq \{a^* \mid E \not\rightarrowtail^R a\} \subseteq E'$, as $E$ is conflict-free by definition. Thus, we have $\left\{a, a^* \mid a \in E_R^+\right\} \subseteq E_{R'}'^+$ and it remains to show that also the arguments $a, a^*$ with $a \in A_F \setminus E_F^+$ are in the range of $E'$. Now for any argument $a \in A_F \setminus E_R^+$ we have that, first $a^* \in E'$ by definition, and, second by completeness of $E$, $a$ is attacked by some undefended argument $b \notin E_R^+$. Hence, $b^* \in E'$ and thus $E_{R'}'^+ = A'$, i.e., $E' \in stb(Tr(F))$.

*From preferred back to complete semantics*, $E' \in prf(Tr(F)) \Rightarrow E = E' \cap A_F \in com(F)$, we strip down a given preferred extension $E$ of $F'$ to its projection in $F$. Due to the embedding property clearly $E$ is conflict-free. Furthermore $E$ even is admissible. To see this take some endangering $b \in A_F$ s.t. $b \rightarrowtail^R E$. Obviously $b$ is also endangering in $F'$, i.e., $b \rightarrowtail^{R'} E'$, and by admissibility of $E'$ in $F'$ we thus have $E' \rightarrowtail^{R'} b$. Due to maximality of $E'$, and the observation that $a^*$ can only be defended in $F'$ if $a$ is defended as well, we have that there is some $a \in A_F$ such that $a \rightarrowtail^R b$ and thus $E \rightarrowtail^R b$. Similarly for $a$ being defended by $E$, we have that $E'$ defends $a$ and due to maximality of $E'$ thus $a \in E$. Hence $E \in com(F)$.

Furthermore, since the difference between $E$ and $E'$ as defined above is to be found among the arguments $A^*$ only, and due to maximality of $E'$ we have $E' = E \cup \left\{a^* \mid E \not\rightarrowtail^R a\right\}$, marking proposed relations as bijections. Using the relations $stb(F) \subseteq sem(F) \subseteq prf(F)$ and equality of stage and stable semantics where the latter is non-empty we obtain the assertion, the given translation interlocks complete semantics of the original framework $F$ with stage, stable, semi-stable and preferred semantics in the translated framework $F' = Tr(F)$. □

We observe that grounded semantics is the minimal complete semantics. By modifying the previous translation such that newly introduced arguments are preferred over their original counterparts, we can use this property for another translation, from grounded to semi-stable semantics. Observe

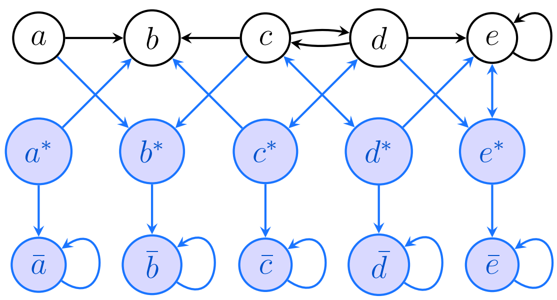



Figure 2 Illustration of Translation 2 ($grd \Rightarrow sem$).

that for various reasons the same trick does not work for the other semantics on the right-hand side of the previous translation.

**Translation 2** ($grd \Rightarrow sem$)**.** *The transformation* $Tr(A, R) = (A', R')$*, with* $A' = A \cup A^* \cup \bar{A}$ *and*

$$R' = R \cup \{(a, b^*), (a^*, b) \mid (a, b) \in R\} \\ \cup \{(\bar{a}, \bar{a}), (a^*, \bar{a}) \mid a \in A\}$$

*is an embedding modular faithful translation for* $grd \Rightarrow sem$*.*

*Proof.* First observe that the difference between this translation $Tr$ and the previous $Tr_s$ (Translation 1: $com \Rightarrow (stage|stb|sem|prf)$) is a difference of additional self-attacking arguments that do not attack anything but themselves. Which means that, by the directionality of preferred semantics [1], $prf(Tr(F)) = prf(Tr_s(F))$ for any AF $F$, which again means that the projection of $prf(Tr(F))$ to $F$ is identical with $com(F)$.

We recall that the unique status grounded semantics has as only extension the minimal complete extension, i.e., for any AF $F$, $grd(F) = \{E\}$, we have that $E \in com(F)$ and also $E = \bigcap com(F)$. We conclude that there is one complete extension $G$ of $F$ that is contained in all other complete extensions of $F$ and therefore has the $\subseteq$-maximal set $\{a \mid E \not\mapsto^R a\}$.

Furthermore, we know that every $E' \in prf(Tr(F))$ is of the form $E' = E \cup \left\{a^* \mid E \not\mapsto^R a\right\}$ for some $E \in com(F)$ and $A \cup A^* \subseteq E'^+_{R'}$. Thus, as by construction $\bar{a} \in E'^+_{R'}$ iff $a^* \in E'$, the range of $E$ is given by $E'^+_{R'} = A \cup A^* \cup \{\bar{a} \mid E \not\mapsto^R a\}$. Now, as the semi-stable extensions are the preferred extensions with maximal range, we obtain that $E_G = G \cup \left\{a^* \mid G \not\mapsto^R a\right\}$ is the unique semi-stable extension of $Tr(F)$. □

Next we consider $cf$ and $naive$ semantics. In the following faithful translation, for each argument $a$ we introduce a new argument $\bar{a}$, encoding that $a$ is not a member of the extension, and subsequently a mutual conflict between $a$ and $\bar{a}$. So each naive set of the translated AF either contains $a$ or $\bar{a}$ and thus each conflict-free set of the original AF corresponds to exactly one naive set of the modified AF.

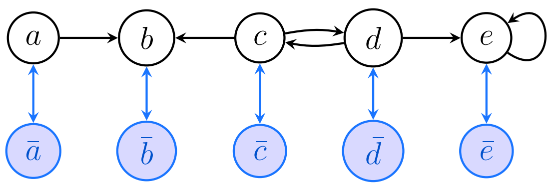



Figure 3 Illustration of Translation 3 ($cf \Rightarrow naive$).

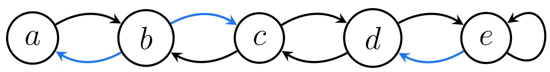



Figure 4 Illustration of Translation 4 ($cf \Rightarrow (cf|adm)$, $naive \Rightarrow (naive|prf)$).

**Translation 3** ($cf \Rightarrow naive$)**.** *The transformation $Tr(A, R) = (A', R')$, with $A' = A \cup \bar{A}$ and*

$$R' = R \cup \{(a, \bar{a}), (\bar{a}, a) \mid a \in A\}$$

*is an embedding modular faithful translation for $cf \Rightarrow naive$.*

*Proof.* For $E \in cf(F)$ define $E' = E \cup \{\bar{a} \mid a \in A_F, a \notin E\}$. Now $E'$ is maximal conflict-free in $Tr(F)$ and thus $E' \in naive(Tr(F))$. On the other hand for $E' \in naive(Tr(F))$ we observe that for each argument $a \in A_F$ either $a \in E'$ or $\bar{a} \in E'$, thus the projection $E = E' \cap A_F$ is unique. Due to the embedding property it follows that $E \in cf(F)$. □

The next translation weakens the attack relation achieving symmetry such that admissibility and conflict-freeness coincide. Then also the maximal admissible sets (preferred extensions) coincide with the maximal conflict-free sets (naive extensions). However, as we introduce new attacks between the original arguments the translation is not embedding anymore but only covering.

**Translation 4** ($cf \Rightarrow (cf|adm)$, $naive \Rightarrow (naive|prf)$)**.** *The transformation $Tr(A, R) = (A, R')$, with $R' = R \cup \{(b, a) \mid (a, b) \in R\}$, is a covering modular exact translation for $cf \Rightarrow (cf|adm)$ and $naive \Rightarrow (naive|prf)$.*

*Proof.* We have that $Tr(F)$ is a symmetric framework with the same conflicts as $F$. The results are immediate by the fact that the notion of admissibility and conflict-freeness coincide on such AFs. □

We now reconsider a translation from [20] and show that it is also a translation for $naive \Rightarrow stage$. The idea is that for each $a \in A$ we introduce a new argument $\bar{a}$ which attacks itself and is otherwise attacked only by $a$. Thus, in the translated AF $\subseteq$-incomparable conflict-free sets have also incomparable ranges.

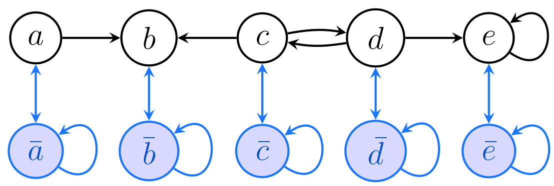



Figure 5 Illustration of Translation 5 ($adm \Rightarrow (adm|com)$, $naive \Rightarrow (naive|stage)$, $prf \Rightarrow (prf|sem)$).

**Translation 5** ($adm \Rightarrow (adm|com)$, $naive \Rightarrow (naive|stage)$, $prf \Rightarrow (prf|sem)$)**.** *The transformation $Tr(A, R) = (A', R')$, defined as $A' = A \cup \bar{A}$ and*

$$R' = R \cup \{(a, \bar{a}), (\bar{a}, a), (\bar{a}, \bar{a}) \mid a \in A\}$$

*is an embedding modular exact translation for $adm \Rightarrow (adm|com)$, $naive \Rightarrow (naive|stage)$ and $prf \Rightarrow (prf|sem)$.*

*Proof.* Observe that by definition $Tr$ equals $Tr_1$ from [20]. A detailed proof of $adm \Rightarrow (adm|com)$ and $prf \Rightarrow (prf|sem)$ is to be found there. We are left with showing that $Tr$ is an exact translation for $naive \Rightarrow (naive|stage)$. In other words for any AF $F$ we have (1) $naive(F) = naive(Tr(F))$ and (2) $naive(Tr(F)) = stage(Tr(F))$.

1. Since $Tr$ is embedding and any $\bar{a} \in \bar{A}$ is self-conflicting we have that any $E \subseteq A_{Tr(F)}$ is conflict-free in $F$ iff it is conflict-free in $Tr(F)$. Thus $naive(F) = naive(Tr(F))$.

2. Recall that any stage extension is also a naive extension. If $E \in naive(Tr(F))$ then $E^+_{Tr(F)} = E^+_F \cup \{\bar{a} \mid a \in E\}$. Considering $E' \in naive(Tr(F))$ such that $E^+_{Tr(F)} \subseteq E'^+_{Tr(F)}$ we receive $E \subseteq E'$ since any $\bar{a}$ with $a \in E$ is attacked only by $a$ and $\bar{a}$. Thus with maximality of naive extensions $E' = E$ and therefore also $naive(Tr(F)) = stage(Tr(F))$.

□

## 3.2 Not quite efficient translations

We now study combinations of semantics where no efficient translation exists and consider translations with arbitrary computational power. We start with an obvious translation from grounded semantics to naive and stage semantics, where we compute the grounded extension and make all arguments not contained in the grounded extension unacceptable by adding self-attacks.

**Translation 6** ($grd \Rightarrow (naive|grd|stage)$)**.** *The transformation $Tr(A, R) = (A, R \cup \{(a, a) \mid a \in A \setminus grd(F)\})$ is a covering exact translation for $grd \Rightarrow (naive|grd|stage)$.*

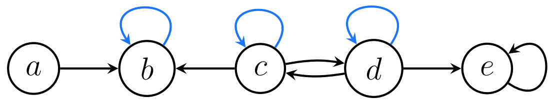



Figure 6   Illustration of Translation 6 ($grd \Rightarrow (naive|grd|stage)$).

*Proof.* Clearly none of the self-attacking arguments $a \in A \setminus grd(F)$ can be a member of any naive set (or any stage extension) of $Tr(A, R)$. Now as the grounded extension is by definition conflict-free we have that $grd(F)$ is also the only maximal conflict-free set in $Tr(A, R)$. □

Notice that although Translation 6 is not efficient in the sense of Definition 5 it can still be computed in polynomial time (but not when limited to logarithmic space).

We finally turn to translations that cannot be computed in polynomial time. First we give a generic translation to $(stage|stb|sem|prf|com)$ semantics. Let it be known that this translation requires to compute all the extensions of the original AF. The basic idea is to introduce a cloud that represents all the desired extensions with new arguments, a cloud that enforces these extensions with unidirectional attacks. We introduce new variables $\widetilde{E}_F$, one for each of the original extensions $E \in \sigma(F)$, and make them mutually conflicting such that each extension of $F'$ picks at most one of them. Further one additional argument $\widetilde{F}$ attacks all the original arguments and thus ensures that none of those can be accepted without one of the $\widetilde{E}_F$ arguments, i.e., each non-empty extension must contain one of the $\widetilde{E}_F$ arguments. Finally, an argument $\widetilde{E}_F$ then attacks all arguments not member of $E$ and thus defends all arguments belonging to $E$.

**Translation 7** ($\sigma \Rightarrow (stb|prf|com|stage|sem)$)**.** *We define* $Tr(A, R) = (A', R')$ *as*

$$
\begin{aligned}
A' &= A \cup \{\widetilde{E} \mid E \in \sigma(F)\} \cup \{\widetilde{F}\} \\
R' &= R \cup \{(\widetilde{F}, \widetilde{F}), (\widetilde{F}, a) \mid a \in A\} \\
&\quad \cup \{(\widetilde{E}, \widetilde{F}) \mid E \in \sigma(F)\} \\
&\quad \cup \{(\widetilde{E}, b) \mid E \in \sigma(F), b \in A, b \notin E\} \\
&\quad \cup \{(\widetilde{E}, \widetilde{D}) \mid E, D \in \sigma(F), E \neq D\}
\end{aligned}
$$

*For semantics with $\sigma(F) \subseteq cf(F)$ $Tr$ is an embedding translation that is faithful for $\sigma \Rightarrow stb$ and weakly faithful (with remainder set $\emptyset$) for $\sigma \Rightarrow (prf|com|sem)$.*

*For strictly non-empty cf-based semantics[2] $\sigma$, i.e., for any AF $F$ we have $\sigma(F) \subseteq cf(F)$ and $|\sigma(F)| \geq 1$, $Tr$ is a faithful translation for $\sigma \Rightarrow (stb|prf|stage|sem)$.*

*Proof.* For AF $F$, extension $E \in \sigma(F)$ and thus $E \in cf(F)$ consider $E' = E \cup \{\widetilde{E}\}$. $E'$ is conflict-free since $\widetilde{E}$ attacks only (but all) those arguments from $A_F$ not being member of $E$. Furthermore $\widetilde{E}$

[2]As far as the semantics introduced in this work are concerned this excludes only stable semantics.

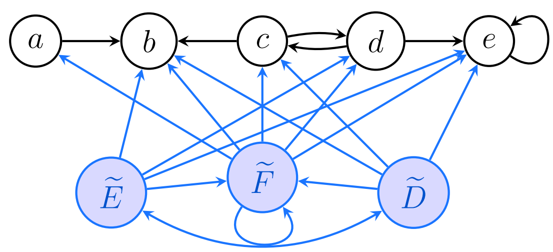



Figure 7   Illustration of Translation 7 for $prf \Rightarrow (stb|prf|com|stage|sem)$, where $E = \{a, c\}$ and $D = \{a, d\}$.

attacks any $\widetilde{D}$ with $D \in \sigma(F)$, $D \neq E$, and $\widetilde{E}$ also attacks the argument $\widetilde{F}$. Hence $E' \in stb(Tr(F))$ and thus by the known relations between semantics also $E' \in stage(Tr(F)) = sem(Tr(F))$, $E' \in com(Tr(F))$ and $E' \in prf(Tr(F))$.

We now consider $E' \in \sigma(Tr(F))$.

(a) If we assume that $|\sigma(F)| \geq 1$ then (by the above observations in the first part of the proof) there exists a stable extension and hence stable, stage and semi-stable extensions coincide. Thus, in the following we will not consider stage semantics explicitly, and conclusively can also assume admissibility. We know that $E' \in com(Tr(F))$ and $E \neq \emptyset$. Due to the attacks $(\widetilde{E}, \widetilde{D})$, at most one $\widetilde{E}$ is member of $E'$. We observe that $E' \cap \{\widetilde{E} \mid E \in \sigma(F)\} \neq \emptyset$, since all arguments from $A$ are attacked by the argument $\widetilde{F}$, which in turn is attacked only by arguments of the form $\widetilde{E}$. We can thus pick the unique $\widetilde{E} \in E' \cap \{\widetilde{E} \mid E \in \sigma(F)\}$. But then $\widetilde{E}$ defends all arguments $a \in E$ and it follows immediately that $E' \cap A = E \in \sigma(F)$.

(b) If we assume that $|\sigma(F)| = 0$, i.e., $\sigma(F) = \emptyset$, then there are no arguments $\widetilde{E}$ and all arguments are attacked by the argument $\widetilde{F}$ while $\widetilde{F}$ is only attacked by itself. Thus in the case of stable semantics we have $stb(Tr(F)) = \emptyset = \sigma(F)$. Furthermore we observe that $com(Tr(F)) = prf(Tr(F)) = sem(Tr(F)) = \{\emptyset\}$. As we have remainder set $\emptyset$ this returns the empty set of extensions, i.e., $\emptyset = \sigma(F)$. □

Next we present an embedding exact translation from $sem \Rightarrow prf$. We already know that each semi-stable extension is also a preferred one. This exact translation thus has to make sure that possible additional preferred extensions are eliminated. The idea behind the following translation is to eliminate preferred extensions that are not semi-stable by modifying the AF such that these extensions and their subsets are no longer admissible. That is for each such extension $E$ we introduce a new argument $\widetilde{E}$ that attacks all arguments in $E$ but is not attacked by any argument in $E$. As we do not want to affect other extensions $\widetilde{E}$ must be attacked by all the other preferred extensions, which is realized by each argument not member of $E$ attacking $\widetilde{E}$.

**Translation 8** $(sem \Rightarrow (prf|sem))$**.** *We use $\mathcal{P}(F)$ to denote the set of less welcome preferred extensions, that is $\mathcal{P}(F) = prf(F) \setminus sem(F)$. The transformation $Tr(A, R) = (A', R')$ with*

$$A' = A \cup \{\widetilde{P} \mid P \in \mathcal{P}(F)\}$$

$$R' = R \cup \{(a, \widetilde{P}), (\widetilde{P}, \widetilde{P}), (\widetilde{P}, b) \mid P \in \mathcal{P}(F), a \in A \setminus P, b \in P\}$$

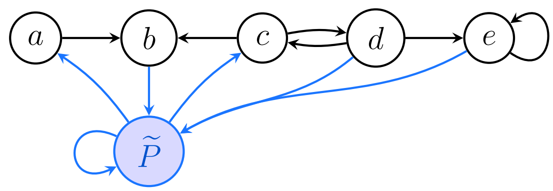


Figure 8 Illustration of Translation 8 ($sem \Rightarrow (prf|sem)$), where $P = \{a, c\}$.

*is an embedding exact translation for* $sem \Rightarrow (prf|sem)$.

*Proof.* Observe that additional arguments of $Tr(F)$ are of the form $\widetilde{P}$ with $P \in \mathcal{P}(F)$ and thus are self-attacking by definition. Now due to the embedding property for a set $E \subseteq A'$ we have that $E$ is conflict-free in $Tr(F)$ iff it is conflict-free in $F$ and further if $E$ is admissible in $Tr(F)$ then it is admissible in $F$.

First direction, $prf(Tr(F)) \subseteq sem(F)$: We look at some $P \in prf(Tr(F))$. We already know that $P \in prf(F)$. But now immediately also $P \in sem(F)$ for otherwise for $P \in prf(F) \setminus sem(F)$ by definition necessarily $\widetilde{P} \rightarrowtail^{R'} P$ while $P \not\rightarrowtail^{R'} \widetilde{P}$.

Second direction, $sem(F) \subseteq prf(Tr(F))$: Take into account some $E \in sem(F)$, and recall that for any $F$ we have $sem(F) \subseteq prf(F)$. Then for any $P \in \mathcal{P}(F)$ we have that $E \setminus P \neq \emptyset$ and hence by definition $E \rightarrowtail \widetilde{P}$. This ensures that admissibility of $E$ in $F$ is preserved in $Tr(F)$, and subsequently $E \in prf(Tr(F))$. □

Towards a translation for $stage \Rightarrow prf$ we first have to recall a translation from [20] for $stage \Rightarrow sem$. This translation is based on two ideas. First, all the original attacks are made symmetric such that conflict-freeness and admissibility coincide. Second, the range of the extensions in the original AF is mirrored on new arguments $a'$.

**Translation 9** ([20] - $stage \Rightarrow (stage|sem)$)**.** *The transformation* $Tr(A, R) = (A', R')$ *with* $A' = A \cup A'$ *and*

$$R' = R \cup \{(b, a), (a, b') \mid (a, b) \in R\} \\ \cup \{(a, b) \mid a \in A, (b, b) \in R\} \\ \cup \{(a, a'), (a', a') \mid a \in A_F\}$$

*is a monotone covering exact translation for* $stage \Rightarrow (stage|sem)$.

Now given an exact translation for $stage \Rightarrow sem$ and another exact translation for $sem \Rightarrow prf$ we can use the transitivity of exact translations to obtain $stage \Rightarrow prf$.

**Corollary 1** ($stage \Rightarrow prf$)**.** *Take into account Translation 8 as* $Tr_8$ *and Translation 9 as* $Tr_9$*. The transformation* $Tr = Tr_8 \circ Tr_9$ *is a covering exact translation for* $stage \Rightarrow prf$.

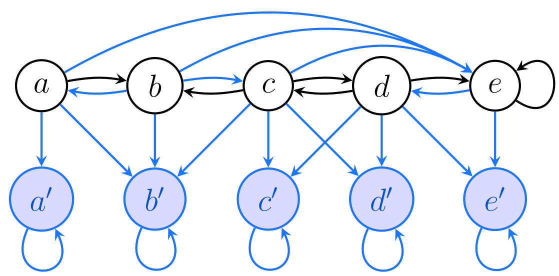



Figure 9 Illustration of Translation 9 ($stage \Rightarrow (stage|sem)$).

# 4 Impossibility Results

In the previous section we discussed a broad range of translations between semantics. It will not have escaped the attentive reader that we did not provide translations for all combinations. In this section we investigate the missing gaps and ask whether it actually is possible to provide translations in these cases. That is we provide a couple of results to show that in certain cases exact or even faithful translations are impossible. This analysis completes our picture of intertranslatability between the studied semantics.

We start with impossibility results that rely on the incompatibility of very basic properties of semantics. These are given by the Theorems 1-6. The first of these impossibility results is a straightforward observation of the unique status property of grounded semantics and can immediately be extended to other unique status semantics.

**Theorem 1.** *There is no translation which is weakly faithful for*

$$(cf|naive|stb|adm|prf|com|stage|sem) \Rightarrow grd.$$

*Proof.* Any semantics of interest but $grd$ can possess more than one extension, for instance for the AF $(\{a, b\}, \{(a, b), (b, a)\})$. As grounded semantics always proposes a single extension there cannot be a translation from any of the stated semantics to $grd$. □

The next theorem exploits that stable semantics cannot realize the empty set as an extension.[3] Thus all semantics that might have the empty set as an extension cannot be weakly exactly translated to stable semantics.

**Theorem 2.** *There is no translation which is weakly exact for*

$$(cf|naive|adm|prf|com|grd|stage|sem) \Rightarrow stb.$$

*Proof.* Consider the AF $(\{a\}, \{(a, a)\})$ where all the semantics from the left hand side propose the empty set as being an extension. Recall that we require AFs to be non-empty, thus w.l.o.g. for some $x$ we have $x \in A_{Tr(F)}$. Now by definition of stable semantics the empty set can not be a stable extension, as $x$ is neither attacked by nor a member of the empty set. □

[3]Notice that we only allow nonempty argumentation frameworks.

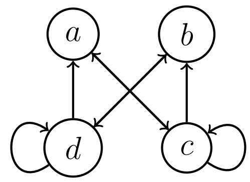



Figure 10    Argumentation framework serving as a counter example for Theorem 4.

Next we use that the empty set is always a conflict-free as well as an admissible set. For more sophisticated semantics however the extension status of the empty set depends on the actual AF.

**Theorem 3.** *There is no translation which is weakly exact for*

$$(naive|prf|com|grd|stage|sem) \Rightarrow (cf|adm).$$

*Proof.* For the semantics on the left hand side it might occur that the empty set is an extension, e.g., for the AF $(\{a\}, \{(a,a)\})$. However this is not always the case, e.g., for the AF $(\{a\}, \{\})$. By the first observation, for any weakly exact translation, we cannot have the empty set as a remainder set. The second observation together with the fact that the empty set is always admissible and thus conflict-free yields that, for any weakly exact translation, the empty set needs to be a remainder set. Hence, there is no weakly exact translation $\sigma \Rightarrow (cf|adm)$. □

Given an arbitrary AF $F$ the set $cf(F)$ of conflict-free sets is what is sometimes called downward-closed, i.e. if a set $S$ is conflict-free than also all subsets of $S$ are conflict-free. This does not hold for any other semantics under our consideration.

**Theorem 4.** *There is no translation which is weakly faithful for*

$$(naive|stb|adm|prf|com|grd|stage|sem) \Rightarrow cf.$$

*Proof.* For $cf$ semantics any subset of any conflict-free set again is a conflict-free set. Now consider the AF $F = (A, R)$ as depicted in Figure 10 with $A = \{a, b, c, d\}$ and $R = \{(a,c), (c,a), (b,d), (d,b), (d,a), (d,d), (c,b), (c,c)\}$. We have that for all of the left hand side semantics the set $\{a, b\}$ is an extension while $\{a\}$ is rebutted as an extension (because of not being admissible and/or because of not being maximal). Now recall that the remainder sets are required to contain only arguments not occurring in the original AF. Any set containing $a$ can thus not be a remainder set. As $\{a\} \subseteq \{a, b\}$, there is no AF $F$ such that there is a set $S$ with $S \cup \{a, b\} \in cf(F)$ and $S \cup \{a\} \notin cf F$. Thus there is no weakly faithful translation $\sigma \Rightarrow cf$. □

For some semantics different extensions may be subsets of each other, while for other semantics this is not possible. This gives further restrictions on exact translations.

**Theorem 5.** *There is no translation which is weakly exact for*

$$(cf|adm|com) \Rightarrow (naive|stb|prf|stage|sem).$$

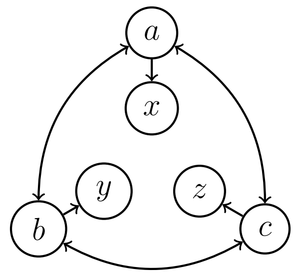



Figure 11 Argumentation framework serving as a counter example for Theorem 7.

*Proof.* For $cf$, $adm$ and $com$ it might be the case that an extension forms a proper subset of another extension. For instance consider the AF $F = (A, R)$ as depicted in Figure 10 with $A = \{a, b, c, d\}$ and $R = \{(a, c), (c, a), (b, d), (d, b), (d, a), (d, d), (c, b), (c, c)\}$. We have $cf(F) = \{\emptyset, \{a\}, \{b\}, \{a, b\}\}$ and $adm(F) = com(F) = \{\emptyset, \{a, b\}\}$. All semantics on the right hand side, $\sigma' \in \{naive, stb, prf, stage, sem\}$, satisfy that their extensions comprise the $\subseteq$-maximality property and thus cannot realize $\emptyset$ and simultaneously $\{a, b\}$ as extensions for the same AF. □

For complete semantics we have that if there are at least two extensions then there is one extension, i.e., the grounded, which is a proper subset of all the other extensions. This causes problems if we want to exactly translate two or more $\subseteq$-maximal extensions to complete semantics.

**Theorem 6.** *There is no weakly exact translation for* $(naive|prf|stage|sem) \Rightarrow com$.

*Proof.* Towards a contradiction we assume that such a translation $Tr$ exists. Now observe that for $\sigma \in \{naive, prf, sem, stage\}$ there are AFs such that $\emptyset \in \sigma(F)$. Thus the empty set is not a member of the remainder sets of $Tr$.

Now take into account the AF $F = (A, R)$ with $A = \{a, b\}$ and $R = \{(a, b), (b, a)\}$. Then $\sigma(F) = \{\{a\}, \{b\}\}$. The grounded extension can be defined as the least complete extension, thus with $\{a\}, \{b\} \in com(Tr(F))$ we need $\emptyset = grd(Tr(F))$ and thus $\emptyset \in com(Tr(F))$. Thus for $Tr$ to be a weakly exact translation we need $\emptyset$ to be a remainder set, contradicting our previous observation. □

The next result makes use of a more sophisticated construction. On a high level we use the fact that often the existence of naive extensions implies that a certain set is conflict-free and thus must be part of an additional naive extension. The idea follows the lines of the impossibility result for $sem \Rightarrow stage$ [20] where an AF $F$ for semi-stable semantics is constructed such that the semi-stable extensions of $F$, when translated to stage semantics, enforce another unwanted stage extension.

**Theorem 7.** *There is no weakly faithful translation for*

$$(stb|adm|prf|com|stage|sem) \Rightarrow (cf|naive).$$

Table 2 Final results for (weakly) faithful/exact translations.

| | $cf$ | $naive$ | $grd$ | $adm$ | $stb$ | $com$ | $prf$ | $sem$ | $stage$ |
|---|---|---|---|---|---|---|---|---|---|
| $cf$ | ✓ | ✓/ - | – | ✓ | ✓/ - | ✓ | ✓/ - | ✓/ - | ✓/ - |
| $naive$ | – | ✓ | – | ✓/ - | ✓/ - | ✓/ - | ✓ | ✓ | ✓ |
| $grd$ | – | ✓ | ✓ | ✓/ - | ✓/ - | ✓ | ✓ | ✓ | ✓ |
| $adm$ | – | – | – | ✓ | ✓/ - | ✓ | ✓/ - | ✓/ - | ✓/ - |
| $stb$ | – | – | – | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ |
| $com$ | – | – | – | ✓/ - | ✓/ - | ✓ | ✓/ - | ✓/ - | ✓/ - |
| $prf$ | – | – | – | ✓/ - | ✓/ - | ✓/ - | ✓ | ✓ | ✓/ - |
| $sem$ | – | – | – | ✓/ - | ✓/ - | ✓/ - | ✓ | ✓ | ✓/ - |
| $stage$ | – | – | – | ✓/ - | ✓/ - | ✓/ - | ✓ | ✓ | ✓ |

*Proof.* To be more specific in the following we show that any weakly faithful translation of the desired kind would require some remainder set $E$ with $E \cap A \neq \emptyset$, violating our definition of remainder sets.

Now for $\sigma \in \{stb, adm, prf, com, stage, sem\}$ and $\sigma' \in \{cf, naive\}$ take a weakly faithful translation $Tr : \sigma \Rightarrow \sigma'$ as given. Consider the AF $F = (A, R)$ as depicted in Figure 11 with $A = \{a, b, c, x, y, z\}$ and $R = \{(\alpha, \beta) \mid \alpha \neq \beta \in \{a, b, c\}\} \cup \{(a, x), (b, y), (c, z)\}$. Observe that for $E_a = \{a, y, z\}, E_b = \{b, x, z\}, E_c = \{c, x, y\}, X = \{x, y, z\}$ we have that $E_a, E_b, E_c$ are $\sigma$-extensions while $X \notin \sigma(F)$. So for any $E_\alpha$ there has to be some $E'_\alpha \in \sigma'(Tr(F))$ such that $E_\alpha \subseteq E'_\alpha$. Thus immediately $X \in cf(Tr(F))$, since pairwise conflict-freeness of $x, y, z$ is granted by $E'_a$, $E'_b$ and $E'_c$. For any conflict-free set $X$ in any AF $F'$ there has to be some extension $E \in naive(F')$ such that $X \subseteq E$. Subsequently $X$ or $E$ need to be remainder sets. □

We summarize all our negative results together with the intertranslatability results from Section 3 in Table 2.

# 5 Intertranslatability & Monotonicity

In Section 3 we provided translations whenever possible. Most of them have the desired properties of being covering (or even embedding) and modular. However, Translations 6, 7, 8, and 1 are not modular and not even monotone. While Proposition 1 gives evidence of the impossibility of corresponding modular translations (since, due to [20], there is no efficient translation at all) it might still be possible to make the translations monotone. Following [20], monotonicity is a desired property in scenarios where it is impossible to withdraw already proposed arguments. For instance consider a setting with several agents interchanging arguments, but using different semantics. An agent might not agree to forget arguments already communicated to him and thus the translation of an augmented AF must respect the already existing translation.

In this section we consider the question of whether we can use above translations to derive monotone versions. That is, whenever possible we provide monotone translations and otherwise we show that it is impossible to get monotone translations. When it comes to impossibility results we also consider the property of being embedding and show that in all cases where our translations are not embedding, in fact comparable embedding translations even are impossible.

## 5.1 Monotone Translations

In the following we give three monotone translations covering a broad range of semantics. We start with a generalization of Translation 7 that gives a monotone faithful translation for $\sigma \Rightarrow (stage|stb|sem|prf)$ and a monotone weakly faithful translation for $\sigma \Rightarrow com$. The idea is to encode each subframework and all of its extensions as arguments in the modified AF. Any argument $\widetilde{E}_{F_i}$ corresponding to some extension $E$ of some subframework $F_i$ then attacks all arguments corresponding to subframeworks $F_j$ of $F_i$, $F_j \subsetneq F_i$, i.e., for $E \in \sigma(F_i)$ we get $\widetilde{E}_{F_i} \rightarrowtail \widetilde{F}_j$ and $\widetilde{E}_{F_i} \rightarrowtail \widetilde{E}'_{F_j}$ for $E' \in \sigma(F_j)$. Thus arguments corresponding to proper subframeworks of $F$ are hindered from building extensions in $Tr(F)$.

**Translation 10** ($\sigma \Rightarrow (stb|prf|com|stage|sem)$). *We define* $Tr(A, R) = (A', R')$ *as*

$$
\begin{aligned}
A' &= A \cup \{\widetilde{F}_i,\ \widetilde{E}_{F_i} \mid F_i \subseteq F,\ E \in \sigma(F_i)\} \\
R' &= R \cup \{(\widetilde{E}_{F_i}, \widetilde{F}_i), (\widetilde{F}_i, \widetilde{F}_i), (\widetilde{F}_i, a) \mid a \in A_{F_i}\} && (1)\\
&\quad \cup \{(\widetilde{E}_{F_i}, b) \mid b \in A_{F_i} \setminus E\} && (2)\\
&\quad \cup \{(\widetilde{E}_{F_i}, \widetilde{E}'_{F_i}) \mid E \neq E'\} && (3)\\
&\quad \cup \{(\widetilde{E}_{F_i}, \widetilde{E}_{F_k}), (\widetilde{E}_{F_i}, \widetilde{F}_k) \mid F_k \subsetneq F_i\} && (4)
\end{aligned}
$$

*For strictly non-empty* $cf$*-based semantics*[4] $\sigma$ *(i.e.,* $|\sigma(F)| \geq 1$ *and* $\sigma(F) \subseteq cf(F)$*)* $Tr$ *is an embedding monotone faithful translation for* $\sigma \Rightarrow (stb|prf|stage|sem)$ *and a weakly faithful with remainder set* $\emptyset$ *translation for* $\sigma \Rightarrow com$.

To achieve monotonicity we introduced arguments $\widetilde{F}_i$ (where $F_i \subseteq F$) to represent subframeworks, and arguments $\widetilde{E}_{F_i}$ (where $E \in \sigma(F_i)$) to encode extensions of those subframeworks. The attacks in (1) and (2) ensure that a selected extension defends its arguments. The mutual attacks in (3) ensure that only one extension is selected while (4) ensures that only extensions of the full original framework are selected. An illustration of this translation as applied to a simple AF of merely two arguments and one attack is shown in Figure 12.

*Proof.* For AF $F$, extension $E_F \in \sigma(F)$ and thus $E_F \in cf(F)$ consider $E = E_F \cup \{\widetilde{E}_F\}$. $E$ is conflict-free since $\widetilde{E}_F$ attacks only (but all) those arguments from $A_F$ not being member of $E_F$. Furthermore $\widetilde{E}_F$ attacks all arguments $\widetilde{F}_i$ for $F_i \subseteq F$ and $\widetilde{E}_F$ attacks all $\widetilde{D}_{F_i}$ for $D_{F_i} \in \sigma(F_i)$ where $D_{F_i} \neq E_F$ (or $F_i \neq F$, as subframeworks might provide the same extension). Hence $E \in stb(Tr(F))$ and thus also $E \in stage(Tr(F))$, $E \in sem(Tr(F))$, $E \in prf(Tr(F))$ and $E \in com(Tr(F))$.

We now consider $\emptyset \neq E \in com(Tr(F))$. For any $F_i \subsetneq F$ and $D_{F_i} \in \sigma(F_i)$ we have that $\widetilde{D}_{F_i}$ is not a member of $E$ since the only arguments defending $\widetilde{D}_{F_i}$ against the by requirement

[4] As far as the semantics introduced in this work are concerned this excludes only stable semantics.

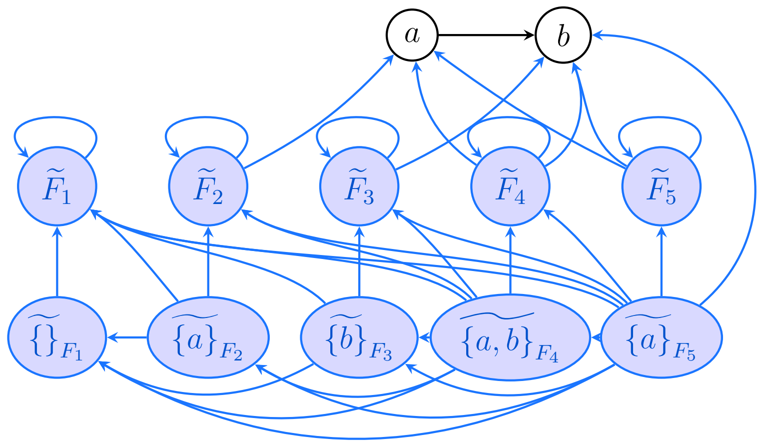



Figure 12 Illustration of Translation 10 for $prf \Rightarrow (stb|prf|com|stage|sem)$. With $F = (\{a,b\},\{(a,b)\})$ and $F_1 = (\{\},\{\})$, $F_2 = (\{a\},\{\})$, $F_3 = (\{b\},\{\})$, $F_4 = (\{a,b\},\{\})$, and $F_5 = F = (\{a,b\},\{(a,b)\})$.

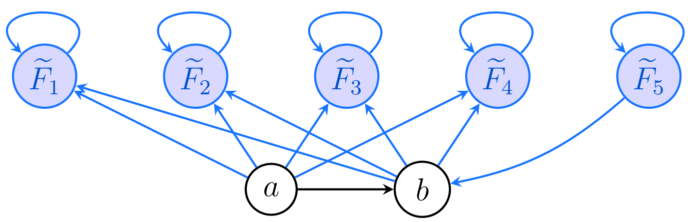



Figure 13 Illustration of Translation 11 ($grd \Rightarrow (prf|com|grd|sem)$). With $F = (\{a,b\},\{(a,b)\})$ and $F_1 = (\{\},\{\})$, $F_2 = (\{a\},\{\})$, $F_3 = (\{b\},\{\})$, $F_4 = (\{a,b\},\{\})$, and $F_5 = F = (\{a,b\},\{(a,b)\})$.

non-empty set $\{\widetilde{E}'_F \mid E' \in \sigma(F)\}$ are members of this set and thus also attacking $\widetilde{D}_{F_i}$. Furthermore, by (3), at most one $\widetilde{E}'_F$ is member of $E$. We observe that there is no $D \in adm(Tr(F))$ such that $D \cap \{\widetilde{E}'_F \mid E' \in \sigma(F)\} = \emptyset$, since all arguments from $A$ are attacked by the argument $\widetilde{F}$, which in turn is attacked only by arguments $\widetilde{E}'_F$. We can thus pick the unique $\widetilde{E}_F \in E \cap \{\widetilde{E}'_F \mid E' \in \sigma(F)\}$. But then $\widetilde{E}_F$ defends all arguments $a \in E$ and it follows immediately that $E \cap A = E_F \in \sigma(F)$. □

Next we give a monotone translation for $grd \Rightarrow (prf|com|sem)$. Again to ensure monotonicity, for a given AF $F$ and all $F_i \subseteq F$ we use arguments $\widetilde{F}_i$ to represent subframeworks. For each $\widetilde{F}_i$ we introduce attacks to ensure that if $\widetilde{F}_i$ is selected then only the arguments from the grounded extension of $F_i$ remain admissible (attack set (1) below). Further, we add attacks that ensure that arguments that are corresponding to proper subframeworks of $F$ are disabled (attack set (2) below). An illustration of the following translation is given in Figure 13.

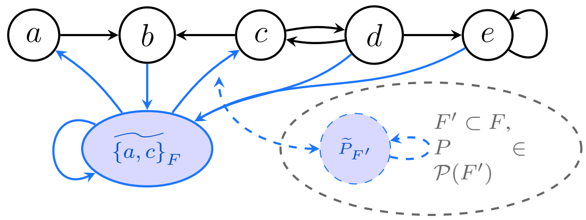



Figure 14  Illustration of Translation 12 ($sem \Rightarrow (prf|sem)$).

**Translation 11** ($grd \Rightarrow (prf|com|grd|semi)$)**.** *We define the transformation* $Tr(A,R) = (A', R')$ *as*

$$A' = A \cup \{\widetilde{F_i} \mid F_i \subseteq F\}$$
$$R' = R \cup \{(\widetilde{F}_i, \widetilde{F}_i), (\widetilde{F}_i, a) \mid F_i \subseteq (A,R), a \in A_{F_i} \setminus grd(F_i)\} \quad (1)$$
$$\cup \{(a, \widetilde{F}_k) \mid F_k \subsetneq F_i \subseteq F, a \in A_{F_i}\} \quad (2)$$

*$Tr$ is an embedding monotone exact translation for $grd \Rightarrow (prf|com|grd|sem)$.*

*Proof.* The argument $\widetilde{F}$ is attacked only by itself yet attacks any argument not being member of the grounded extension of $F$, thus disabling the arguments $A_F \setminus grd(F)$ for inclusion in any admissible extension. If $grd(F) = \emptyset$ we clearly have $\emptyset$ as only extension for all the semantics of interest. Now consider the case where $grd(F) \neq \emptyset$. Then there is an argument $a \in A$ that is not attacked at all in $F$ and therefore in all subframeworks of $F$. Hence $a$ is in the grounded extension of all subframeworks containing $a$ and thus in the grounded extension of $Tr(F)$ (as it is not attacked in $Tr(F)$). Now we can ignore all arguments $\widetilde{F_i}$ with $F_i \subsetneq F$ since in $Tr(F)$ they are attacked by $a$. It follows that the grounded extension of $F$ is also the grounded extension of $Tr(F)$. Further as all the other arguments are unacceptable the semantics of interest collapse. □

To achieve a monotone exact translation for $sem \Rightarrow prf$ we generalize Translation 8. The idea is to apply the procedure from Translation 8 to each subframework $F_i$ of $F$ and additionally add attacks from each argument in $A_{F_i}$ to each argument produced for a proper subframework of $F_i$.

**Translation 12** ($sem \Rightarrow (prf|sem)$)**.** *We use $\mathcal{P}(F_i)$ to denote $\mathcal{P}(F_i) = prf(F_i) \setminus sem(F_i)$. The transformation $Tr(A,R) = (A', R')$ with*

$$A' = A \cup \{\widetilde{P}_{F_i} \mid F_i \subseteq (A,R), P \in \mathcal{P}(F_i)\}$$
$$R' = R \cup \{(a, \widetilde{P}_{F_i}), (\widetilde{P}_{F_i}, \widetilde{P}_{F_i}), (\widetilde{P}_{F_i}, b) \mid a \in A_{F_i} \setminus P_{F_i}, b \in P_{F_i}\} \quad (1)$$
$$\cup \{(a, \widetilde{P}_{F_k}) \mid F_k \subsetneq F_i \subseteq F, a \in A_{F_i}\} \quad (2)$$

*is an embedding monotone exact translation for $sem \Rightarrow (prf|sem)$.*

*Proof.* Observe that any conflict-free set in $Tr(F)$ (for $F = (A, R)$) consists of arguments $a \in A$ only. For $F_i \subsetneq F$ we have that additional arguments of the form $\widetilde{P}_{F_i}$ for $P \in \mathcal{P}(F_i)$ are attacked by all $a \in A$, we can thus restrict ourselves to the set $S \subseteq A \cup \{\widetilde{P}_F \mid P \in \mathcal{P}(F)\}$. Now assume $E \in sem(A, R)$. Since $E \setminus P \neq \emptyset$ for all $P \in \mathcal{P}(F)$ by definition, we have that $E$ attacks all $\widetilde{P}_F$ for $P \in Tr(F)$ and thus $E \in prf(Tr(F))$. On the other hand we might look at some $E \in prf(Tr(F))$ and assume for a contradiction that $E \notin sem(F)$. As admissibility of $E$ in $Tr(F)$ implies admissibility of $E$ in $F$ there has to be some $P \in \mathcal{P}(F)$ such that $E \subseteq P$. But now $E$ is attacked by the argument $\widetilde{P}_F$ and defended only by arguments $a \in A \setminus P$ and thus cannot be admissible. □

We can again use the transitivity of exact translations to obtain a monotone translation for $stage \Rightarrow prf$.

**Corollary 2** ($stage \Rightarrow prf$)**.** *Considering Translation 12 as $Tr_{12}$ and Translation 9 as $Tr_9$, then the transformation $Tr = Tr_{12} \circ Tr_9$ is a covering monotone exact translation for $stage \Rightarrow prf$.*

## 5.2 Impossibility Results

In the following we give impossibility results for several intertranslatabilities regarding the embedding and monotonicity properties, that is we show that the presented translations are optimal w.r.t. these two translational properties.

First consider translations from grounded semantics to naive and stage semantics. This impossibility result relies on the fact that an argument that is not self-attacking appears always in at least one conflict-free set and thus also in at least one naive set.

**Theorem 8** ($grd \Rightarrow (naive|stage)$)**.** *There is no translation which is*

1. *embedding or monotone weakly* faithful *for $grd \Rightarrow naive$.*

2. *embedding or monotone weakly* exact *for $grd \Rightarrow stage$.*

*Proof.* For the semantics of interest we observe that for embedding or monotone translations $Tr$ with $a \in A_F$ immediately also $a \in A_{Tr(F)}$. Furthermore, we can allow $(a, a) \in R_{Tr(F)}$ if and only if $(a, a) \in R_F$. For embedding this is right by definition for monotone due to expandability, i.e. by the fact that we can extend each AF such that argument $a$ becomes part of the grounded extension as long $(a, a) \notin R_F$. We refer to these observations by the term inheritance for the realm of this proof.

Take into account the AFs $F = (A, R)$ and $F' = (A, R')$ with $A = \{a, b, c\}$, $R = \{(b, c), (c, b)\}$ and $R' = R \cup \{(a, b)\}$. Now $grd(F) = \{a\}$ and $grd(F') = \{a, c\}$. For a contradiction we assume existence of a translation $Tr : grd \Rightarrow \sigma$ of the desired kind. For $\sigma = naive$ due to inheritance we deduce that $(c, c) \notin R_{Tr(F)}$. Hence there exists an extension $E \in naive(Tr(F))$ with $c \in E$ the latter implying that $E$ cannot be a remainder set, a contradiction. For $\sigma = stage$ we observe that due to inheritance and exactness there has to be some conflict between $a$ and $c$ in $Tr(F)$, thus $Tr$ cannot be embedding. If $Tr$ is monotone then from $F \subseteq F'$ we conclude that the conflict between $a$ and $c$ also occurs in $Tr(F')$, a contradiction to $\{a, c\} \in stage(Tr(F))$. □

Next we consider translations from $cf$ and $naive$ to admissibility based semantics. We show that there is no translation which is embedding and (weakly) exact.

**Theorem 9.** *There is no translation for* $(cf|naive) \Rightarrow (stb|adm|prf|com|sem)$ *which is embedding and weakly exact.*

*Proof.* For a contradiction we assume that such a translation $Tr$ happens to exist. We take into account the AF $F = (\{a, b\}, \{(a, b)\})$. We have $cf(F) = \{\{a\}, \{b\}, \emptyset\}$ and $naive(F) = \{\{a\}, \{b\}\}$. Since $b$ is attacked by $a$ in $F$ and with the premise of embedding in mind for any admissible set $E$ with $b \in E$ we need $E$ to attack $a$. So either $b$ attacks $a$ and the translation is not embedding or some from $b$ different argument $c \in E$ attacks $a$ and the translation is not exact. With the observation that stable, semi-stable, preferred and complete semantics are all based on admissibility we finish this proof. □

**Remark 1.** *The AF* $F = (A, R)$ *with* $A = \{a, b, c\}$ *and* $R = \{(a, b), (b, c), (c, c)\}$ *is used in [20] to show impossibility of embedding weakly exact translations* $stage \Rightarrow sem$*. The stage extensions of F are given by* $stage(F) = \{\{a\}, \{b\}\}$*. For an embedding translation* $Tr$ *we have that in the AF* $Tr(F)$ *the set* $\{b\}$ *is attacked by* $a$ *but* $a$ *is not counterattacked by* $\{b\}$*. Hence* $\{b\}$ *is not admissible and thus not semi-stable. Hence,* $Tr$ *cannot be exact for* $stage \Rightarrow sem$*.*

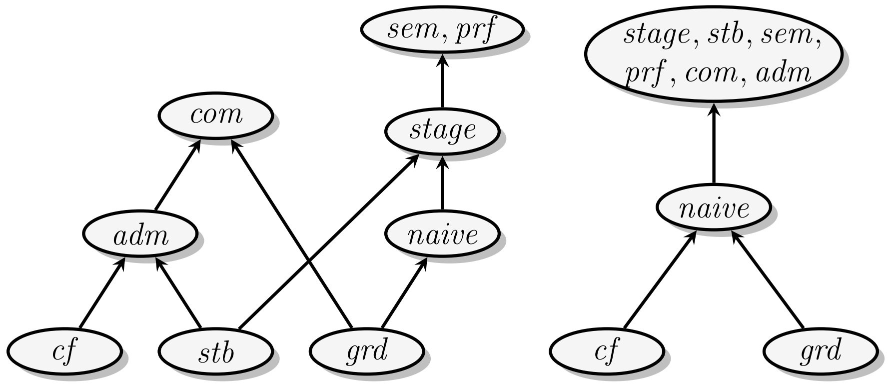
*Immediately, by the same argument we get that there is no embedding weakly exact translation for* $stage \Rightarrow prf$*.*

In summary, one can say that in most cases where we have a translation one can also find one that is embedding and monotone, with some notable exceptions as listed below. Notice, that embedding seems to be harder to achieve as whenever we have an embedding translation then we also have a monotone one.

- Concerning monotone translations we have that there is no weakly faithful for $grd \Rightarrow naive$ and no weakly exact for $grd \Rightarrow stage$.
- For embedding translations further there are no weakly exact translations from $cf$ or $naive$ to any other semantics under our considerations and no weakly exact translations for $stage \Rightarrow sem$ and $stage \Rightarrow prf$.

# 6 Discussion

We studied expressiveness of argumentation semantics via the intertranslatability of argumentations semantics. Driven by the goal of understanding expressiveness, in contrast to previous work on intertranslatability, we did not restrict ourselves to efficiently computable transformations but all our translations have access to arbitrary computational resources. In Table 2 we gave a full picture of intertranslatability between the semantics under our considerations (when neglecting computational costs). This allows us to draw hierarchies of expressiveness for argumentation semantics. Our results complement and strengthen results from previous work in the following ways. First it appears that certain translations remain impossible regardless of the computational effort one is willing to




(a) (weakly) exact (b) (weakly) faithful

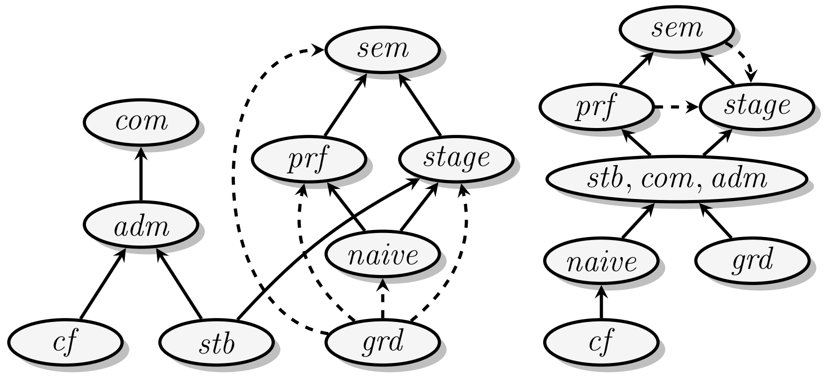
Figure 15 Results for inefficient intertranslatability.

put into the translation. Second, in several cases where no efficient translation exists, we showed that if only we accept high computational costs (time and space) then translations satisfying in fact still nice properties become possible.

Figure 15 visualizes the hierarchies of expressiveness of the chosen argumentation semantics for (a) (weakly) exact translations and (b) (weakly) faithful translations. A solid path from a semantics $\sigma$ to a semantics $\sigma'$ expresses that there is an exact, or faithful respectively, translation for $\sigma \Rightarrow \sigma'$. Furthermore, if for two semantics $\sigma, \sigma'$ there is no path from $\sigma$ to $\sigma'$ then it is proven that there is no such exact, faithful respectively, translation for $\sigma \Rightarrow \sigma'$. If several semantics are in the same composite-node they are equivalent w.r.t. the notion of translation, that is each of the semantics in the node can be translated to all the other semantics in the node. For instance we have that there is a weakly exact translation for $cf \rightarrow com$ as there is a path in Figure 15 (a) while there is no weakly faithful translation for $prf \rightarrow naive$ as there is no such path in Figure 15 (b).

For a better comparison in Figure 16 we restate the results on efficient intertranslatability from [20] together with our results on conflict-free and naive semantics. Again a solid path from a semantics $\sigma$ to a semantics $\sigma'$ expresses that there is an exact, or faithful respectively, translation for $\sigma \Rightarrow \sigma'$, and the absence of a path tells us that there is no translation $\sigma \Rightarrow \sigma'$. Additionally, in Figure 16, we have dashed lines that correspond to open problems, i.e., the cases where we do not yet know whether there exists an efficient translation or not.

Let us highlight some differences between the hierarchies for general translations as opposed to efficient translations. When neglecting computational costs we have exact translations for $grd \Rightarrow (com|stage|prf|sem)$; for efficient translations $grd \Rightarrow com$ was shown to be incompatible with common complexity assumptions on the polynomial hierarchy while the other cases still represent open questions. Concerning (weakly) faithful translations we have that $stage, stb, sem, prf, com, adm$ can be translated to each other while upon the condition of (weakly) faithful translations being efficient these semantics form at least three levels of intertranslatability. Our interpretation of these results on faithful translations is the following: When allowing projections to the arguments of interest the mentioned semantics have the same capabilities of expressing things, however certain constructs can be more succinctly represented for semantics




(a) (weakly) exact (b) (weakly) faithful

Figure 16 Results for efficient intertranslatability. Results from [20] complemented by the results on $cf$ and $naive$.

which are higher in the hierarchy of efficient (weakly) faithful translations. For instance preferred and semi-stable semantics have the same expressiveness when it comes to inefficient translations, but there is no efficient translation from semi-stable to preferred semantics. This suggests that certain sets of extensions can be represented easier in terms of semi-stable semantics than in terms of preferred semantics.

## 6.1 Related Work

As already mentioned our work builds on previous work on the intertranslatability of argumentation semantics [20]. The main difference being that [20] focuses on efficient translations, which mirrors the computational motivation behind it. In this work we are interested in expressiveness of argumentation semantics and thus also consider translations for the cases where no efficient translation is possible. We also refer the reader back to Section 2, in particular Table 1, where we list the results of [20] that also apply to our setting.

Recently, a different approach to characterize expressiveness of argumentation semantics via so called signatures has been proposed [15, 16]. While driven by a similar motivation there are significant differences to our approach. The aim there is to characterize the sets of extensions (not a single extension) that are possible for a certain semantics, i.e., sets of extensions that can be expressed by a semantics, with so called signatures, while our work is motivated by comparing the expressiveness of different semantics. Due to the similar motivation there are certain relations between signatures of semantics and translations between semantics, which we will discuss below together with the points that distinguish these approaches. Given the signatures of semantics one can compare them, which also results in an expressiveness hierarchy of semantics. However, as signatures require that exactly the given extensions are realized in an AF, they are more restrictive than translations where we allow to exclude extensions and arguments. If we consider translations that do not allow exclusion of extensions or arguments we end up with exact translations which are closely related to signatures in the following sense. If for two semantics $\sigma$ and $\sigma'$ there are exact translations $\sigma \Rightarrow \sigma'$ and $\sigma' \Rightarrow \sigma$ then these two semantics also have the same signature [16]. Also when each set of extensions that are possible for a semantics $\sigma$ can also be realized by a semantics $\sigma'$ then there is an exact translation. However, this translation might not be efficient or satisfy any of the desired properties. Summarized one can say that, when it comes to expressiveness, the strength of the approach with signatures lies in the additional structural information about the sets of extensions they provide, while the strength of the approach with translations is that one can study the expressiveness of semantics in a more flexible manner that make different semantics better comparable.

Dyrkolbotn [21] studies signatures for preferred and semi-stable labellings. But in contrast to [16] when introducing additional arguments only the projection of the labellings to the original arguments must coincide with the labellings, while new arguments can have arbitrary labels. This idea is somehow similar to our notion of faithful translations, although stated for labellings and in terms of signatures.

The work of Strass [27] compares the expressiveness of several formalisms, i.e., abstract argumentation, normal logic programs, abstract dialectical frameworks (ADFs) and the satisfiability problem of propositional logic and gives an expressiveness hierarchy of them. In terms of abstract argumentation however his analysis is limited to stable semantics.

## 6.2 Future research directions

Several open research questions arise when it comes to the intertranslatability of argumentation semantics.

The whole work on intertranslatability considers extension-based semantics, but often one is more interested in semantics that are characterized by argument labellings (see, e.g., [1, 9]). So one issue is to find a meaningful notion of translations in the setting of (3-valued) labelling based semantics and to study how the results of the extension-based setting change when it comes to the equivalent labelling based semantics.

One limitation of existing work is that only finite AFs are considered. While certain results and translations will immediately extend when one also allows infinite AFs others do not. For example, when it comes to semi-stable and stage semantics there are infinite AFs which do not provide any extension [4, 10], which flaws several of the arguments used for finite AFs. In particular these cases open a direction for further research.

While this paper covers the most prominent semantics in abstract argumentation there are several other semantics around. An obvious approach for future research would be to extend the existing studies to other semantics of interest. In general, translations should be considered as an additional tool when introducing new semantics or analyzing existing ones. Translations provide a way of implementing a semantics using existing systems for other semantics, a comparison of the expressiveness, and also a way to obtain complexity bounds.

Unique status semantics, like grounded, play a special role in the studies of intertranslatability. As they always provide exactly one extension no multiple status semantics can be translated to them. So it would be of interest to study the relation between established unique status semantics like grounded, ideal [14] and eager [6]. Another perspective of these three semantics is not to consider them as semantics in their own right but as special reasoning modes of complete, preferred and semi-stable semantics [17]. Given this perspective it would be interesting to study notions of intertranslatability that also faithfully maintain these kinds of reasoning.

# References


[1] Pietro Baroni, Martin Caminada, and Massimiliano Giacomin. An introduction to argumentation semantics. *Knowledge Eng. Review*, 26(4):365–410, 2011.

[2] Pietro Baroni, Federico Cerutti, Massimiliano Giacomin, and Giovanni Guida. AFRA: Argumentation framework with recursive attacks. *Int. J. Approx. Reasoning*, 52(1):19–37, 2011.

[3] Pietro Baroni and Massimiliano Giacomin. On principle-based evaluation of extension-based argumentation semantics. *Artif. Intell.*, 171(10-15):675–700, 2007.

[4] Ringo Baumann and Christof Spanring. Infinite argumentation frameworks - on the existence and uniqueness of extensions. In Thomas Eiter, Hannes Strass, Miroslaw Truszczynski, and Stefan Woltran, editors, *Advances in Knowledge Representation, Logic Programming, and Abstract Argumentation - Essays Dedicated to Gerhard Brewka on the Occasion of His 60th Birthday*, volume 9060 of *Lecture Notes in Computer Science*, pages 281–295. Springer, 2015.

[5] Gerhard Brewka, Paul E. Dunne, and Stefan Woltran. Relating the semantics of abstract dialectical frameworks and standard AFs. In *Proceedings of the 22nd International Joint Conference on Artificial Intelligence (IJCAI 2011)*, pages 780–785. AAAI Press, 2011.

[6] Martin Caminada. Comparing two unique extension semantics for formal argumentation: ideal and eager. In *Proceedings of the 19th Belgian-Dutch Conference on Artificial Intelligence (BNAIC 2007)*, pages 81–87, 2007.

[7] Martin Caminada and Leila Amgoud. On the evaluation of argumentation formalisms. *Artif. Intell.*, 171(5-6):286–310, 2007.

[8] Martin Caminada, Walter A. Carnielli, and Paul E. Dunne. Semi-stable semantics. *Journal of Logic and Computation*, 2011.

[9] Martin Caminada and Dov M. Gabbay. A logical account of formal argumentation. *Studia Logica*, 93(2):109–145, 2009.

[10] Martin Caminada and Bart Verheij. On the existence of semi-stable extensions. In *Proc. BNAIC*, 2010.

[11] Claudette Cayrol and Marie-Christine Lagasquie-Schiex. Bipolar abstract argumentation systems. In I. Rahwan and G. Simari, editors, *Argumentation in Artificial Intelligence*, pages 65–84. Springer, 2009.

[12] Marc Denecker, Wiktor Marek, and Miroslaw Truszczyński. Uniform semantic treatment of default and autoepistemic logics. *Artif. Intell.*, 143(1):79–122, 2003.

[13] Phan Minh Dung. On the acceptability of arguments and its fundamental role in nonmonotonic reasoning, logic programming and n-person games. *Artif. Intell.*, 77(2):321–358, 1995.

[14] Phan Minh Dung, Paolo Mancarella, and Francesca Toni. Computing ideal sceptical argumentation. *Artif. Intell.*, 171(10-15):642–674, 2007.

[15] Paul E. Dunne, Wolfgang Dvořák, Thomas Linsbichler, and Stefan Woltran. Characteristics of multiple viewpoints in abstract argumentation. In *Proc. DKB*, pages 16–30, 2013. Available under `http://www.dbai.tuwien.ac.at/staff/linsbich/pubs/dkb_2013.pdf`.

[16] Paul E. Dunne, Wolfgang Dvořák, Thomas Linsbichler, and Stefan Woltran. Characteristics of multiple viewpoints in abstract argumentation. In *KR*, pages 72–81, 2014.

[17] Paul E. Dunne, Wolfgang Dvořák, and Stefan Woltran. Parametric properties of ideal semantics. *Artificial Intelligence*, 202(0):1 – 28, 2013.

[18] Paul E. Dunne and Michael Wooldridge. Complexity of abstract argumentation. In Guillermo Simari and Iyad Rahwan, editors, *Argumentation in Artificial Intelligence*, pages 85–104. Springer US, 2009.

[19] Wolfgang Dvořák and Christof Spanring. Comparing the expressiveness of argumentation semantics. In Bart Verheij, Stefan Szeider, and Stefan Woltran, editors, *Computational Models of Argument - Proceedings of COMMA 2012, Vienna, Austria, September 10-12, 2012*, volume 245 of *Frontiers in Artificial Intelligence and Applications*, pages 261–272. IOS Press, 2012.

[20] Wolfgang Dvořák and Stefan Woltran. On the intertranslatability of argumentation semantics. *J. Artif. Intell. Res. (JAIR)*, 41:445–475, 2011.

[21] Sjur Kristoffer Dyrkolbotn. How to argue for anything: Enforcing arbitrary sets of labellings using afs. In *KR*, pages 626–629, 2014.

[22] Georg Gottlob. Translating default logic into standard autoepistemic logic. *J. ACM*, 42(4):711–740, 1995.

[23] Tomi Janhunen. On the intertranslatability of non-monotonic logics. *Ann. Math. Artif. Intell.*, 27(1-4):79–128, 1999.

[24] K. Konolige. On the relation between default and autoepistemic logic. *Artif. Intell.*, 35(3):343–382, 1988.

[25] W. Marek and M. Truszczyński. *Nonmonotonic Logic: Context Dependent Reasoning*. Springer, 1993.

[26] Sanjay Modgil and Trevor J. M. Bench-Capon. Metalevel argumentation. *J. Log. Comput.*, 21(6):959–1003, 2011.

[27] Hannes Strass. On the relative expressiveness of argumentation frameworks, normal logic programs and abstract dialectical frameworks. In *Proceedings of the 15th International Workshop on Non-Monotonic Reasoning (NMR 2014)*, 2014.

[28] Bart Verheij. Two approaches to dialectical argumentation: admissible sets and argumentation stages. In J. Meyer and L. van der Gaag, editors, *Proceedings of the 8th Dutch Conference on Artificial Intelligence (NAIC'96)*, pages 357–368, 1996.